A theoretical note on converted wave amplitude variation with offset in viscoelastic media

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SUMMARY

Approximate converted PS- reflection coefficients for weak contrast interfaces separating isotropic viscoelastic media have been derived in this paper. This problem has been investigated by others but the changes in the attenuation angles across the boundary have been neglected. In this paper, we remove this limitation by deriving a formula for the PS-wave reflection coefficient considering the Snell's law which relates the transmitted attenuation angle for S waves in terms of incident attenuation angle. The AVO equation that we obtained is a complex function which for small angles has four terms from zeroth order up to the third order in $\sin\theta_{\rm P}$. Viscoelastic converted PS-wave AVO comparing to the elastic converted PS-wave has two more term which are zeroth and second order in $\sin\theta_{\rm P}$. These extra terms come from the inhomogeneity of the waves. We also compare the approximate linearized solutions with the exact solutions of Zoeppritz equations for three rock contrast model.

INTRODUCTION

The aim of reflection seismology is to estimate the rock property from the reflected seismic waves from subsurface interfaces (Castagna and Backus, 1994). In fact, it is very difficult to extract the sensitivity of the reflected waves to the changes in medium properties. This obstacle can be removed by linearization of the reflection coefficients respect to the changes in physical properties of the earth. This linearization procedure is based upon two assumptions, the properties between two layers change slightly and the incident angle significantly be smaller that the critical angle. In the case of isotropic elastic medium weak contrast reflection coefficients linearly depend to the fractional changes in density, P-wave and S-wave velocities weighted by trigonometric functions of incident angle. In the case that the medium is viscoelastic the reflection coefficients get more complicated, and due to the complexity of ray parameter and polarizations the coefficients are complex (Ursin and Stovas, 2002; Moradi and Innanen, 2016; Krebes, 1984, 1983). In this case linearized AVO equations not only depend on the changes in elastic properties across the boundary but also depend on the changes in P- and S-wave quality factors weighted by trigonometric functions of incident phase and attenuation angles. One feature of our approach in deriving the linearized AVO equation is that we apply Snell's law and its linearized form in the linearization. This is typically ignored and it is assumed that attenuation angle not to change across the boundary (Behura and Tsvankin, 2009a,b).

AVO analysis of converted PS-wave is extensively used in seismic exploration and reservoir characterization (Ata et al., 1994; Stewart et al., 2003, 2002). In this paper we show that in contrast to PP-wave AVO analysis, viscoelastic amplitude variation with offset for PS-wave is sensitive only to the fractional changes in density, S-wave velocity and S-wave quality factor. The resulted AVO equation is a complex function with four term proportional to the orders in $\sin\theta_{\rm P}$ from zero to third. The significance of our result is that it is possible to estimate the quality factor and the attenuation angle dependency of reservoir rocks. This is expected to be of practical important in the characterization of viscosity in unconventional reservoirs.

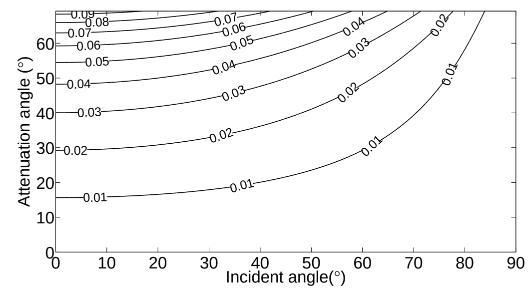

Figure 1: Diagram illustrating the imaginary part of complex ray parameter scales by 10^3 as a function of phase and attenuation angles for $V_{\rm P} = 2{\rm Km}/s$ and $Q_{\rm P} = 8$.

VISCOELASTIC WAVES

In viscoelastic media there are three types of waves: P, Type-I S, and Type-II S. In the case that propagation and attenuation vectors are not in the same direction wave is called inhomogeneous otherwise called homogeneous (Borcherdt, 2009). Polarization vectors for inhomogeneous P and SI-waves is elliptical and for homogeneous is linear. This elliptical motion reduces to a linear motion in the limit of homogenous case. SI-wave is the generalization of the elastic SV waves. Similarly, SII-wave in the case that attenuation goes to zero reduces to the elastic SH-wave. SII-wave has a linear motion perpendicular to the propagation attenuation plane for both homogeneous and inhomogeneous cases. In the case that the incident wave is an inhomogeneous P-wave, the reflected wave can be the inhomogeneous P- or SI-wave. For an inhomogeneous wave, ray parameter and slowness vector not only depend to the phase angle but also depend to the attenuation angle. For example ray parameter is given by

$$\mathrm{p} = \frac{1}{V_{\rm E}}\left[\sin\theta\left(1 - i\frac{Q^{-1}}{2}\right) + \frac{i}{2}Q^{-1}\cos\theta\tan\delta\right], \qquad (1)$$

where, $V_{\rm E}$ is either P-wave or S-wave velocity, θ is the phase angle and δ is the attenuating angle between propagation and attenuation vectors. In Figure 1 we plot the imaginary part of the inhomogeneous ray parameter versus phase and attenuation angles.

DECOMPOSED LOW CONTRAST PS-REFLECTION COEFFICIENT

By solving the Zoeppritz equation for two half spaces involving low-loss viscoelastic media we can obtain the exact PP- and PS-reflection coefficients (Moradi and Innanen, 2015, 2016). To linearize the reflectivity in terms of changes in elastic and anelastic properties we consider to the incident angle smaller that 30° and weak contrast, also in the case of converted P-wave, using the Snell's law, average in S-wave attenuation angle for small angle of incident is written as a function of incident phase and attenuation angles (Appendix A). In this case the weak contrast converted PS-wave reflectivity is given by

$$\mathrm{R}_{\mathrm{PS}}^{\mathrm{VE}} = i\mathrm{A}_{\mathrm{PS}}^{\mathrm{AIH}} + (\mathrm{A}_{\mathrm{PS}}^{\mathrm{E}} + i\mathrm{A}_{\mathrm{PS}}^{\mathrm{AH}})\sin\theta_P + i\mathrm{B}_{\mathrm{PS}}^{\mathrm{AIH}}\sin^2\theta_P + (\mathrm{B}_{\mathrm{PS}}^{\mathrm{E}} + i\mathrm{B}_{\mathrm{PS}}^{\mathrm{AH}})\sin^3\theta_{\mathrm{P}}. \quad (2)$$

Here, superscripts VE, E, AH and AIH respectively refer to the viscoelastic, elastic, anelastic-homogeneous and anelastic-inhomogeneous terms, θ_{P} is the angle of incident inhomogeneous P-wave and δ_{P} is the incident attenuation angle. Additionally coefficients in (2) are

$$A_{\mathrm{PS}}^{\mathrm{E}} = -\left(\frac{1}{2} + \frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)\frac{\Delta\rho}{\rho} - 2\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}},$$

$$B_{\mathrm{PS}}^{\mathrm{E}} = \frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\left[\left(\frac{1}{2} + \frac{3}{4}\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)\frac{\Delta\rho}{\rho} + 2\left[\frac{1}{2} + \frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right]\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right],$$

$$A_{\mathrm{PS}}^{\mathrm{AH}} = \frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\left\{Q_S^{-1}\frac{\Delta Q_S}{Q_S} - \frac{1}{2}(Q_S^{-1} - Q_P^{-1})\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right)\right\},$$

$$B_{\mathrm{PS}}^{\mathrm{AH}} = -\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\left[\frac{1}{2} + \frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right]Q_S^{-1}\frac{\Delta Q_S}{Q_S} - \frac{1}{4}\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2(Q_S^{-1} - Q_P^{-1})\frac{\Delta\rho}{\rho} + \frac{1}{4}\frac{V_S}{V_P}\left(1 + 4\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)(Q_S^{-1} - Q_P^{-1})\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right),$$

$$A_{\mathrm{PS}}^{\mathrm{AIH}} = -\frac{1}{2}\frac{V_S}{V_P}\left[\left(1 + \frac{1}{2}\frac{V_P}{V_S}\right)\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right]Q_P^{-1}\tan\delta_P,$$

$$B_{\mathrm{PS}}^{\mathrm{AIH}} = \frac{1}{8}\left[1 - 3\left(\frac{V_S}{V_P}\right)^2\right]\frac{\Delta\rho}{\rho}Q_P^{-1}\tan\delta_P + \frac{V_S}{V_P}\left(1 + \frac{3}{2}\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right)Q_P^{-1}\tan\delta_P,$$

comparing to the PP-wave reflection coefficient, we observe that the converted PS-wave is not sensitive to the changes in P-wave velocity and P-wave quality factor. We can also decompose the PS-reflectivity into elastic, homogenous and inhomogeneous terms as follows

$$\mathrm{R}_{\mathrm{PS}}(\theta_{\mathrm{P}},\delta_{\mathrm{P}}) = \mathrm{R}_{\mathrm{PS}}^{\mathrm{E}}(\theta_{\mathrm{P}}) + i\mathrm{R}_{\mathrm{PS}}^{\mathrm{AH}}(\theta_{\mathrm{P}}) + i\mathrm{R}_{\mathrm{PS}}^{\mathrm{AIH}}(\theta_{\mathrm{P}},\delta_{\mathrm{P}}), \quad (3)$$

with following components

$$\mathrm{R}_{\mathrm{PS}}^{\mathrm{E}}(\theta_{\mathrm{P}}) = A_{\mathrm{PS}}^{\mathrm{E}}\sin\theta_{\mathrm{P}} + B_{\mathrm{PS}}^{\mathrm{E}}\sin^3\theta_{\mathrm{P}},$$
$$\mathrm{R}_{\mathrm{PS}}^{\mathrm{AH}}(\theta_{\mathrm{P}}) = A_{\mathrm{PS}}^{\mathrm{AH}}\sin\theta_{\mathrm{P}} + B_{\mathrm{PS}}^{\mathrm{AH}}\sin^3\theta_{\mathrm{P}},$$
$$\mathrm{R}_{\mathrm{PS}}^{\mathrm{AIH}}(\theta_{\mathrm{P}},\delta_{\mathrm{P}},\delta_{\mathrm{S}}) = A_{\mathrm{PS}}^{\mathrm{AIH}} + B_{\mathrm{PS}}^{\mathrm{AIH}}\sin^2\theta_P.$$

In above equations, $\Delta\rho/\rho$ is fractional change in density, with $\Delta\rho = \rho_2 - \rho_1$ and $\rho = (\rho_2 + \rho_1)/2$; $\Delta V_{\mathrm{S}}/V_{\mathrm{S}}$ is fractional change in S-wave velocity, with $\Delta V_{\mathrm{S}} = V_{\mathrm{S2}} - V_{\mathrm{S1}}$ and $V_{\mathrm{S}} = (V_{\mathrm{S2}} +$

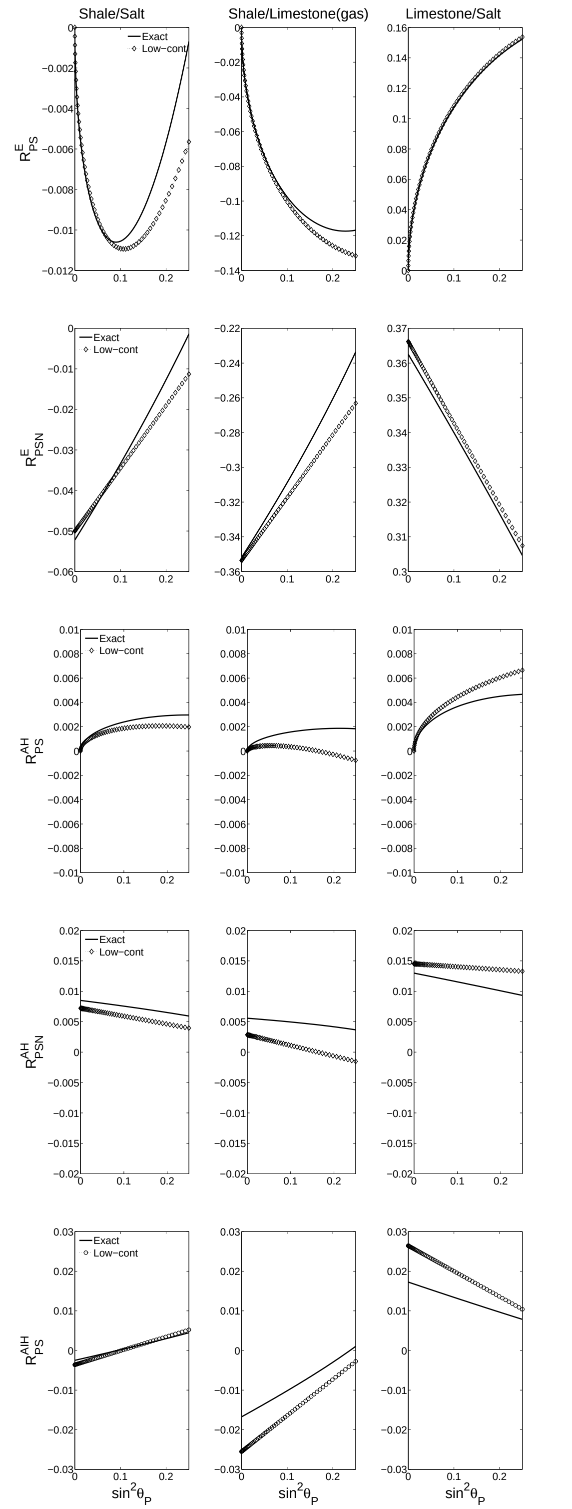

Figure 2: Comparison of the exact solutions of viscoelastic Zoeppritz's equation(solid line) and a linearized(circle-dot line) for three two-half-space models introduced in table 1. In all cases we assume that $\delta_P = 60°$.

Rock Type	Material	$V_P(m/s)$	$V_S(m/s)$	$\rho(g/cm^3)$
Shale/Salt	Shale	3811	2263	2.40
	Salt	4573	2729	2.05
Shale/Limestone(gas)	Shale	3811	2263	2.40
	Limestone	5043	2957	2.49
Limestone/Salt	Limestone	5335	2957	2.65
	Salt	4573	2729	2.05

Table 1: Elastic parameters for three half-space models: shale/salt, gas shale/limestone, and limestone/salt (Jerez, 2003; Ikelle and Amundsen, 2005). For all three models $\Delta Q_P = \Delta Q_S = 2$.

$V_{S1})/2$; $\Delta Q_S/Q_S$ is fractional change in S-wave quality factor, with $\Delta Q_S = Q_{S2} - Q_{S1}$ and $Q_S = (Q_{S2} + Q_{S1})/2$. In addition $\theta_P = (\theta_{P2} + \theta_{P1})/2$ where θ_{P1} is the incident phase angle and θ_{P2} is the transmitted phase angle; $\delta_P = (\delta_{P2} + \delta_{P1})/2$ where δ_{P1} is the incident attenuation angle and δ_{P2} is the transmitted attenuation angle. Subscript 1 refers to the upper layer and subscript 2 refers to the lower layer.

Elastic term is sensitive to changes in density and S-wave velocity. Anelastic-homogeneous term is sensitive to the changes in density, S-wave velocity and its quality factor. These two terms are zero in the case of normal incident. The anelastic inhomogeneous term is affected by only changes in density and S-wave velocity, this term also depends on the incident attenuation angle and is non zero at the normal incidence case. In Figure 2, we plot the exact versus linearized elastic, anelastic homogeneous and anelastic inhomogeneous terms for three two-half space models in table 1. We can see that elastic and homogenous terms are not linear in terms of $\sin^2\theta_P$, however the inhomogeneous terms is linear. We also defined the normalized elastic and homogeneous reflectivities as

$$\mathrm{R}^{\mathrm{E}}_{\mathrm{PSN}}(\theta_{\mathrm{P}}) = \frac{\mathrm{R}^{\mathrm{E}}_{\mathrm{PS}}(\theta_{\mathrm{P}})}{\sin\theta_{\mathrm{P}}}, \qquad \mathrm{R}^{\mathrm{AH}}_{\mathrm{PSN}}(\theta_{\mathrm{P}}) = \frac{\mathrm{R}^{\mathrm{AH}}_{\mathrm{PS}}(\theta_{\mathrm{P}})}{\sin\theta_{\mathrm{P}}}, \qquad (4)$$

It can be seen from the Figure 2 that the exact and approximate normalized elastic and homogenous reflection coefficients are linear in terms of $\sin^2\theta_P$. These normalized function are essential for the nonlinear PP-PS joint inversion.

CONCLUSIONS

In contrast to the elastic case, the linearization in an attenuative media is more complicated in two ways. First, due to the seismic amplitude damping, the polarization and slowness vectors are complex. As a result the reflectivity is complex. Second, besides the incident phase angle the attenuation angle across the boundary is changed, this fact is demonstrated by the Snell's law. Taking into account these facts, the linearized AVO equations include the terms related to the changes in S-wave quality factors and the attenuation angle. In this research, the variation of the converted seismic PS-wave with offset is obtained for an isotropic low-loss viscoelastic media. We show that complex amplitude variation with angle of incident is sensitive to the changes in elastic and anelastic rock properties. The real part of the PS-wave AVO equation is a linear function of fractional changes in density and S-wave velocity. The imaginary part which is due to the anelasticity in medium is sensitive to changes in density, S-wave velocity and S-wave quality factor.

In terms of powers of $\sin\theta_P$, converted PS-wave has four terms from zeroth to third order. Comparing to the elastic case, the extra terms are due to the inhomogeneity of the waves. We examine our AVO equation with three two-half space models. As a result the elastic and homogeneous terms are not linear respect to $\sin^2\theta_P$, however the inhomogeneous term for small angles ($\theta_P < 30^\circ$) is perfectly linear for both exact and approximate cases.

APPENDIX A

LINEARIZED SNELL'S LAW FOR VISCOELASTIC WAVES

Snell's law relates the reflected and transmitted phase and attenuation angles to the incident phase and attenuation angles (Moradi and Innanen, 2016)

$$\tan\delta_S = \left(\frac{V_S}{V_P}\right)\frac{\sin\theta_P - \frac{Q_S}{Q_P}\left[\sin\theta_P - \cos\theta_P\tan\delta_P\right]}{\sqrt{1-\left(\frac{V_S}{V_P}\right)^2\sin^2\theta_P}}. \qquad \text{(A-1)}$$

In the case of converted P-wave, using the Snell's law, average in S-wave attenuation angle for small angle of incident can be written as a function of incident phase and attenuation angles

$$\begin{aligned} Q_S^{-1}\tan\delta_S = {} & \frac{V_S}{V_P}Q_P^{-1}\tan\delta_P \\ & + \frac{V_S}{V_P}(Q_S^{-1} - Q_P^{-1})\sin\theta_P \\ & - \frac{1}{2}\frac{V_S}{V_P}\left[1-\left(\frac{V_S}{V_P}\right)^2\right]Q_P^{-1}\tan\delta_P\sin^2\theta_P \\ & + \frac{1}{2}\left(\frac{V_S}{V_P}\right)^3(Q_S^{-1} - Q_P^{-1})\sin^3\theta_P \qquad \text{(A-2)} \end{aligned}$$

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