Weak contrast inhomogeneous PP-reflection coefficient in low-loss attenuative media

Shahpoor Moradi and Kristopher A. Innanen
Department of Geoscience, University of Calgary, Calgary, Canada.

SUMMARY

A useful four term Amplitude Variation with Offset (AVO) equation for inhomogeneous PP-wave reflection coefficient for a weak contrast separating two isotropic low-loss viscoelastic media has been derived. This coefficient is complex and linearly depends on the fractional changes in elastic properties as well as sensitive to P- and S-wave quality factors. Comparing to the elastic PP-reflectivity, which for small angles has contributions at zeroth and second order in $\sin\theta_{\rm P}$, we show that the viscoelastic reflectivity is function of $\sin\theta_{\rm P}$ from zeroth up to third order, where the first and third orders are related to the case that waves are inhomogeneous. In a second decomposition the reflectivity can be split up into elastic, anelastic-homogeneous and anelastic-inhomogeneous terms. For incident angles sufficiently smaller than the critical angles, we compare our four term AVO equations to the exact reflectivity as a solution of Zoeppritz equations.

INTRODUCTION

Amplitude variation with offset equations are utilized to determine fluids and fracture properties of subearth hydrocarbon reservoirs (Castagna and Backus, 1994). The linearized reflection/transmission coefficients for a weak-contrast interface between two isotropic media have been in use for a long time (Aki and Richards, 2002). The problem of exact and approximate reflection and transmission coefficients at a plane interface between two viscoelastic media for homogenous waves was studied by Ursin and Stovas (2002). The authors concluded that the approximate PP and PS reflectivities are very similar to the exact solutions of Zoeppritz equations. Furthermore they generalized the problem to transversely isotropic viscoelastic case (Stovas and Ursin, 2003). The effects of attenuation on PP- and PS-wave reflection coefficients for anisotropic viscoelastic media with the main emphasis on transversely isotropic models with a vertical symmetry axis is studied by Behura and Tsvankin (2009a). They considered inhomogeneous waves, and assumed the attenuation angles across the boundary does not change. However, the inhomogeneity angle can make a substantial contribution to the AVO response for strongly attenuative media (Behura and Tsvankin, 2009b). Formulas of the kind we will derive can be used to drive anelastic inversion procedures, both linear and nonlinear (Innanen, 2011); possibly, via examination of the frequency rate of change of reflection coefficients (Innanen, 2012). Techniques of this kind become increasingly relevant as evidence accrues that anelastic amplitude signatures provide direct information about reservoir fluids (Ostrander, 1984; Chapman et al., 2006; Odebeatu et al., 2006; Schmalholz and Podladchikov, 2009; Ren et al., 2009; Wu et al., 2014).

Scattering of seismic waves from viscoelastic inclusions in the context of Born approximation recently is investigated. This mathematical framework for scattering which is based on the Bordcherdt's formalism of the viscoelastic wave propagation has been developed for the purposes of modeling, processing, and inversion of seismic data exhibiting nonegligible intrinsic attenuation (Moradi and Innanen, 2015b). Along these lines, linearized forms of PP, PS, and SS reflection coefficients for low-contrast interfaces separating two arbitrary low-loss viscoelastic media for arbitrary incident angle were derived by Moradi and Innanen (2016, 2015a). These AVO responses relate the AVO response to the anelastic parameters. It is shown that the reflectivity not only depends upon the perturbations in elastic properties but also on perturbations in quality factors for P- and S-waves.

In the presence of anelasticity, AVO equations need to be modified. In this case the AVO equations include the changes in elastic and anelastic parameters. In this brief note we consider the problem of reflection of an inhomogeneous P-wave in a viscoelastic layered media, considering Snell's law. Linearization of Snell's law in viscoelastic medium accounts for changes in phase and attenuation angles in terms of velocities and quality factors. Comparing to our previous results (Moradi and Innanen, 2016), we obtain the four term AVO equations for angles smaller than critical. We also decompose the complex reflectivity into three parts, elastic, homogenous and inhomogeneous parts, which is useful in linear and nonlinear inversion of multicomponent seismic data (Margrave et al., 2001; Lehocki et al., 2014; Jerez, 2003).

COMPLEX RAY PARAMETER AND SNELL'S LAW

Linearized AVO analysis requires the definition of polarization and slowness vectors. In a viscoelastic medium, the wavenumber vector is a complex vector whose real part characterizes the direction of wave propagation and imaginary part characterizes the attenuation of the wave. Borcherdt (2009) has presented a complete theory for seismic waves propagating in layered anelastic media, assuming a viscoelastic model to hold. Borcherdt's formulation is particularly powerful in that it predicts a range of transverse, inhomogeneous wave types unique to viscoelastic media (Type I and II S waves), and develops rules for conversion of one type to another during interactions with planar boundaries. As a result complexity of the wavenumber vector, slowness and polarization vectors are complex functions. The complex wave-number vector is given by

$$\mathbf{K} = \mathbf{P} - i\mathbf{A}, \tag{1}$$

where, propagation vector $\mathbf{P}$ is perpendicular to the wavefront and attenuation vector $\mathbf{A}$ is perpendicular to the plane of constant amplitudes and specified the direction of the maximum attenuation medium. The angle between these two vectors is called attenuation angle, δ, which is always less that 90°. In the case that the attenuation and propagation vectors are parallel the wave is called homogenous. Otherwise it is inhomogeneous. In the case of low-loss viscoelastic media as $Q^{-1} \ll 1$, we have the propagation and attenuation vectors as

$$\begin{aligned} \mathbf{P} &= \frac{\omega}{V_{\mathrm{E}}}(\mathbf{x}\sin\theta + \mathbf{z}\cos\theta), \\ \mathbf{A} &= \frac{\omega}{V_{\mathrm{E}}}Q^{-1}\sec\delta(\mathbf{x}\sin(\theta-\delta) + \mathbf{z}\cos(\theta-\delta)). \end{aligned} \tag{2}$$

Where, V_{E} is either P-wave or S-wave velocity, θ is the phase angle and δ is the attenuating angle between propagation and attenuation vectors. For inhomogeneous P-wave, ray parameter and vertical slowness are complex function depending on the quality factor and attenuation angles (Moradi and Innanen, 2016)

$$\begin{aligned} p &= \frac{1}{\omega}(\mathrm{PE}_x - i\mathrm{A}_x) = \\ &\frac{1}{V_{\mathrm{P}}}\left[\sin\theta_{\mathrm{P}}\left(1 - i\frac{Q_{\mathrm{P}}^{-1}}{2}\right) + \frac{i}{2}Q_{\mathrm{P}}^{-1}\cos\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\right], \\ q_{\mathrm{P}} &= \frac{1}{\omega}(\mathrm{P}_z - i\mathrm{A}_z) = \\ &\frac{1}{V_{\mathrm{PE}}}\left[\cos\theta_{\mathrm{P}}\left(1 - i\frac{Q_{\mathrm{P}}^{-1}}{2}\right) - \frac{i}{2}Q_{\mathrm{P}}^{-1}\sin\theta_{\mathrm{P}}\tan\delta_{\mathrm{P}}\right]. \end{aligned} \tag{3}$$

In Figure 1, we plot the ray parameter in complex plane versus phase angle for various values of attenuation angle. Diagram displays that the ray parameter is an ellipse whose eccentricity decline as attenuation angle gets smaller. The same interpretation is valid for polarization and slowness vectors.

Snell's law is used to determine the reflected and transmitted angles in terms of incident angle when an elastic waves hits the boundary of two layered media. The linearization of Snell's law is essential key of the linearization procedure when the difference between the incident and transmitted wave is expressed in terms of changes in velocity between the layers. Snell's law for viscoelastic materials is discussed by Wennerberg (1985) and Borcherdt (2009). In a viscoelastic media besides the phase angle for inhomogeneous waves the attenuation angle between propagation and reflected waves is changed. Study of Snell's law in attenuative media has important features; among them we can express the attenuation angle in terms of incident phase and attenuation angles (Moradi and Innanen, 2015a). Furthermore linearization of the anelastic part of Snell's law gives the changes in attenuation angle in terms of changes in velocity and corresponding quality factor which essential key part of the linerization of the reflection coefficients.

AVO EQUATIONS

There are two main assumptions in linear AVO analysis, firstly, the relative changes in properties either elastic or anelastic across the interface are small. Secondly the incident angle is smaller than the critical angle. An incident inhomogeneous wave at a boundary between two viscoelastic media can generate reflected inhomogeneous-P-waves and inhomogeneous-SI-waves as well as transmitted inhomogeneous P-waves and SI-

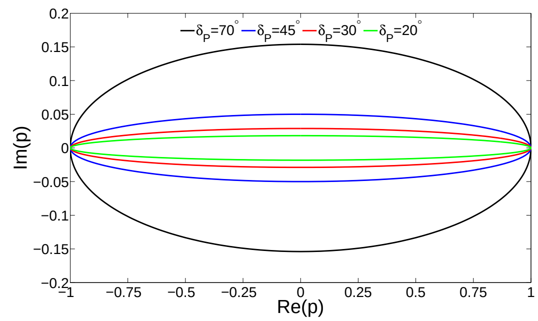

Figure 1: Digram illustrating the ray parameter versus phase angle in complex plane for different values of attenuation angle δ.

waves (Borcherdt, 2009). In general, the reflection and transmission coefficients vary with offset and attenuation angles. AVO signatures can be detected by three-dimensional seismic surveys and is a useful seismic attribute for reservoir characterization. In this section we present the small offset linearized P-to-P reflection coefficients for an inhomogeneous seismic waves reflected from boundary of two isotropic viscoelastic media under the assumption of small contrast interface. In contrast to elastic medium, in a viscoelastic medium the slowness vector and ray parameter are not real. As a result, reflectivity is complex-valued function

$$\begin{aligned} \mathrm{R}_{\mathrm{PP}}^{\mathrm{VE}} =& (\mathrm{A}_{\mathrm{PP}}^{\mathrm{E}} + i\mathrm{A}_{\mathrm{PP}}^{\mathrm{AH}}) + i\mathrm{A}_{\mathrm{PP}}^{\mathrm{AIH}}\sin\theta_{\mathrm{P}} \\ &+ (\mathrm{B}_{\mathrm{PP}}^{\mathrm{E}} + i\,\mathrm{B}_{\mathrm{PP}}^{\mathrm{AH}})\sin^2\theta_{\mathrm{P}} + i\,\mathrm{B}_{\mathrm{PP}}^{\mathrm{AIH}}\sin^3\theta_{\mathrm{P}}. \end{aligned} \tag{4}$$

Here, superscripts VE, E, AH and AIH respectively refer to the viscoelastic, elastic, anelastic-homogeneous and anelastic-inhomogeneous terms, θ_{P} is the angle of incident inhomogeneous P-wave and δ_{P} is the incident attenuation angle. It should be noted that the aforementioned angles can be considered either incident or average of incident and transmitted angles as long as we consider to the small angles. Additionally, the complex coefficients in Eq.(4) are given by

$$\begin{aligned} A_{\mathrm{PP}}^{\mathrm{E}} &= \frac{1}{2}\left[\frac{\Delta\rho}{\rho} + \frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}\right], \\ B_{\mathrm{PP}}^{\mathrm{E}} &= -2\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right] + \frac{1}{2}\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}, \\ A_{\mathrm{PP}}^{\mathrm{AH}} &= -\frac{1}{4}Q_{\mathrm{P}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}}, \\ B_{\mathrm{PP}}^{\mathrm{AH}} &= 2\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2\left[(Q_{\mathrm{P}}^{-1} - Q_{\mathrm{S}}^{-1})\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right) + Q_{\mathrm{S}}^{-1}\frac{\Delta Q_{\mathrm{S}}}{Q_{\mathrm{S}}}\right] \\ &\quad - \frac{1}{4}Q_{\mathrm{P}}^{-1}\frac{\Delta Q_{\mathrm{P}}}{Q_{\mathrm{P}}}, \\ A_{\mathrm{PP}}^{\mathrm{AIH}} &= Q_{\mathrm{P}}^{-1}\tan\delta_P\left\{-2\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right] + \frac{1}{2}\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}\right\}, \\ B_{\mathrm{PP}}^{\mathrm{AIH}} &= Q_{\mathrm{P}}^{-1}\tan\delta_P\left\{\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{P}}}\right)^2\left[\frac{\Delta\rho}{\rho} + 2\frac{\Delta V_{\mathrm{S}}}{V_{\mathrm{S}}}\right] + \frac{3}{4}\frac{\Delta V_{\mathrm{P}}}{V_{\mathrm{P}}}\right\}. \end{aligned}$$

In above equations, $\Delta\rho/\rho$ is fractional change in density, with $\Delta\rho = \rho_2 - \rho_1$ and $\rho = (\rho_2 + \rho_1)/2$; $\Delta V_{\mathrm{P}}/V_{\mathrm{P}}$ is fractional change in P-wave velocity, with $\Delta V_{\rm P} = V_{\rm P2} - V_{\rm P1}$ and $V_{\rm P} = (V_{\rm P2} + V_{\rm P1})/2$; $\Delta V_{\rm S}/V_{\rm S}$ is fractional change in S-wave velocity, with $\Delta V_{\rm S} = V_{\rm S2} - V_{\rm S1}$ and $V_{\rm S} = (V_{\rm S2} + V_{\rm S1})/2$; $\Delta Q_{\rm P}/Q_{\rm P}$ is fractional change in P-wave quality factor, with $\Delta Q_{\rm P} = Q_{\rm P2} - Q_{\rm P1}$ and $Q_{\rm P} = (Q_{\rm P2} + Q_{\rm P1})/2$; $\Delta Q_{\rm S}/Q_{\rm S}$ is fractional change in S-wave quality factor, with $\Delta Q_{\rm S} = Q_{\rm S2} - Q_{\rm S1}$ and $Q_{\rm S} = (Q_{\rm S2} + Q_{\rm S1})/2$. In addition $\delta_{\rm P} = (\delta_{\rm P2} + \delta_{\rm P1})/2$ where $\delta_{\rm P1}$ is the incident attenuation angle and $\delta_{\rm P2}$ is the transmitted attenuation angle. Subscript 1 refers to the upper layer and subscript 2 refers to the lower layer.

On the other hand the reflectivity can be written as

$$\mathrm{R}_{\rm PP}(\theta_{\rm P}, \delta_{\rm P}) = \mathrm{R}^{\rm E}_{\rm PP}(\theta_{\rm P}) + i\mathrm{R}^{\rm AH}_{\rm PP}(\theta_{\rm P}) + i\mathrm{R}^{\rm AIH}_{\rm PP}(\theta_{\rm P}, \delta_{\rm P}), \quad (5)$$

with elastic term

$$\mathrm{R}^{\rm E}_{\rm PP}(\theta_{\rm P}) = A^{\rm E}_{\rm PP} + B^{\rm E}_{\rm PP} \sin^2\theta_{\rm P}, \quad (6)$$

anelastic homogenous term

$$\mathrm{R}^{\rm AH}_{\rm PP}(\theta_{\rm P}) = A^{\rm AH}_{\rm PP} + B^{\rm AH}_{\rm PP} \sin^2\theta_{\rm P}, \quad (7)$$

anelastic inhomogeneous term

$$\mathrm{R}^{\rm AIH}_{\rm PP}(\theta_{\rm P}, \delta_{\rm P}) = A^{\rm AIH}_{\rm PP} \sin\theta_P + B^{\rm AIH}_{\rm PP} \sin^3\theta_P. \quad (8)$$

The coefficients $A^{\rm E}_{\rm PP}$ and $A^{\rm AH}_{\rm PP}$ are called intercepts or normal incident reflectivity and $B^{\rm E}_{\rm PP}$ and $B^{\rm AH}_{\rm PP}$ are called gradients. The parameter B describes the variation at intermediate offsets.

The elastic part of the reflectivity is sensitive to changes in density, P- and S-wave velocities and has a non zero value for waves at normal incidence. The anelastic-homogeneous term is sensitive to changes in density, S-wave velocity, P-wave quality factor and S-wave quality factor. At normal incidence this term is not zero. In this case, only a change in P-wave quality factor influences this term. The inhomogeneous term is nonzero in the case that wave is inhomogeneous. This term is sensitive to changes in density, P- and S-wave velocities and is zero at normal incidence. We show later that the inhomogeneous term is a function of incident angle similar to the elastic and anelastic converted P-wave. In Figure 2 we plot the exact versus linearized P-to-P reflectivity for three models of the two half-space rock types in table 1. For elastic and anelastic-homogeneous parts of reflectivity the linear behavior can be seen explicitly, but the anelastic-inhomogeneous term $\mathrm{R}^{\rm AIH}_{\rm PP}$ is not a linear function of $\sin^2\theta_{\rm P}$. However we can enforce linearity by normalization of this term as follows

$$\mathrm{R}^{\rm AIH}_{\rm PPN} = \frac{\mathrm{R}^{\rm AIH}_{\rm PP}}{\sin\theta_{\rm P}}. \quad \theta_{\rm P} \neq 0 \quad (9)$$

In Figure 2 we plot the approximate reflectivity versus the exact solutions for three terms introduced in Eq. (5). It can be seen that the normalized anelastic inhomogeneous term is perfectly linear with respect to $\sin^2\theta_{\rm P}$ for $0 \leq \theta_{\rm P} \leq 25°$. Using the decomposition defined in Eq. (5) and the plotted terms in Figure 2, we can obtain the zero offset coefficients A and gradient terms B from the exact reflectivity. Using these coefficients within a nonlinear inversion we can estimate the changes

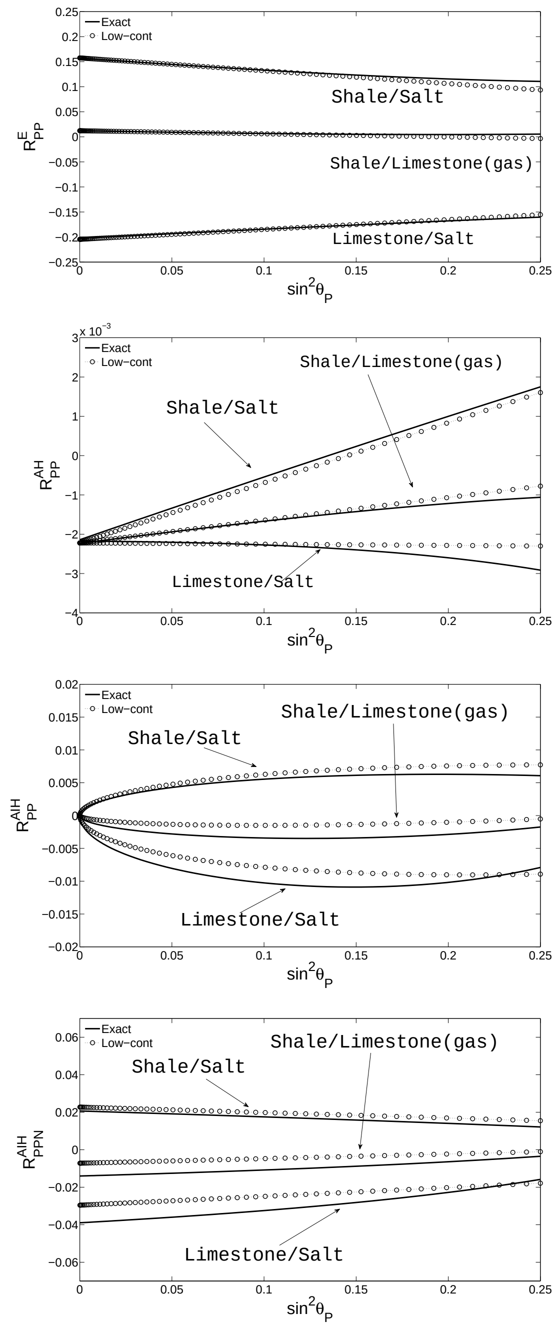

Figure 2: Comparison of the exact solutions of viscoelastic Zoeppritz's equation(solid line) and a linearized(circle-dot line) for three two-half-space models introduced in table 1. The first diagram is elastic part of reflectivity, the second diagram is anelastic-homogeneous term, third diagram is anelastic-inhomogeneous term and the fourth diagram is normalized anelastic-inhomogeneous term. In all diagrams we assumed that $\delta_{\rm P} = 60°$.

Rock Type	Material	$V_P(m/s)$	$V_S(m/s)$	$\rho(g/cm^3)$
Shale/Salt	Shale	3811	2263	2.40
	Salt	4573	2729	2.05
Shale/Limestone(gas)	Shale	3811	2263	2.40
	Limestone	5043	2957	2.49
Limestone/Salt	Limestone	5335	2957	2.65
	Salt	4573	2729	2.05

Table 1: Elastic parameters for three half-space models: shale/salt, gas shale/limestone, and limestone/salt (Jerez, 2003; Ikelle and Amundsen, 2005). For all three models $\Delta Q_P = \Delta Q_S = 2$

in density, P- and S-wave velocities and their quality factors, V_S/V_P ratio and attenuation angle. To display the effects of attenuation angles on AVO equations, the maps for inhomogeneous component of P-to-P reflectivity versus phase and attenuation angles are plotted in Figure 3.

CONCLUSIONS

Explicit forms of linearized AVO equations for a weak contrast separating two viscoelastic media are presented. To derive an approximate reflection coefficient which is linear in fractional changes in medium properties, we used a perturbation approach. The main assumption in this study is that the variations in elastic and anelastic properties across the boundary are small and the incident angle is adequatlly smaller than the critical angle. The approximate viscoelastic AVO equation has four terms from zeroth order up to third order of $\sin\theta_P$. It is shown that the AVO equation are complex function, the real part is the elastic linearized AKi-Richards equation including the terms proportional to changes in density and velocities. The complex part of the AVO equations is due to the changes in density, velocities and quality factors and depends to the incident attenuation angle. We show that the linearized and exact reflectivities trend quite well up to 30°. To exploit the non linear inversion we decompose the reflectivity into three terms, elastic, homogeneous and inhomogeneous terms. Linearity of the elastis and homogeneous parts are visible however the inhomogeneous part should be normalized to be linear respect to the incident angle. Our results for viscoelastic AVO are particularly relevant for the analysis of the sensitivity of seismic data to the inhomogeneity of the seismic waves and jumps in the quality factors. The result presented in this research indicate that linearized reflection coefficients for inhomogeneous PP-wave match for most inverse schemes the exact reflection coefficients with adequate accuracy. More important, the new approximations and the decomposition of reflectivity into three terms indicate that intercepts and gradients can be used to determine the quality factor and attenuation angle in an appropriate inversion strategy.

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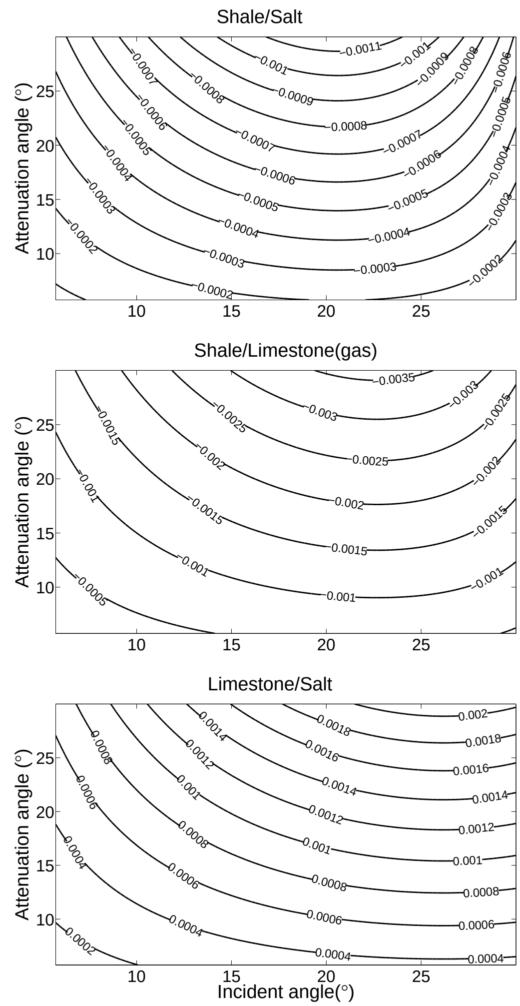

Figure 3: The maps of the inhomogeneous part of the PP-reflectivity for three models in table 1. The vertical axis indicates the attenuation angle and horizontal axis incident angle.

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