

# Effects of discrepancies between modeled and true physics in anacoustic FWI

Scott D. Keating and Kristopher A. Innanen University of Calgary, CREWES

## Summary

The most successful descriptions of attenuation are empirical, and may be inaccurate in certain environments. This can have severe implications for anacoustic FWI. A flexible anacoustic FWI approach is investigated here, where a strategy for coping with unknown attenuation physics is adopted. This approach is shown to provide results superior to those of a conventional anacoustic FWI approach in numerical examples on a simple model.

#### Introduction

Full waveform inversion (FWI) is a technique which seeks to recover the properties of the subsurface by minimizing the mismatch between measured seismic data and synthetic data. Crucially, it is assumed that the wave physics which give rise to the observed data are adequately accounted for in the modeling which generates the synthetic data. While accurately modeling all of the aspects of seismic wave propagation is an extremely demanding task, sufficient complexity in the modeling needs to be present to account for the major features of the measured data.

Where the mechanisms at play in the subsurface are well understood, generating synthetic data which display the same effects is an achievable goal. The alternate case, where the physics associated with the observed data are not as well understood, presents significant obstacles in FWI. Attenuative and dispersive effects may be more appropriately grouped into this second case. No single attenuative-dispersive model is held to be correct for the general case of seismic wave propagation (Ursin and Toverud (2002), Liu et al. (1976)), and this poses a difficult problem for the use of FWI on data where these effects play a significant role. If the attenuative-dispersive model assumed in the FWI is different from that which best describes the true behaviour of the earth, then the model which best matches the data will not necessarily be similar to the true subsurface, and could introduce significant errors.

This raises important questions about a possible anacoustic or anelastic FWI. Specifically, it is important to know whether attenuation compensation still takes place, and whether deviations from a background value in the assumed attenuation-dispersion variable occur at the same spatial location as the anomalies in the attenuative-dispersive behaviour of the true subsurface. The first of these questions is important insofar as an anacoustic or anelastic FWI is being pursued with the objective of improving estimates of acoustic or elastic parameters. If the improved recovery of these non attenuative parameters is the goal, then the second question is of little importance, provided that the effective dispersion and attenuation characteristics of the medium are accurately accounted for. The second question is important if an anacoustic or anelastic FWI is used in the hopes of recovering the locations of attenuation or dispersion changes in the subsurface, likely a significantly more difficult problem.

## **Theory**

Unlike many other aspects of seismic wave propagation, the physical causes of seismic attenuation are not well understood. Consequently, the most successful descriptions of seimic attenuation are empirically based.

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One observation which motivates many of the most successful descriptions is that the quality factor Q, which characterises attenuation, is often nearly independent of frequency. The resulting nearly constant Q model types are often useful for describing seismic attenuation, but are not physically motivated. Additionally, there are many observations of frequency dependent Q behaviour. These facts are problematic when considering including attenuation in FWI, where the assumption of known physics is critical. A significant possibility is that this assumption may fail with respect to attenuation, and negatively impact the inversion.

If the true physics at play in the subsurface are well described by a velocity and an attenuation, then any anacoustic model type which can specify these parameters independently can accurately reproduce the observed physics at a fixed frequency. Only at nonzero frequency bands will discrepancies between the true and assumed model type prevent the data from being reproduced. An intuitive solution might then be to overcome the problem of unknown physics by recovering anacoustic model parameters at each considered frequency independently, creating an effective medium which reproduces the true model parameters. This approach is problematic, as multiple frequencies are necessary to distinguish between the effects of velocity and attenuation model changes in the FWI problem (Keating and Innanen (2016)). An alternate strategy is pursued here: anacoustic model parameters are recovered on small frequency bands independently. If these bands are sufficiently small, the possibility of approximately matching unknown model physics is retained. Provided these bands are sufficiently large, cross talk between model parameters can be minimized. This strategy will be referred to as the flexible anacoustic FWI approach.

In the numerical examples that follow, two different anacoustic model types were used to demonstrate the method described: the Kolsky-Futterman (KF) nearly constant Q model and the standard linear solid (SLS) model.

The KF model (Kolsky (1956), Futterman (1962)) is based on the empirical observation that Q is often nearly constant in the seismic frequency range, and the requirement of causality. In this model, Q is treated as constant, and velocity is given by

$$c(\omega) = c(\omega_0) \left[ 1 + \frac{1}{\pi Q} \log \left( \frac{\omega}{\omega_0} \right) - \frac{i}{2Q} \right] ,$$

where  $c(\omega)$  is the wave velocity and  $\omega_0$  is a reference frequency.

The standard linear solid is a viscoelastic model with a constitutive relation linear in stress, strain and their derivatives (Casula and Carcione (1992), Liu et al. (1976)). In effect, it models a viscoelastic material as consisting of spring and dash-pot in series, in parallel with a second spring. The Q value given by this model is not constant or nearly so, but is instead given by

$$Q(\omega) = \frac{1 + \omega^2 \tau_{\epsilon} \tau_{\sigma}}{\omega (\tau_{\epsilon} - \tau_{\sigma})} \quad .$$

where  $\tau_{\epsilon}$  and  $\tau_{\sigma}$  are relaxation times related to the constants of the effective springs and dash-pot of the model (Casula and Carcione (1992), Liu et al. (1976)). This function is sharply peaked at  $\omega = \tau^{-1}$ , where  $\tau = \sqrt{\{\tau_{\epsilon}\tau_{\sigma}\}}$ . The real part of the phase velocity for this model is given by

$$c(\omega) = c(\omega_0) \left[ 1 + \frac{(\omega \tau)^2}{Q(1 + (\omega \tau)^2)} - \frac{i}{2Q(\omega)} \right] .$$

## **Examples**

Synthetic examples were generated using an anacoustic SLS model type. The true model used is shown in figure 1. The starting model in each example was constant, equal to the background of the true model. Figure 2 shows the result of a conventional FWI approach, with assumed KF physics. Clearly, the failure

of the assumption of known physics fails significantly here, and prevents a coherent model from being created, both for Q, and for velocity below the Q anomaly.

Figures 3 and 4 demonstrate the result of FWI using the flexible approach described above, once again assuming KF physics. While significant artifacts remain, the recovery of both model parameters is significantly improved.

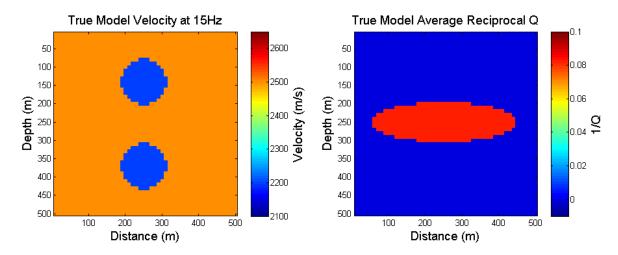


Figure 1: True SLS model. Peak Q at 15Hz.

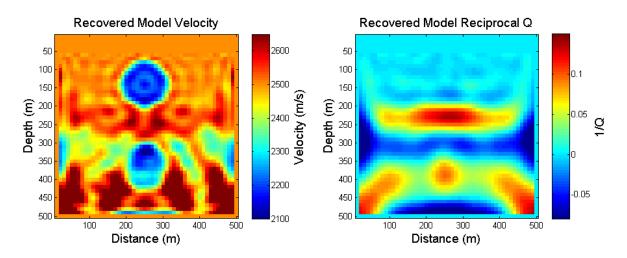


Figure 2: FWI result with KF model assumed.

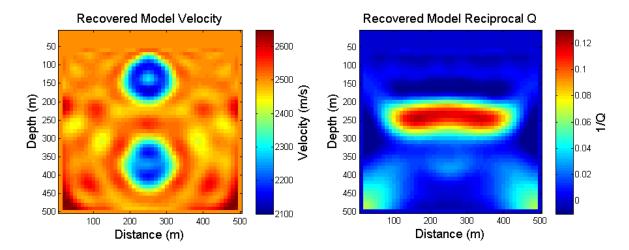


Figure 3: FWI result with KF model assumed, flexible approach at 14-16Hz band.

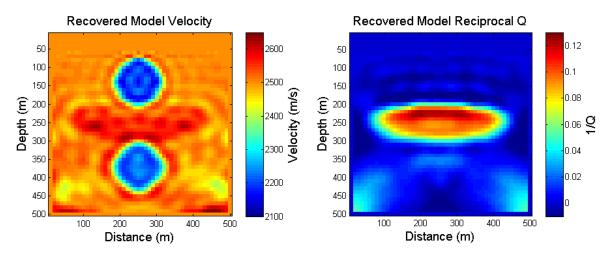


Figure 4: FWI result with KF model assumed, flexible approach at 24-26Hz band.

#### **Conclusions**

The physical mechanisms which give rise to seismic attenuation are not well understood and consequently the most successful descriptions of attenuation are empirical and not universally applicable. This can lead to the situation where the assumed attenuative and dispersive behaviour differ from their true behaviour, which can have severe consequences in FWI. In a flexible anacoustic FWI strategy this error is minimized by assuming consistent anacoustic behaviour only on small frequency bands. This approach was demonstrated on a simple model to provide significant advantages over a traditional FWI approach.

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