Characterizing and mitigating FWI modelling errors due to uncertainty in attenuation physics

Scott Keating and Kristopher A. Innanen, Department of Geoscience, University of Calgary

SUMMARY

A key assumption in seismic FWI is the adequacy of the wave propagation physics model used in simulation and sensitivity calculations. The wide variety of available seismic attenuation and dispersion models makes the risk of modelling errors in QFWI high. We examine the consequences of unknown attenuation physics for QFWI, and propose an alternate updating strategy to alleviate some of them. By relaxing the requirement that the frequency dependence of the assumed attenuation model be self-consistent across the full spectrum, significant improvement in the fidelity of models, inferred from data obeying one attenuation model using methods assuming another holds, is found.

INTRODUCTION

Seismic full waveform inversion (FWI) is a technique which attempts to recover subsurface properties by iteratively minimizing a measure of the discrepancy between observed data and modelled data (e.g., Lailly, 1983; Tarantola, 1984; Virieux and Operto, 2009). Multiparameter FWI (Operto et al., 2013; Plessix et al., 2013; Pan et al., 2016), by involving multiple physical properties, offers the potential to recover not only this larger list of properties but to better match observed data. Broad application of FWI technology in, for instance, reservoir characterization and monitoring, will require methods which are tuned to the multi-parameter problem. Significant challenges remain in bringing multiparameter FWI to the same levels of practicality and sophistication currently occupied by mono-parameter FWI. A particularly pressing issue is the stable inclusion of anelasticity/an-acousticity (e.g., Hicks and Pratt, 2001; Hak and Mulder, 2011; Malinowski et al., 2011; Kamei and Pratt, 2013; Métivier et al., 2015). In this paper we consider one of the more difficult aspects of an-acoustic FWI (hereafter QFWI), the problem of management of modelling error. In a companion paper the interrelation between V_P and Q_P cross-talk, frequency-band selection in multiscale FWI, and efficiency in truncated Newton optimization, are also discussed (Keating and Innanen, 2017).

A crucial assumption in FWI is that the wave physics giving rise to the observed data are adequately accounted for in the simulation component of the procedure. If the wave propagation equations miss, or incorrectly model, important features of the data, FWI will seek to match those data features through often dramatically un-physical spatial arrangements of the available model parameters. QFWI is especially prone to modelling errors, because (1) even small changes in the Q model-type can lead to large differences in, for instance, wave velocities at low frequencies, and (2) many model-types exist, and which is suitable in any given instance may not be clear.

Innanen (2016) points out that, ideally, uncertainty in attenuation physics would be managed by being maximally non-committal – framing FWI to solve for a complex, frequency-dependent velocity at each point in space; but, that is not possible because seismic data cannot in general constrain this many parameters. On the other hand, while more decisively parameterized models are much more completely constrained by data, choosing one a priori risks serious modelling errors.

In this paper, we formulate frequency-domain QFWI such that a "middle-ground" between the two above extremes is occupied. In other words, a parameterization in which seismic data are maximally non-committal regarding model-type within the bounds of what can be constrained by seismic data. The idea of relaxing the constraint that the assumed physics be exactly obeyed is investigated by allowing a band-wise frequency-dependence in the recovered model. This increased flexibility offers important benefits when the assumed physical model involves different frequency dependence of wave propagation from that of the true (or at least a different, more appropriate) physical model.

THEORY AND APPROACH

Constant density an-acoustic physics models can be characterized by two parameters: a Q term specifying attenuation, and a term specifying P-wave phase velocity, both of which can be functions of frequency. Many different physics models exist, differing in the frequency dependence of Q and V_P (Ursin and Toverud, 2002). QFWI, like standard FWI, involves an objective function based on least-squares data misfit,

$$\Phi(\mathbf{m}) = \frac{1}{2}||\mathbf{d}_{obs} - \mathbf{d}_{mod}||_2^2, \tag{1}$$

where $\mathbf{d}_{obs}$ and $\mathbf{d}_{mod}$ are, respectively, measured and modelled wavefields evaluated on a measurement surface, and $\mathbf{m}$ is the set of an-acoustic model parameters giving rise to $\mathbf{d}_{mod}$. This objective function is minimized subject to the condition that a prior-defined wave equation is satisfied by these wavefields. In the framework of a frequency domain finite difference approximation, for instance, data are measurements of a field $\mathbf{u}$ which satisfies

$$\mathbf{S}_1(\omega,\mathbf{m})\mathbf{u}(\omega) = \mathbf{f}(\omega), \tag{2}$$

where $\mathbf{f}$ is a source term and $\mathbf{S}_1(\omega,\mathbf{m})$ is a matrix that applies a finite difference stencil based on the an-acoustic physics relevant to the problem.

In any FWI problem, but of special concern to QFWI, there exists the possibility that wave propagation in the unknown medium is better represented by

$$\mathbf{S}_2(\omega,\mathbf{m})\mathbf{u}(\omega) = \mathbf{f}(\omega), \tag{3}$$

where $\mathbf{S}_2(\omega,\mathbf{m})$ invokes an attenuation model differing from that in $\mathbf{S}_1(\omega,\mathbf{m})$. Differences between, for instance, low-frequency velocity dispersion from one Q model to the next can vary significantly, so the two operators cannot be assumed to be similar. Therefore, concern about the kinds of parameter values a model belonging to $\mathbf{S}_2$ will require, in order to minimize an objective function based on $\mathbf{S}_1$, is high.

We must assume that one of these, say $\mathbf{S}_1$, holds in order to begin the process of inverting the data. This means adopting equation (2) as a constraint. Our approach to managing QFWI modelling errors is to relax this constraint to instead read

$$\mathbf{S}_1(\omega,\mathbf{m}_N)\mathbf{u}(\omega) = \mathbf{f}(\omega), \text{ for } \omega_N < \omega < \omega_{N+1}, \tag{4}$$

where $\mathbf{m}_N$ is a subsurface model for the angular frequency range (ω_N, ω_{N+1}). This allows greater freedom in matching the attenuation behaviour of the measured data, because it requires that the assumed physics be satisfied exactly only on a certain frequency band. As the bandwidths ω_{N+1}-ω_N decrease, modelling errors within any given band become less significant. Piecewise application of $\mathbf{S}_1$ can, in other words, closely mimic a model belonging to $\mathbf{S}_2$.

The lower limit of this process involves bands containing single frequency components. Because the simultaneous determination of velocity and Q requires several frequencies to be compared (Innanen and Weglein, 2007; Keating and Innanen, 2017), this limit should not in practice be approached. In the QFWI approach we consider here, it is in fact necessary to treat the width of the frequency bands as a tradeoff parameter, balancing the suppression of modelling error with the suppression of parameter cross-talk.

Mitigation of FWI modelling errors due to uncertainty in attenuation physics

KF and SLS models of an-acoustic wave propagation

To study attenuation-based modelling errors in isolation, we formulate a constant density anacoustic FWI, in which wave propagation is governed by

$$\left[\omega^2 s(\mathrm{r},\omega)+\nabla^2\right]u(\mathrm{r},\omega)=f(\mathrm{r},\omega), \tag{5}$$

where u is the pressure field, f is a source term, and the model parameter s depends on the dispersive velocity and attenuation:

$$s(\mathbf{r},\omega)\approx\left[c(\mathbf{r},\omega)-\frac{ic_0(\mathbf{r})}{2Q(\mathbf{r},\omega)}\right]^{-2}, \tag{6}$$

where c is the phase velocity, c_0 is the phase velocity at a reference frequency, and Q is a quality factor.

Many models of attenuation and dispersion exist and are in regular use for processing, imaging and inverting seismic data. They tend to agree in their general reproduction of the amplitude and phase features of dissipating waves, but in their detailed predictions of, e.g., phase velocities at low frequencies they may differ widely. We select as benchmark models the the Kolsky-Futterman (KF) nearly constant Q model, and the standard linear solid (SLS) model.

Kolsky-Futterman (KF) model

In certain attenuation models, the quality factor Q, defined as

$$\frac{1}{Q(\omega)}=\frac{\Delta E}{2\pi E}, \tag{7}$$

where E and ΔE are the peak strain energy stored and strain lost during a given cycle (Aki and Richards, 2002), is forced to be constant over a given frequency range. A constant Q in a non-dispersive medium violates causality (Aki and Richards, 2002), so in many models a frequency-dependent Q, which is nearly constant over the range of seismic frequencies, and a dispersion term are adopted. There are many ways to create a function which is nearly constant over the range of seismic frequencies, so there are many different nearly constant Q model types (Ursin and Toverud, 2002; Liu et al., 1976). We select the nearly constant Q model due to Kolsky and Futterman (Kolsky, 1956; Futterman, 1962), hereafter the KF model, in which

$$c(\omega)=c_0\left[1+\frac{1}{\pi Q}\log\left(\frac{\omega}{\omega_0}\right)\right], \tag{8}$$

where $c(\omega)$ is the wave velocity, ω_0 is a reference frequency and $c_0 = c(\omega_0)$.

Standard linear solid (SLS) model

The standard linear solid (SLS) model is based on viscoelastic considerations, with a constitutive relation that is linear in the stress, the strain, and their derivatives (Casula and Carcione, 1992; Liu et al., 1976). Continua are treated as consisting of a spring and dash-pot in series, in parallel with a second spring. The Q value given by this model is not constant, but is instead given by

$$Q(\omega)=\frac{1+\omega^2\tau_\varepsilon\tau_\sigma}{\omega(\tau_\varepsilon-\tau_\sigma)}, \tag{9}$$

where τ_ε and τ_σ are relaxation times related to the constants of the effective springs and dash-pot of the model (Casula and Carcione, 1992; Liu et al., 1976). This function is sharply peaked at $\omega=\tau^{-1}$, where $\tau=\sqrt{\tau_\varepsilon\tau_\sigma}$. The P-wave phase velocity for this model is given by

$$c(\omega)=c_0\frac{\mathbb{R}\mathrm{e}\sqrt{\frac{1+i\omega_0\tau_\sigma}{1+i\omega_0\tau_\varepsilon}}}{\mathbb{R}\mathrm{e}\sqrt{\frac{1+i\omega\tau_\sigma}{1+i\omega\tau_\varepsilon}}}, \tag{10}$$

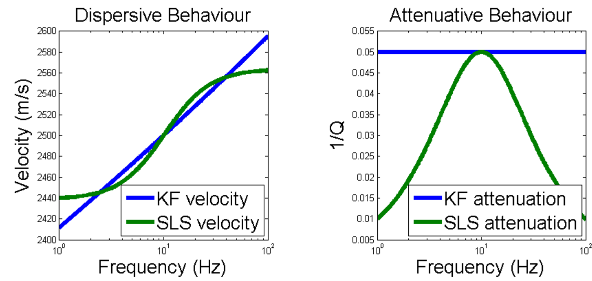

Figure 1: Comparison of SLS and KF models for velocity 2500m/s at reference frequency 15Hz, and Q=20. Left: Velocity comparison. Right: Attenuation comparison. Note the semilog scale.

Many physical processes which could have significant impact on seismic wave attenuation are well modelled by the standard linear solid (Liu et al., 1976). Furthermore it has been pointed out (Liu et al., 1976) that the SLS and KF models are not necessarily at odds with one another. A general standard linear solid can be introduced by considering several standard linear solid systems arranged in parallel. This introduces several relaxation mechanisms, and several attenuation peaks. If the amplitudes and peak frequencies of these individual SLS components are chosen correctly, a general SLS with approximately constant Q over a given bandwidth can be constructed. In this case the dispersive behaviour of the velocity reduces to equation 8 over the nearly constant Q frequency band. In a situation like this a KF-based QFWI procedure would suffer from little modelling error.

Our purpose in this paper is to develop a methodology which limits modelling errors when the QFWI model (e.g., KF) and the actual model operating in the Earth are dissimilar. So, the SLS model considered in the following examples is based on a single spring/dashpot system and does not reduce to KF behaviour. A comparison of KF and SLS Q and P-wave velocity is shown in Figure 1, where the models have the same Q and P-wave velocity at 15Hz.

Flexible FWI with unknown attenuation physics

The discrepancies between the KF and SLS models illustrated in Figure 1 will have strong negative consequences for a QFWI procedure, if the KF model is assumed and the SLS model (or something like it) actually holds. But, the consequences can be significantly reduced if in the QFWI procedure the KF model is not forced to be self-consistent over the full frequency range. The additional flexibility afforded QFWI by imposing the relaxed constraint in equation (4) is illustrated in Figure 2. An example SLS profile for Q and P-wave velocity is illustrated in this figure as a black dashed line, along with the KF model which most closely matches it in blue. Although both models are evaluated using the same parameters, the highly dissimilar frequency-dependence of these parameters in the different physics models mean that the matching is very poor. The red line shows the best match which can be obtained using a relaxed KF model, with different parameters on each 1 Hz band. Clearly, this step offers considerable improvement in the ability to match the observed behaviour, despite having assumed physics different from the SLS. Adopting an FWI strategy which allows for this better matching should improve the quality of the results in the case where the true attenuation model is unknown.

While the flexible strategy outlined above in principle has the capacity to match unknown an-acoustic physics, the question of whether a QFWI procedure based on this idea works in practice is settled neither by simply stating it nor by Figure 4. Two significant challenges may present themselves in inversion using this strategy. First, while the overall dispersive character of an ideal recovered model will closely match the true model, these behaviours may differ significantly within the small bands on which the inversion occurs. This means that insofar as the inversion considers the dispersive character of the observed

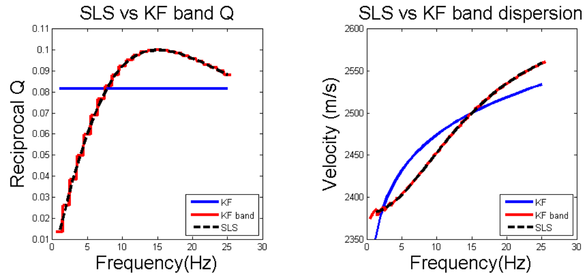

Figure 2: Comparison of SLS with best fitting KF, and band-defined KF. Due to the highly dissimilar behaviour of the model types, the KF result is a poor approximation of the SLS behaviour. The band-defined KF is capable of matching the SLS behaviour much more closely, though still differs in dispersive behaviour on each band.

data, it may lead the estimated model away from the best approximation. The second problem is that it is not straightforward to predict what spatial arrangement of (e.g.) KF model parameters will be settled on by a QFWI procedure, if those parameters vary widely and non-self consistently over the full spectrum, and whether or not these structures will tend to be realistic. It is difficult to address the impact of these concerns without the aid of synthetic examples, which are considered in the next section.

NUMERICAL EXAMPLES

In this section, two distinct QFWI approaches are implemented numerically and used to explore the an-acoustic modelling-error suppression strategy. In the first, which will be referred to as 'traditional' QFWI, equation 2 is strictly adhered to, and the updating strategy used is designed to minimize an-acoustic cross-talk, following Keating and Innanen (2017). The second, which will be called 'flexible' QFWI, obeys instead the constraint in equation 4 and uses an updating strategy consistent with this constraint. The QFWI procedure is built around a KF an-acoustic model in both cases. The phase velocity at the reference frequency, c_0, and Q, are the two unknown parameters to be recovered in all cases. The reference frequency is fixed at 30 Hz.

In order to avoid introducing complicating optimization issues, rather than use the more efficient truncated Newton algorithms, the examples calculated in this paper were chosen to be sufficiently small that exact Gauss-Newton numerical optimization could be employed. Frequency domain finite difference modelling is used for two-dimensional models on a 50 by 50 grid, with grid spacing of 10 meters in both the horizontal and vertical dimensions. 24 sources at 20m intervals are arrayed laterally, at 30m depth, from 10m to 470m. 48 receivers at 10m intervals are placed at 20m depth from 10m to 480m. Frequencies from 1Hz to 25Hz are assumed to be available, and the source function is considered to have a uniform amplitude spectrum over this range. First order Engquist boundary conditions are implemented at every boundary.

Traditional QFWI is applied here with a multiscale approach, considering 24 frequency bands, beginning with 1Hz-2Hz, and increasing the upper limit by 1Hz at each band, up to the final band of 1Hz-25Hz. At each stage of the inversion, 6 frequencies spaced evenly from the minimum to the maximum frequency of the band are simultaneously inverted. The frequency band is changed every 2 iterations.

The flexible FWI approach used here uses 12 instances of f_N, in 2Hz steps from 1Hz to 23Hz. To recover each $\mathbf{m}_N$, six evenly spaced frequencies from f_N to f_{N+1} were inverted simultaneously. 4 iterations were performed for each $\mathbf{m}_N$, resulting in a total number of iterations equal to the number used in the traditional FWI approach. The $\mathbf{m}_N$ were solved for sequentially, beginning with the smallest f_N and then increasing. The initial model used for $\mathbf{m}_1$ was identical to that used in the traditional FWI approach. The initial model for every other $\mathbf{m}_N$ was set equal to the final $\mathbf{m}_{N-1}$.

For the first example, the model in Figure 3, and KF an-acoustic physics, are used to generate the synthetic observed data. The initial model is a uniform velocity of 2500 m/s and uniform Q^{-1} of 0, matching the background of the true model. The QFWI procedure assumes (in this case, correctly) an KF an-acoustic model. The result of traditional QFWI with an exact Gauss-Newton optimization is illustrated in Figure 4, where the recovered velocity at reference frequency and Q are shown. This result acts as a kind of benchmark, reflecting the ideal case of a simple model, dense acquisition and exact Gauss-Newton numerical optimization.

The result of applying the flexible QFWI for two example bands is illustrated in Figures 5 and 6. Results comparable to the benchmark are obtained here, however comparison of the results generated using different bands make clear that variance in the recovered model parameters is introduced from band to band. The left panel of Figure 6 is suggestive that cross-talk issues can appear for certain frequency bands, and that therefore the issues discussed in this paper and those discussed by Keating and Innanen (2017) are not independent. This is suggestive that self-consistency of the an-acoustic model across the full frequency range is optimal, if the correct an-acoustic model type is well established in advance.

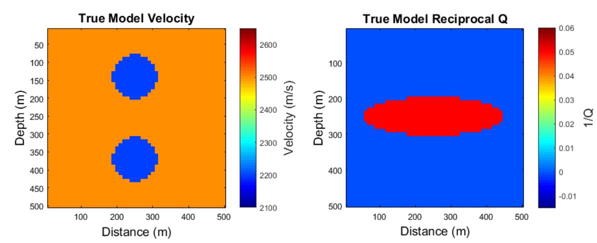

Figure 3: True model velocity at reference frequency 30 Hz (left) and reciprocal Q (right) for KF model type.

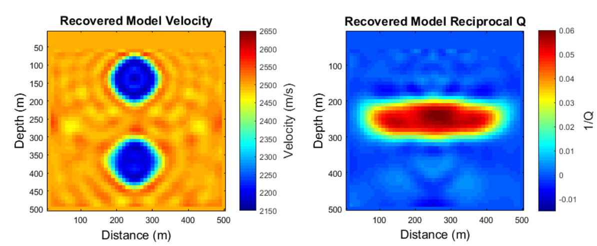

Figure 4: Final result of conventional QFWI using the correct KF model type.

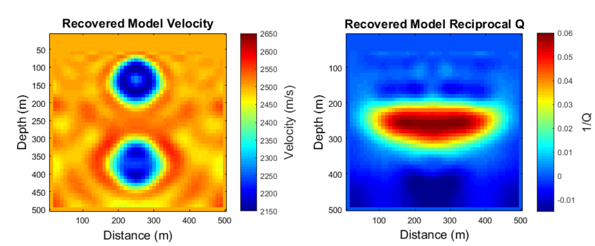

Figure 5: Final result of flexible QFWI approach with a 13-15 Hz maximum band, using the correct KF model type.

In a second example the central problem of the current study, in which

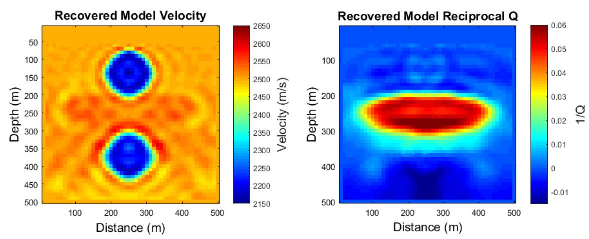

Figure 6: Final result of flexible QFWI approach with a 23-25 Hz maximum band, using the correct KF model type.

the assumed attenuation model is incorrect, is explored. The model shown at example frequencies in Figures 7 and 8 was used to generate the data. In this model, a SLS model was used, with a peak Q^{-1} at 15 Hz. The acquisition geometry and FWI strategy are unchanged from the previous example. The peak reciprocal Q is appreciably higher than for the KF model, in order to introduce non negligible attenuation away from the peak.

First we examine the result of applying to the SLS-type data a traditional QFWI procedure in which the KF an-acoustic model is assumed to hold. The results are illustrated in Figure 9. Evidently QFWI fails to recover a meaningful velocity or Q model — despite benefiting from the simple model geometry, dense acquisition and powerful numerical optimization which allowed for strong recovery in the previous example. This highlights the hazards associated with uncertainty in the QFWI attenuation model, adding further incentive for a flexible approach.

Results produced by applying the flexible QFWI approach, based on the KF model, again on SLS data, are shown for two example bands in Figures 10 and 11, where the recovered velocity at the example frequency and Q are shown. These example results correspond to the true model at frequencies shown in figures 7 and 8, respectively. The less restrictive constraints used in this approach allow for a significantly improved recovery of the true model behaviour. The recovered models effectively identify the position and shape of the anomalies. Generally our survey of the modelling error problem as summarized here supports the use of a flexible QFWI strategy of the type we present here.

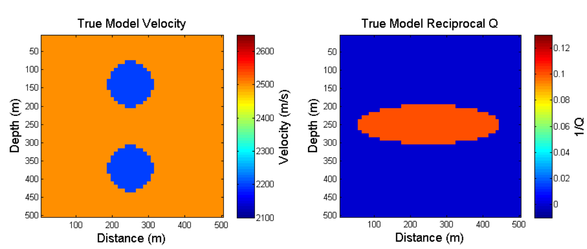

Figure 7: True model velocity (left) and reciprocal Q (right) at 15 Hz for SLS attenuation physics.

The computational cost of the two QFWI approaches is identical, the greater number of models recovered in the flexible approach being offset by the smaller number of iterations used to invert for each. The reason for this similarity is that the flexible approach can be interpreted as an alternative multiscale strategy in conventional FWI, with the caveat that the final result is an approximation of the model behaviour only within the highest frequency band considered, and that the intermediate steps themselves provide an estimate of the model behaviour at their respective frequency ranges.

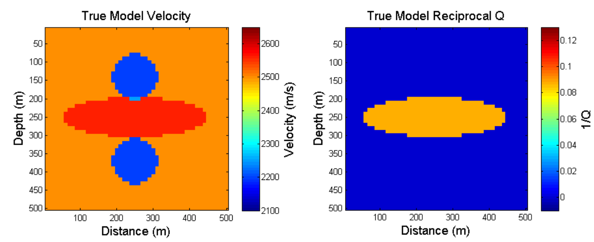

Figure 8: True model velocity (left) and reciprocal Q (right) at 25 Hz for SLS attenuation physics.

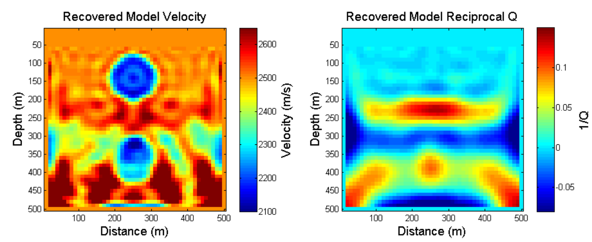

Figure 9: Final result of conventional QFWI; KF inversion carried out on SLS data..

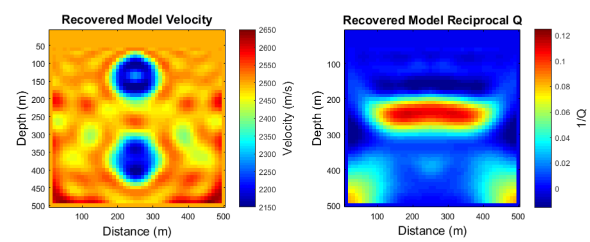

Figure 10: Final result of flexible band approach for 13-15 Hz maximum band; KF inversion carried out on SLS data.

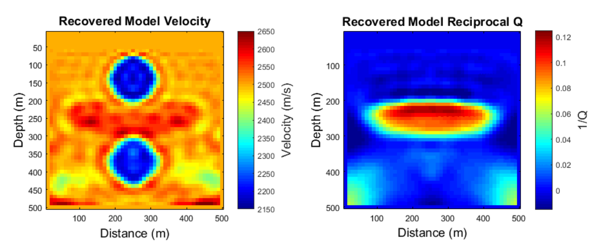

Figure 11: Final result of flexible band approach for 23-25 Hz maximum band; KF inversion carried out on SLS data.

CONCLUSIONS

The inclusion of attenuation in seismic FWI offers the potential for improved recovery of subsurface parameters of interest, but presents unique risks associated with modelling error. The flexible QFWI approach suggested here relaxes the FWI constraint that the modelled wavefield strictly obeys an assumed physics model across all experimental variables. This allowed for significant improvements over traditional FWI strategies as applied to dissipative problems.

ACKNOWLEDGMENTS

We thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13.