Synthetic seismograms in stratified media: a theoretical overview

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Outline

- Why the ‘reflectivity’ method
- Plane waves: basics
- Plane waves: propagation, reflection / transmission
- “Kennett’s” reflectivity matrix
- Computing the reflectivity matrix
- Wavefield constriction
- Conclusions

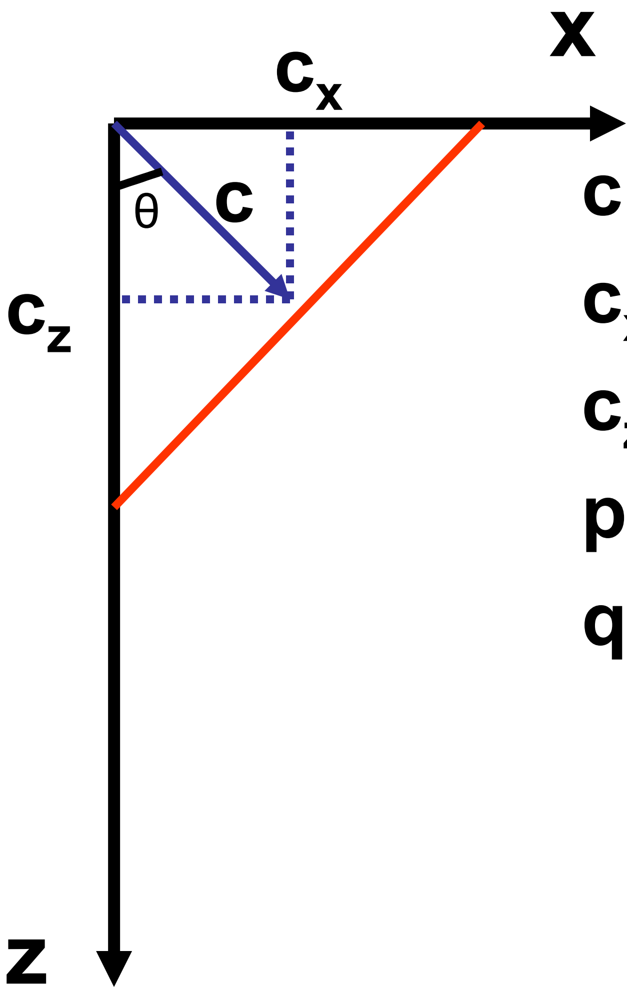
Why the reflectivity method

- Generates the ‘complete’ seismic response
- Includes all primaries and multiples
- Includes all mode conversions
- Easy to implement attenuation using complex velocities
- Can generate partial response, for example primaries only
- Can generate surface waves
- ‘Stratigraphic’ filtering is incorporated
- Fluid over solid layers – marine seismograms
- Source can be buried – ghost effects

Thomson-Haskell method

- matrix method for generation surface waves
- Gilbert and Backus: propagator matrix method
  - solution of the first-order ordinary differential equations describing a media varying in depth
- the method suffers of severe numerical stability
- most of later publications are dedicated to improving the numerical stability
- Kennett’s approach avoids altogether the problem

Plane waves



c - velocity

$c_x = c / \sin\theta$ - horizontal velocity

$c_z = c / \cos\theta$ - vertical velocity

$p = 1 / c_x$ - horizontal slowness

$q = 1 / c_z$ - vertical slowness

Plane waves

$$u = Ae^{i(k_x x + k_z z - \omega t)} = Ae^{i\omega(px + qz - t)}$$

$$u = Ae^{i\omega qz} e^{i\omega(px - t)}$$

$$k^2 = k_x{}^2 + k_z{}^2 = \frac{\omega}{c}$$

$$k_z = \frac{\omega}{c_z} = \frac{\omega}{c}\cos\theta = \omega q \qquad k_x = \frac{\omega}{c_x} = \frac{\omega}{c}\sin\theta = \omega p$$

Plane waves: P and S

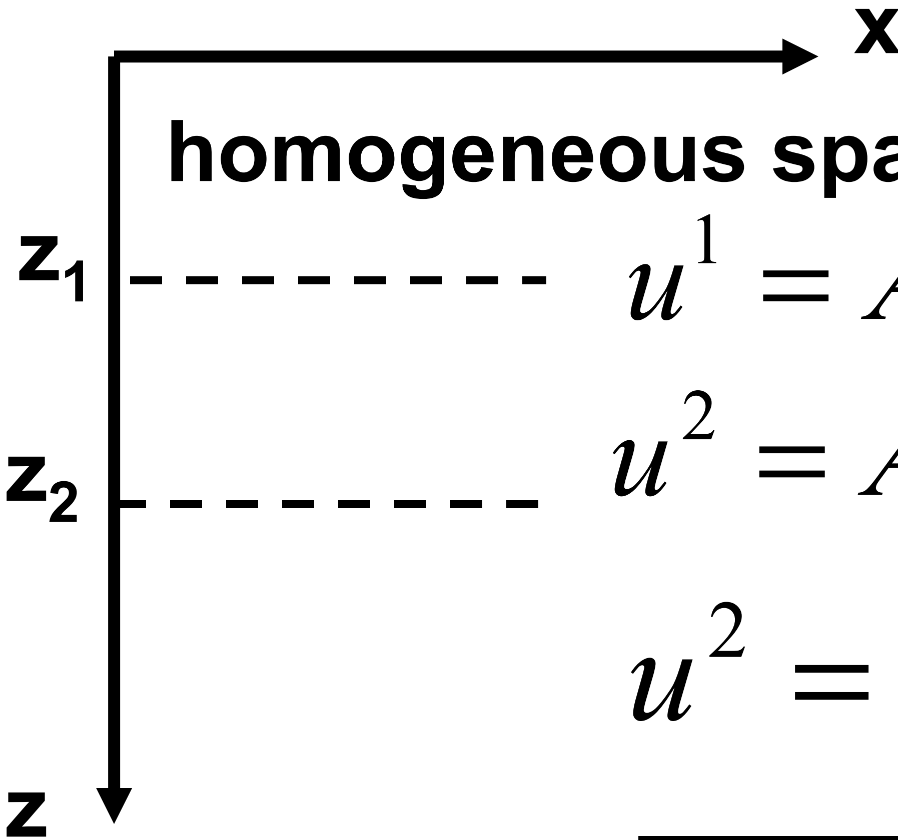
Snell's low: horizontal slowness $p_P = p_S = p$

P and S waves have distinct vertical slowness's

$$q = \frac{k_z}{\omega} = \frac{1}{\omega}\sqrt{\frac{\omega^2}{c^2} - \omega^2 p^2} = \sqrt{\frac{1}{c^2} - p^2}$$

$$q_p = \sqrt{\frac{1}{\alpha^2} - p^2} \qquad q_s = \sqrt{\frac{1}{\beta^2} - p^2}$$

Plane waves propagation



$$u^1 = Ae^{i\omega q z_1} e^{i\omega(px - t)}$$

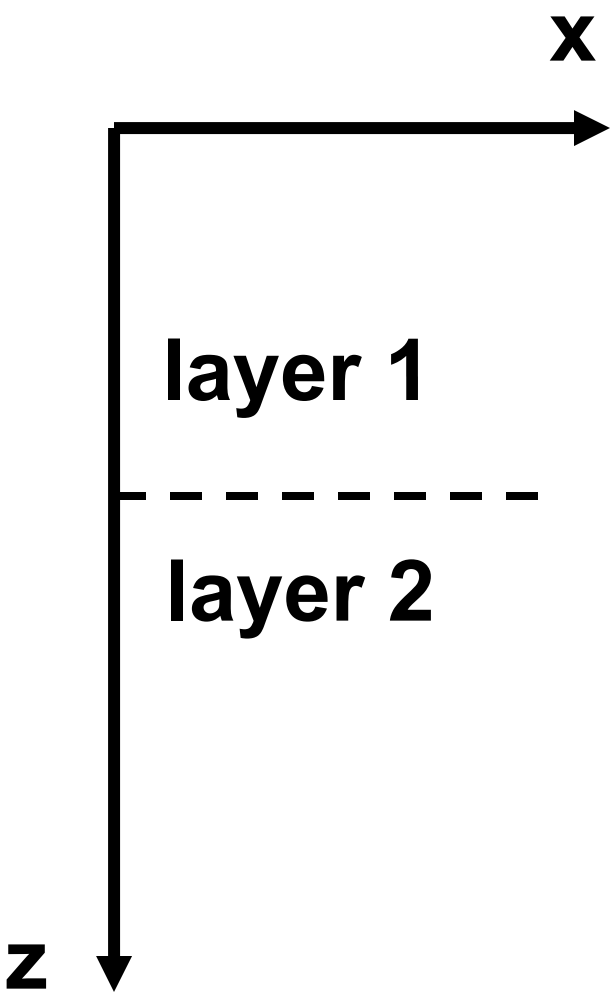
$$u^2 = Ae^{i\omega q z_2} e^{i\omega(px - t)}$$

$$u^2 = u^1 e^{i\omega q (z_2 - z_1)}$$

$$SH : E = e^{i\omega q_s (z_2 - z_1)}$$

$$P - SV : E = \begin{bmatrix} e^{i\omega q_p (z_2 - z_1)} & 0 \\ 0 & e^{i\omega q_s (z_2 - z_1)} \end{bmatrix}$$

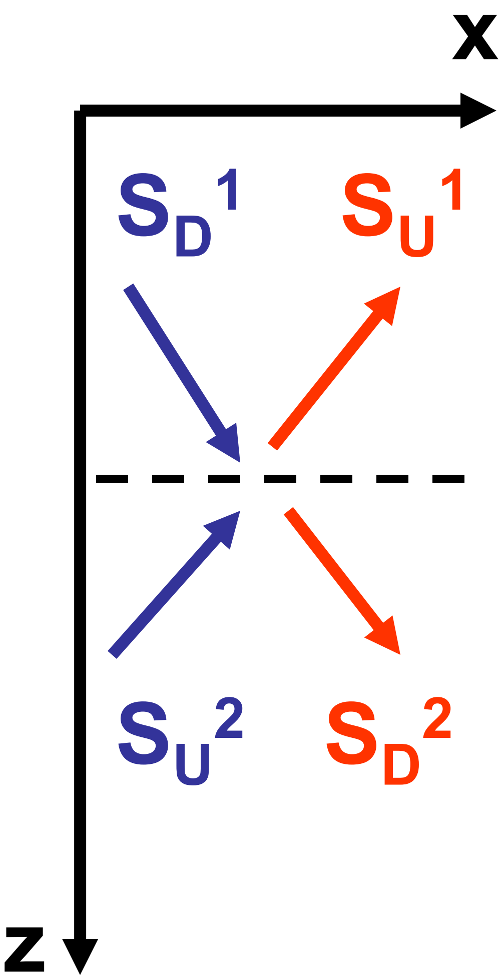
Plane waves at an interface



$$\begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_1 = \begin{bmatrix} u_y \\ \sigma_{yz} \end{bmatrix}_2$$

$$\sigma_{yz} = \mu \frac{\partial u_y}{\partial z} = i\omega\mu q u_y$$

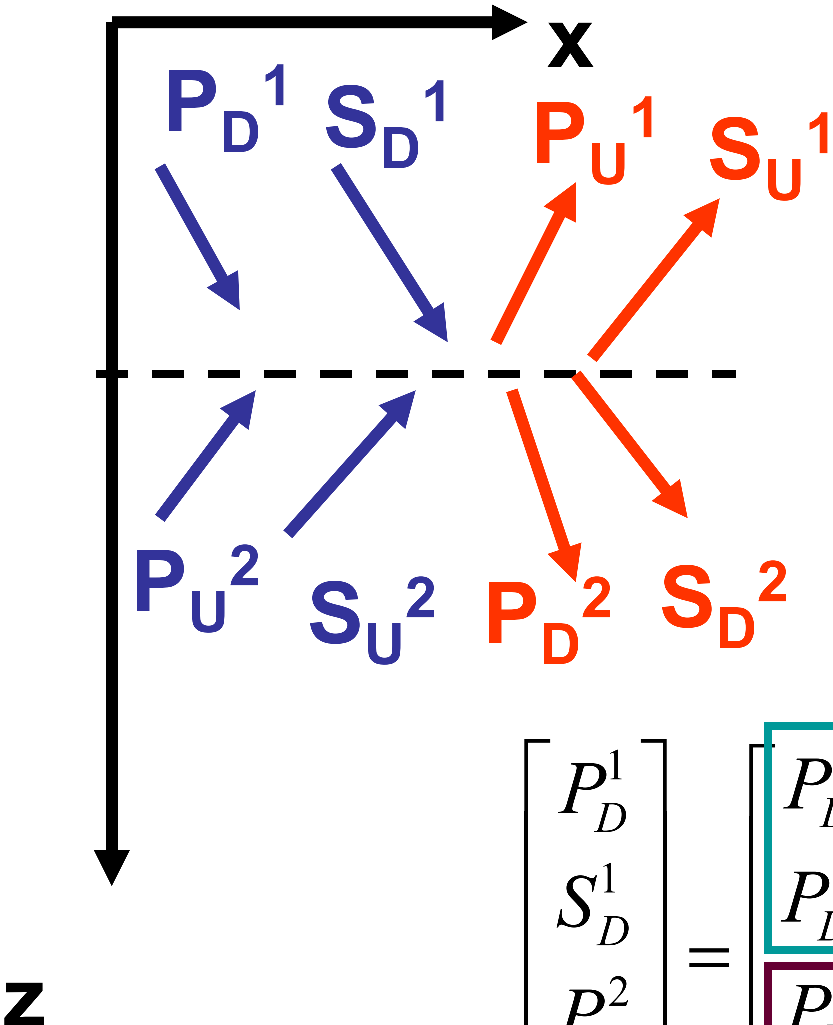
Reflection / transmission of SH waves



$$\begin{bmatrix} 1 & 1 \\ \mu_1 q_1 & -\mu_1 q_1 \end{bmatrix} \begin{bmatrix} S_D^1 \\ S_U^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \mu_2 q_2 & -\mu_2 q_2 \end{bmatrix} \begin{bmatrix} S_D^2 \\ S_U^2 \end{bmatrix}$$

$$\begin{bmatrix} S_U^1 \\ S_D^2 \end{bmatrix} = \begin{bmatrix} R_D & T_U \\ T_D & R_U \end{bmatrix} \begin{bmatrix} S_D^1 \\ S_U^2 \end{bmatrix}$$

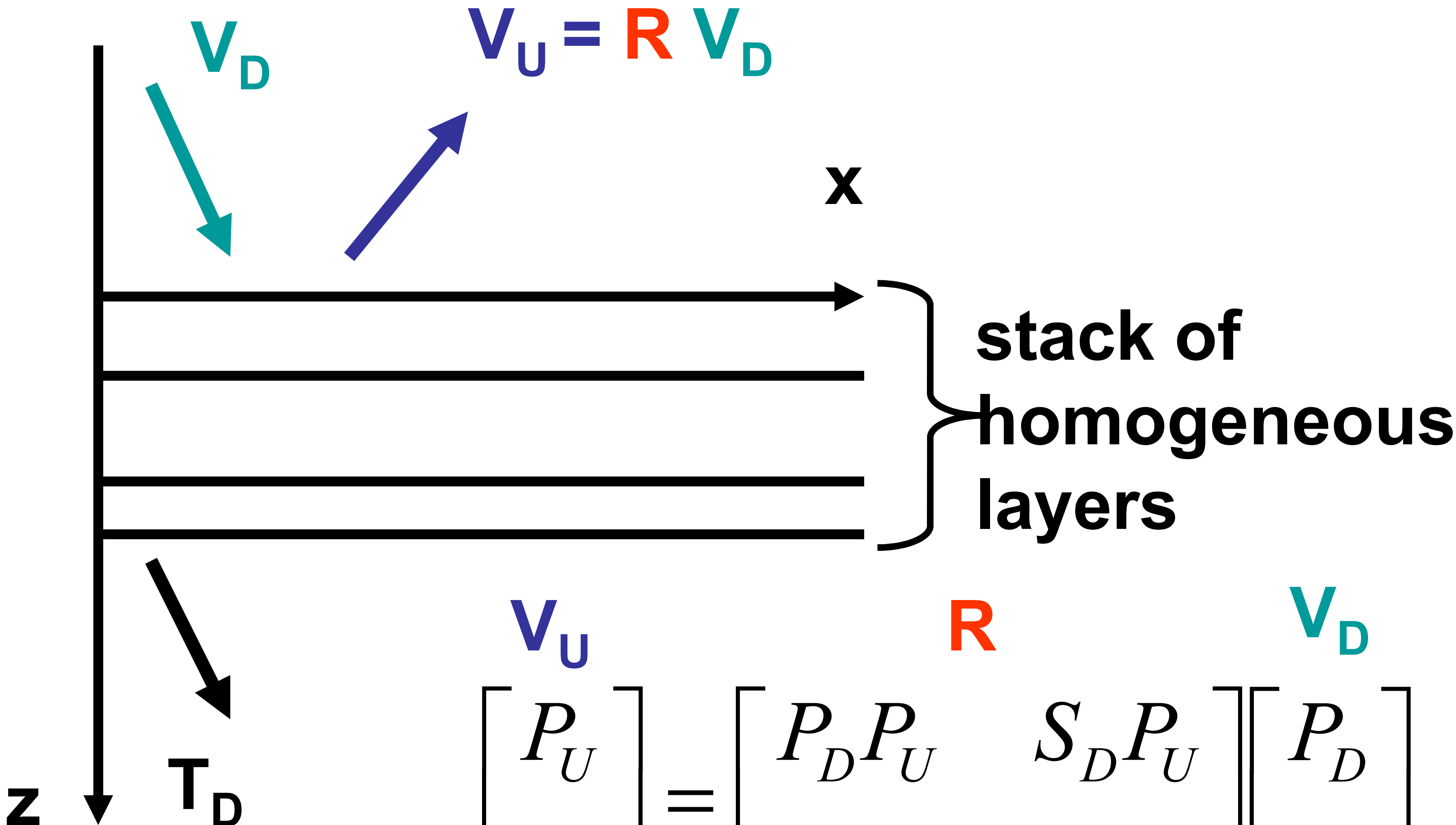
Reflection / transmission of P-SV waves



$$\begin{bmatrix} R_D & T_U \\ T_D & R_U \end{bmatrix}$$

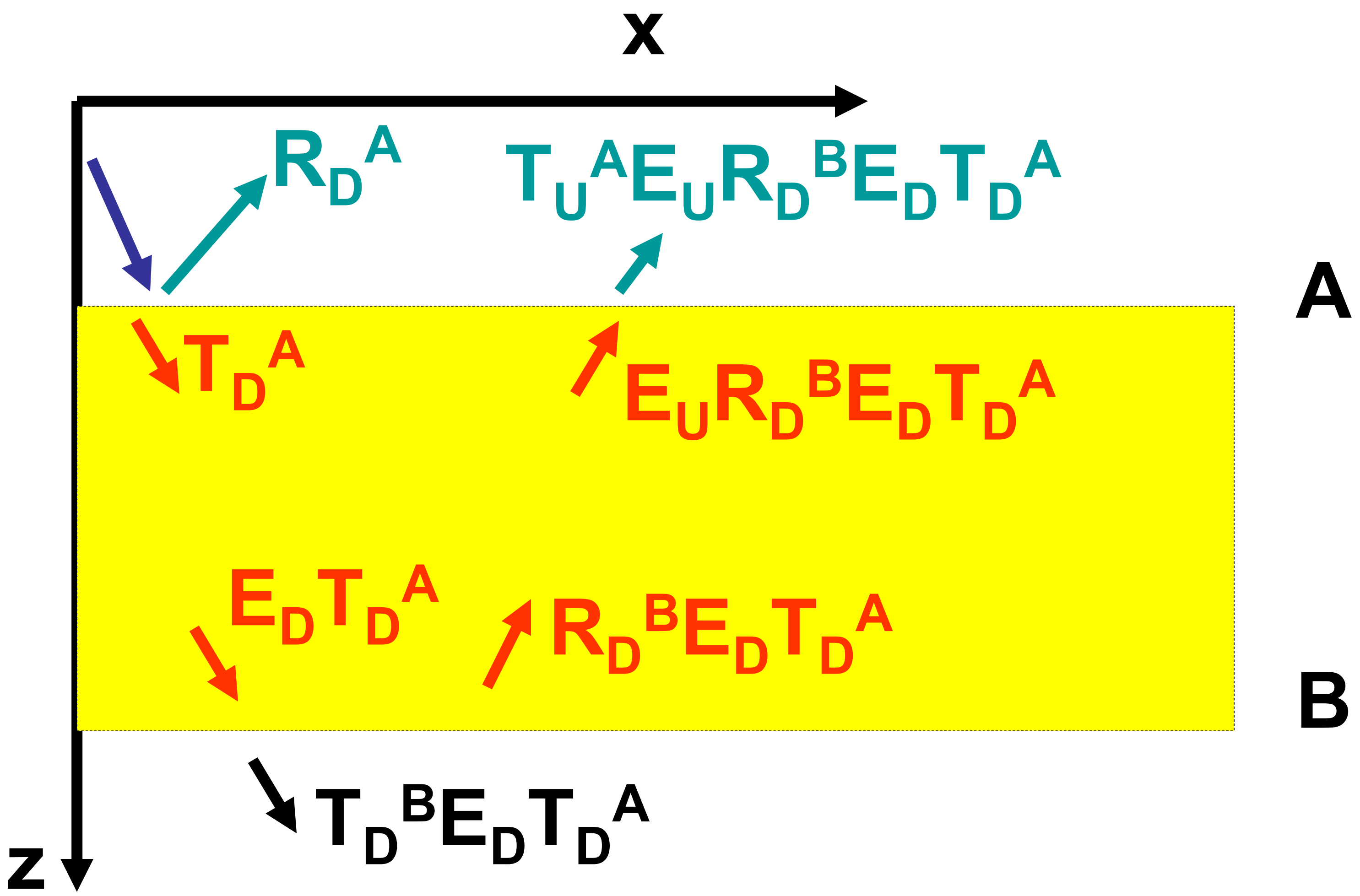
$$\begin{bmatrix} P_D^1 \\ S_D^1 \\ P_U^2 \\ S_U^2 \end{bmatrix} = \begin{bmatrix} P_D P_U & S_D P_U & P_U P_U & S_U P_U \\ P_D S_U & S_D S_U & P_U S_U & S_U S_U \\ P_D P_D & S_D P_D & P_U P_D & S_U P_D \\ P_D S_D & S_D S_D & P_U S_D & S_U S_D \end{bmatrix} \begin{bmatrix} P_U^1 \\ S_U^1 \\ P_D^2 \\ S_D^2 \end{bmatrix}$$

Reflectivity matrix R

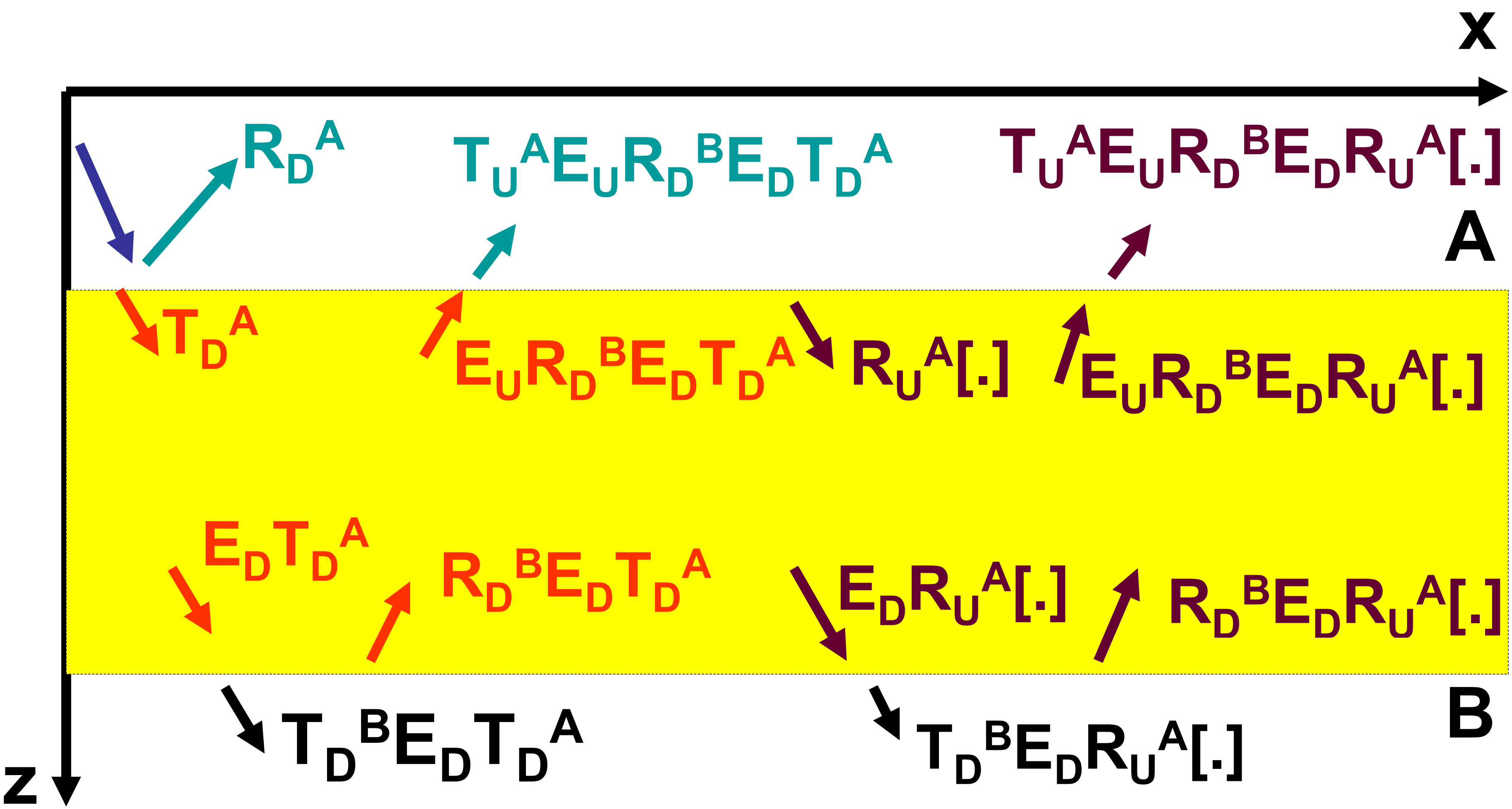


$$\underbrace{\begin{bmatrix} P_U \\ S_U \end{bmatrix}}_{V_U} = \underbrace{\begin{bmatrix} P_D P_U & S_D P_U \\ S_D S_U & P_D S_U \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} P_D \\ S_D \end{bmatrix}}_{V_D}$$

Reflectivity matrix: single layer



Adding multiples



Reflectivity Matrix

$$\mathbf{R} = R_D^A + T_U^A E_U R_D^B E_D T_D^A$$

$$+ T_U^A E_U R_D^B E_D R_U^A E_U R_D^B E_D T_D^A$$

$$+ T_U^A E_U R_D^B E_D R_U^A E_U R_D^B E_D R_U^A E_U R_D^B E_D T_D^A$$

$$+ \ldots\ldots =$$

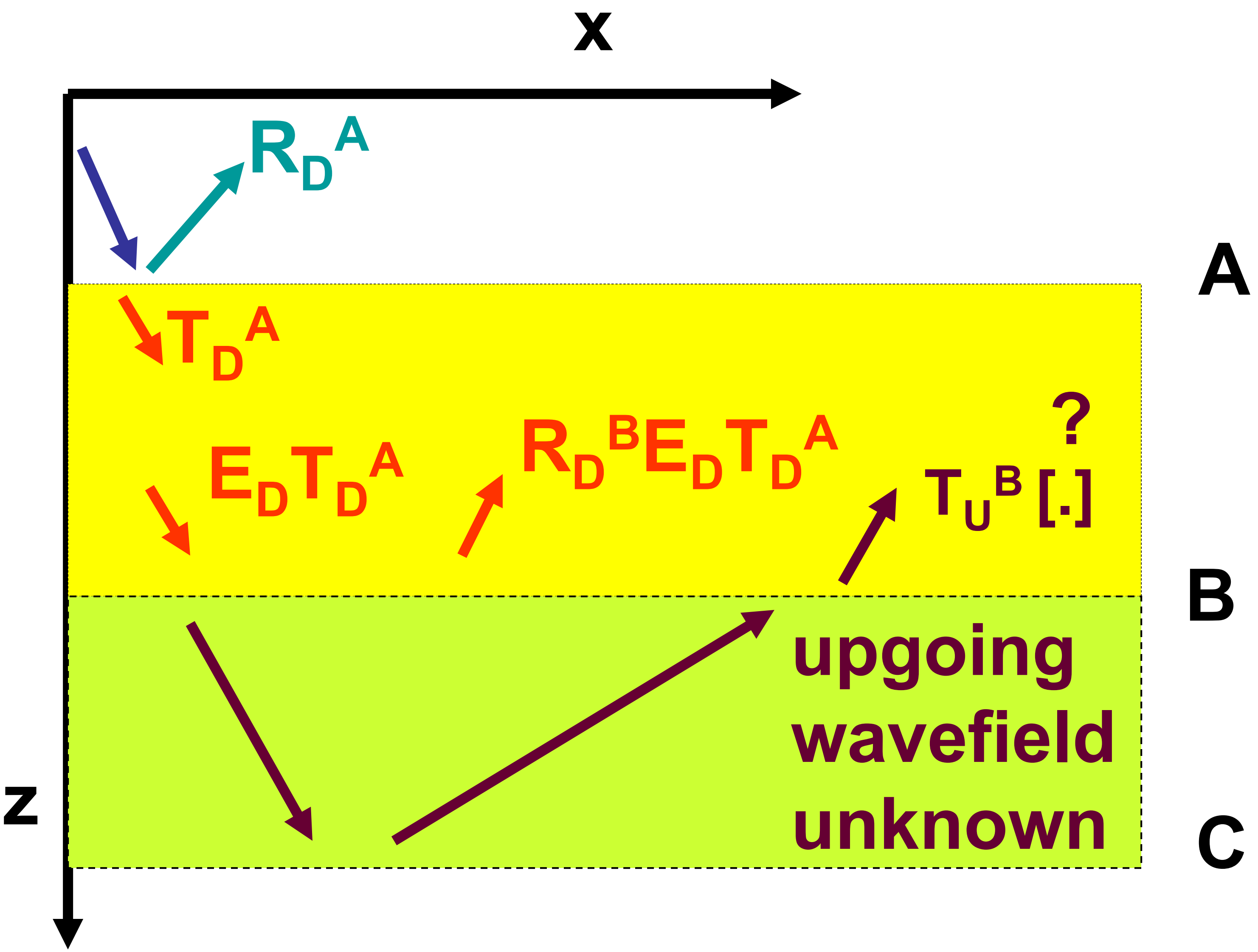
$$R_D^A + T_U^A E_U R_D^B E_D \left[1 + R_U^A E_U R_D^B E_D + \right.$$

$$\left. + R_U^A E_U R_D^B E_D R_U^A E_U R_D^B E_D + \ldots\ldots \right] T_D^A$$

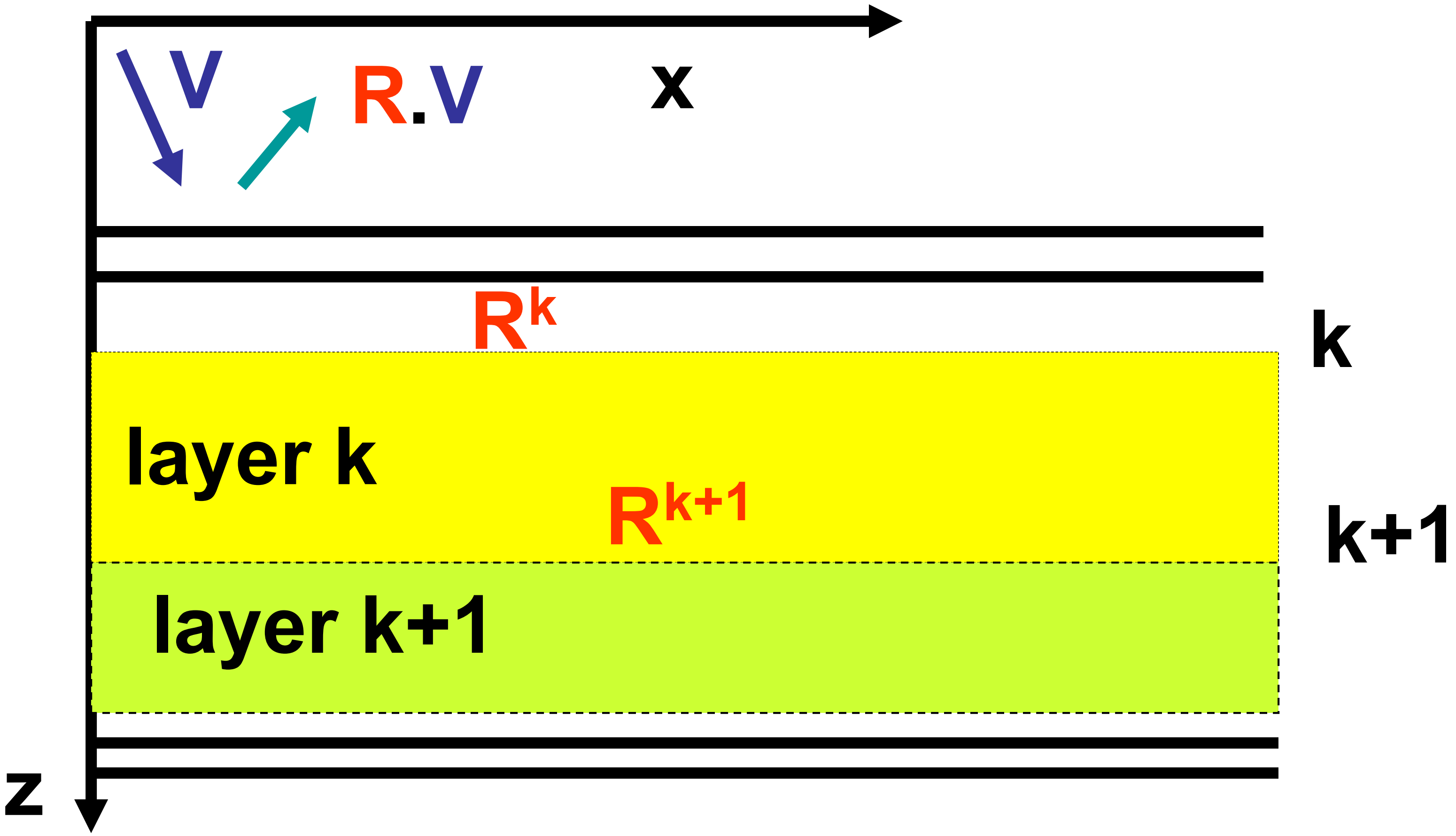
$$\mathbf{R} = R_D^A + T_U^A E_U R_D^B E_D \left[1 - R_U^A E_U R_D^B E_D\right]^{-1} T_D^A$$

reverberation operator

Reflectivity matrix: three layers

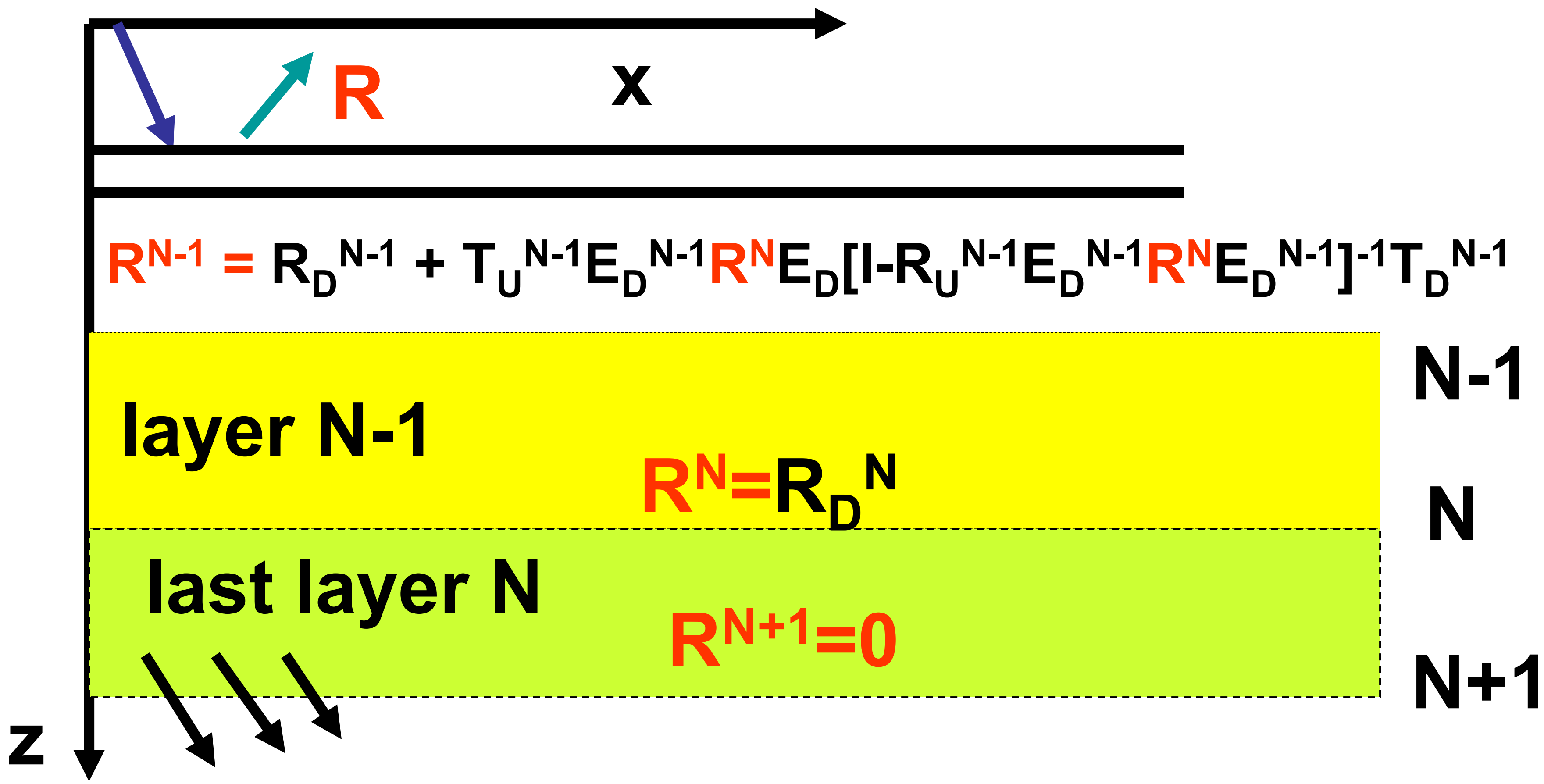


$\mathbf{R}$: recursive computation



$$R^k = R_D^k + T_U^k E_D^k R^{k+1} E_D^k [I - R_U^k E_D^k R^{k+1} E_D^k]^{-1} T_D^k$$

Recursive computation of R from bottom



$$R^{N-1} = R_D^{N-1} + T_U^{N-1}E_D^{N-1}R^N E_D[I-R_U^{N-1}E_D^{N-1}R^N E_D^{N-1}]^{-1}T_D^{N-1}$$

$R^N = R_D^N$

$R^{N+1} = 0$

transmission only,
there is no upgoing wavefield

Displacements from wave amplitudes

$$\begin{bmatrix} U_H(k_x,\omega) \\ U_V(k_x,\omega) \end{bmatrix} = [M][N]^{-1} S(k_x,\omega)$$

$$M = i\omega \begin{bmatrix} p(1+R_{PP}) + q_s R_{SP} & pR_{SP} + q_p(1+R_{SS}) \\ -q_p(1-R_{PP}) + pR_{PS} & -p(1-R_{SS}) - q_p R_{SP} \end{bmatrix}$$

$$N = \mu\omega^2 \begin{bmatrix} 2pq_P(R_{PP}-1) + rR_{SP} & 2pq_P R_{SP} + r(R_{SS}-1) \\ -(r(1+R_{PP}) - 2pq_S R_{PS} & 2pq_S(1+R_{SS}) - rR_{SP} \end{bmatrix}$$

$$r = \frac{1}{\beta^2} - 2p^2$$

The source term

explosion at the surface

$$S(k_x,\omega) = -\mu\omega^2 \begin{bmatrix} 2pq_P \\ r \end{bmatrix} \bar{s}(\omega)$$

explosion at depth h

$$S(k_x,\omega) = 2\omega\frac{\beta^2}{\alpha^2} \begin{bmatrix} p\cos(\omega q_p h) \\ -i(r/q_P)\sin(\omega q_P h) \end{bmatrix} \bar{s}(\omega)$$

Constructing the wavefield

- **vertical component**

$$u_V = \iint U_V(k_x, \omega) e^{i(k_x x - \omega t)} dk_x d\omega$$

- **horizontal component**

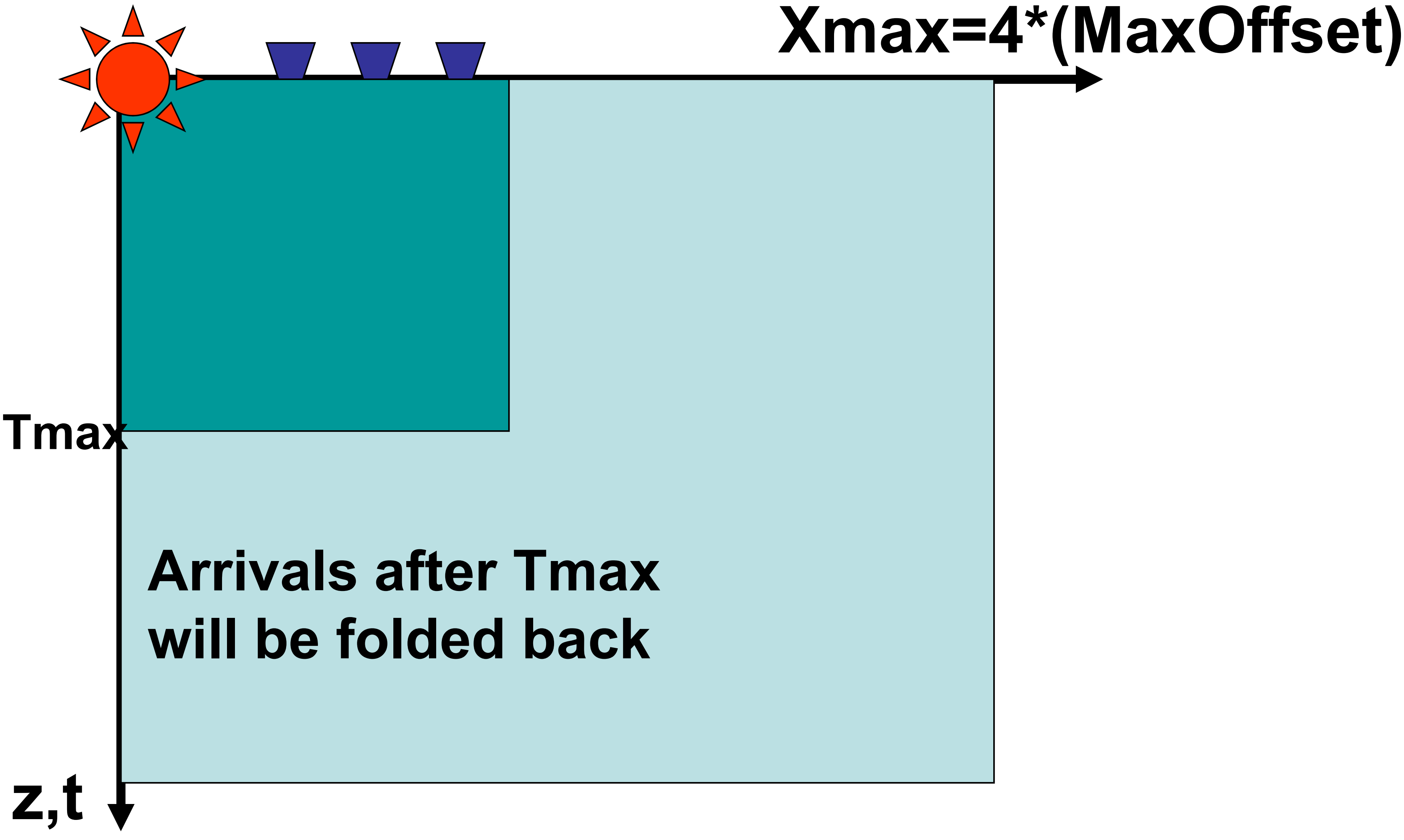
$$u_H = \iint U_H(k_x, \omega) e^{i(k_x x - \omega t)} dk_x d\omega$$

Attenuation: complex velocity

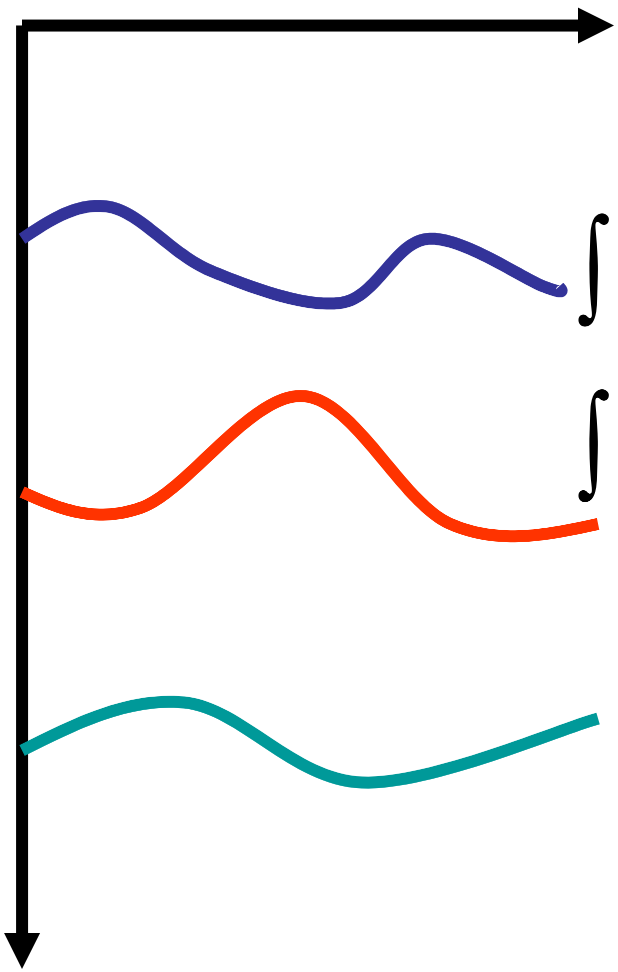
$$\overline{\alpha}(\omega) = \alpha - i\alpha \frac{\mathrm{sgn}(\omega)}{2Q_P(\omega)}$$

$$\overline{\beta}(\omega) = \beta - i\beta \frac{\mathrm{sgn}(\omega)}{2Q_S(\omega)}$$

Watch out: aliasing !!!



Curved interfaces
(Koketsu 1987, Koketsu and Kennett (1991)



The continuity of displacement, stress are integral equations.

$$\int u_1[k_x, z(x)]e^{ik_x x} dk_x = \int u_2[k_x, z(x)]e^{ik_x x} dk_x$$

$$\int \sigma_1[k_x, z(x)]e^{ik_x x} dk_x = \int \sigma_2[k_x, z(x)]e^{ik_x x} dk_x$$

Conclusions

- The ‘reflectivity’ modeling method is the preferred approach for synthetic seismogram generation in stratified media due to its ‘complete’ solution and ability to turn on / off desired features

- Looks like it can be extended to 2-D media with curved interfaces