Velocity Model Sampling and Interpolation: The Gradient Problem

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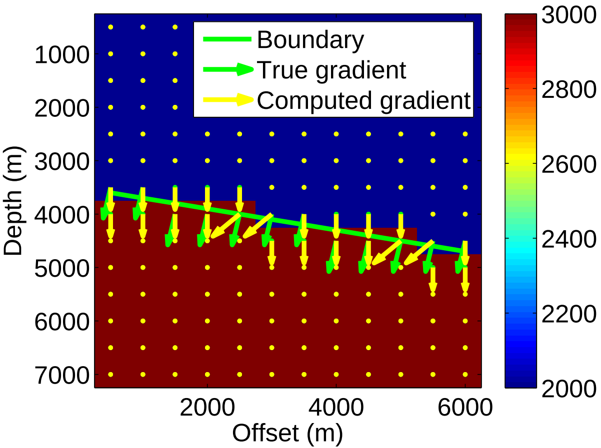
Motivation

Kirchhoff migration uses raytracing equations for prestack depth migration.

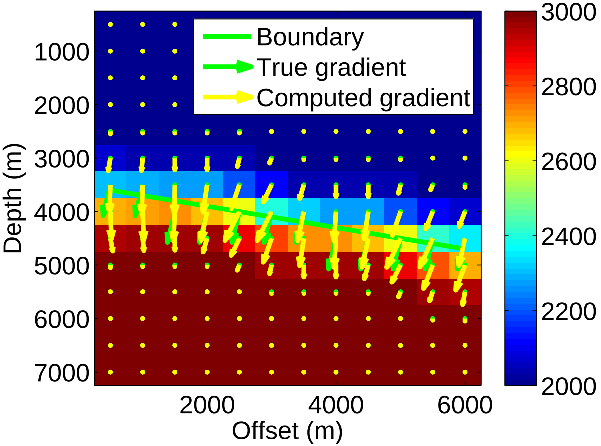
$$\frac{d\vec{x}}{dt} = v(\vec{x})^2\vec{p} \qquad \frac{d\vec{p}}{dt} = -\frac{\nabla v(\vec{x})}{v(\vec{x})} \tag{1}$$

These equations estimate the paths of seismic energy through the subsurface as a function of a known velocity model and its gradient.

Motivation


Boundary
True gradient
Computed gradient
1000
2000
3000
4000
5000
6000
7000
Depth (m)
2000
4000
6000
Offset (m)
3000
2800
2600
2400
2200
2000

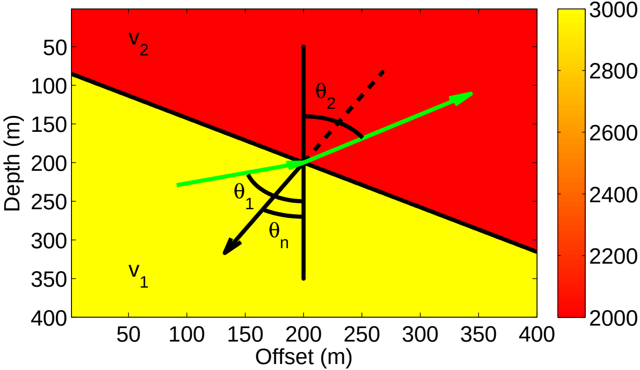
Motivation


Boundary
True gradient
Computed gradient
Depth (m)
1000
2000
3000
4000
5000
6000
7000
2000
4000
6000
Offset (m)
3000
2800
2600
2400
2200
2000

Motivation

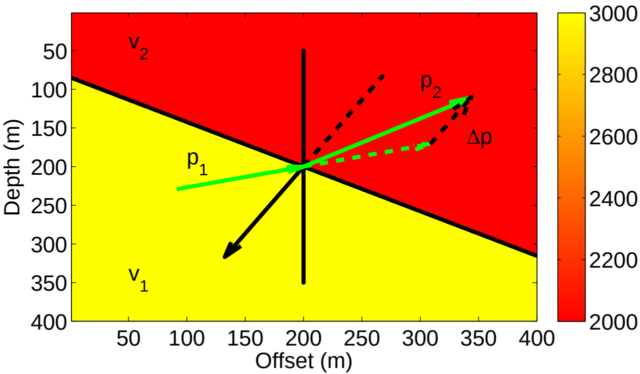
- Velocity models are generally smoothed before Kirchhoff migration.
- Moser (2011) Uses a smoothed velocity model to compute raypaths, and then computes traveltimes using the original blocky model.
- The result is a sharper image with more continuous reflectors than using the smooth model everywhere.

Snell's Law



$$\frac{\sin(\theta_1 - \theta_n)}{v_1} = \frac{\sin(\theta_2 - \theta_n)}{v_2} \quad (2)$$

Snell's Law



$$\frac{d\vec{p}}{dt} = -\frac{\nabla v(\vec{x})}{v(\vec{x})} \implies p_2 = p_1 - \frac{\Delta t}{v}\nabla v \qquad (3)$$

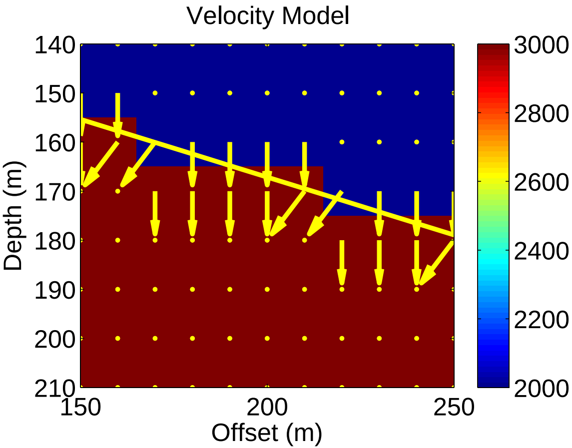
Regression

Processing delivers a velocity model sampled on a grid:

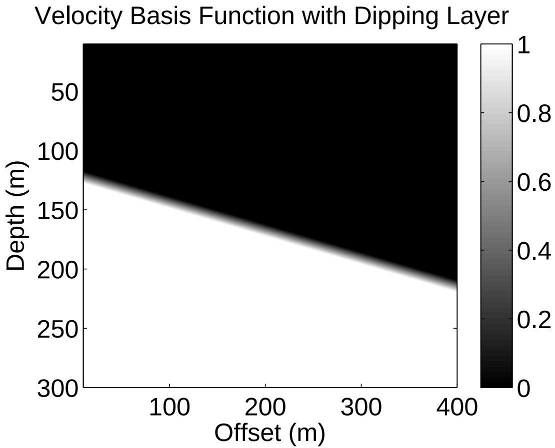
- Rays travel between these nodes, so we need to interpolate
- Sampling of the velocity grid makes it hard to accurately compute gradient vectors
- Some sort of regression would be ideal!

$$v(x,z) = \sum_{k=1}^{N} \alpha_k \tilde{v}_k(x,z) \tag{4}$$

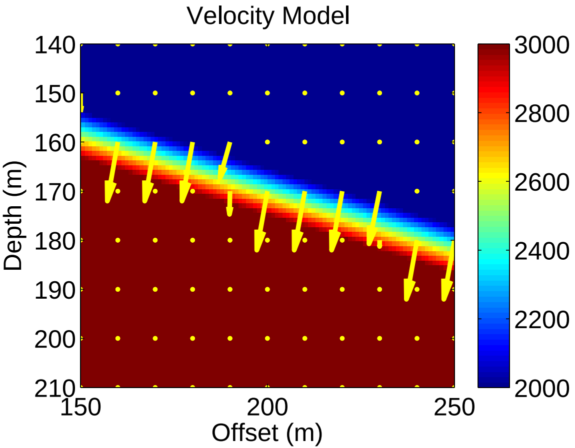
Regression


Velocity Model
140
150
160
170
180
190
200
210
Depth (m)
150
200
250
Offset (m)
3000
2800
2600
2400
2200
2000

Regression


Velocity Basis Function with Dipping Layer
Depth (m)
50
100
150
200
250
300
Offset (m)
100
200
300
400
1
0.8
0.6
0.4
0.2
0

Regression


Velocity Model
140
150
160
170
180
190
200
210
Depth (m)
150
200
250
Offset (m)
3000
2800
2600
2400
2200
2000

Regression

We can easily construct infinitely many basis functions:

- Dipping layers
- Anticlines
- Salt structures
- Random functions

Key Observation:

- The dipping layers are invariant from trace to trace except for a constant dip.
- This is the same assumption Spitz interpolation works on.

Spitz Method

- Model the traces as a sum of L events with constant dip

$$g_n(k_z) = \sum_{j=1}^{L} G_j(k_z) e^{2\pi i k_z (n-1) d_j}$$

- Unknown d_j gives the dip of the event.
- Interpolate between known traces using PEFs, or solve for d_j directly.

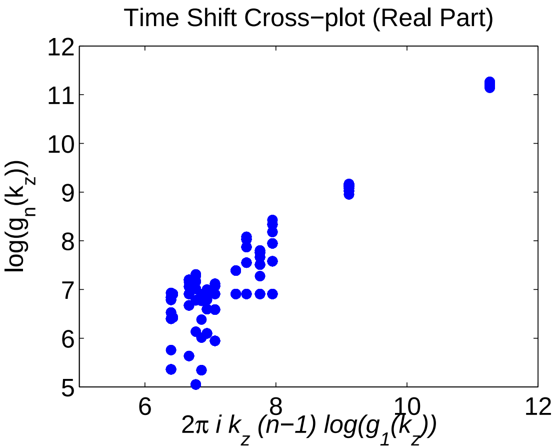
Caveats

- Velocity models are not band limited in z.
- For multiple dipping layers, this is a multiple nonlinear regression problem.

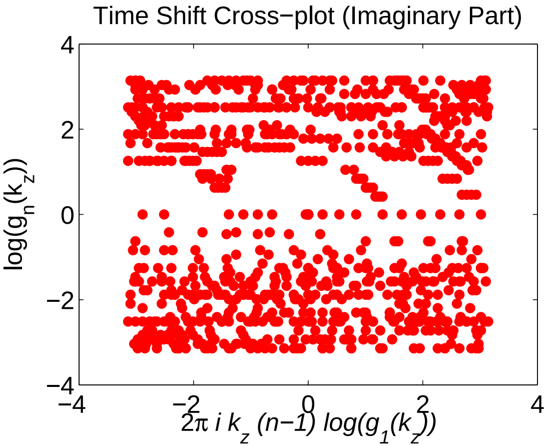
For a single dipping interface, this is a simple linear regression, so we can recover the dip directly by least-squares.

$$\log\left(g_n(f)\right) = \log\left(G_1(k_z)\right) 2\pi i k_z (n-1) d_1$$

Spitz Method



Spitz Method



Results

- Recovered $d_1 = 0.7870$
- Actual dip is -2.3571 (difference of π radians)

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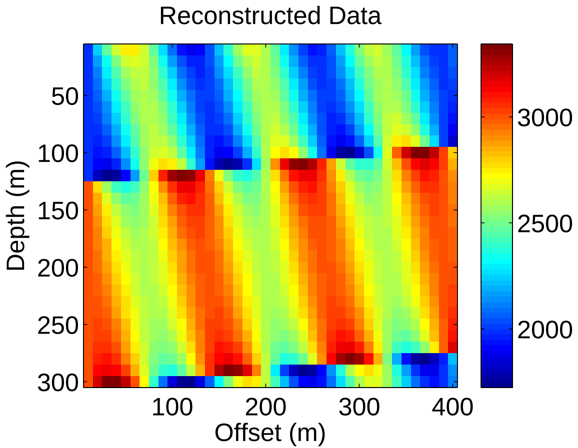
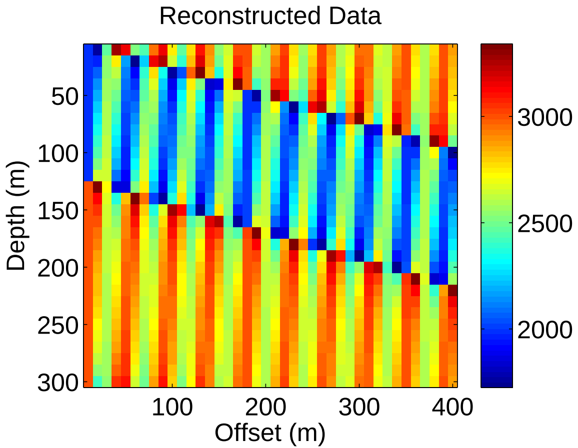
Results



Results


Reconstructed Data
Depth (m)
50
100
150
200
250
300
Offset (m)
100
200
300
400
3000
2500
2000

Conclusions

- Velocity model interpolation is similar to seismic interpolation.
- Nonlinear regression may be able to recover multiple dipping events.

Future work

- Implement seismic interpolation techniques to upsample velocity models for better lateral continuity and gradients.
- Use accurate gradients to perform Kirchhoff migration, to see if we can improve on the results of Moser (2011).

Acknowledgements

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Any Questions?