Investigating the Sharpe Hollow Cavity Model: potential limitations and modeling tests

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March 22nd, 2013

Outline

I A brief introduction and derivation of the Sharpe Hollow Cavity Model

II Exploring the effects of certain variables in the SHCM which include:

- The choice of pressure pulse
- Cavity radius
- Rigidity

III Investigating the frequency content of the data obtained in the Hussar 2011 and Priddis 2012 experiments

IV Comparison of the frequency spectra from both experiments with the predictions of the SHCM

V Potential limitations of the SHCM and future tests

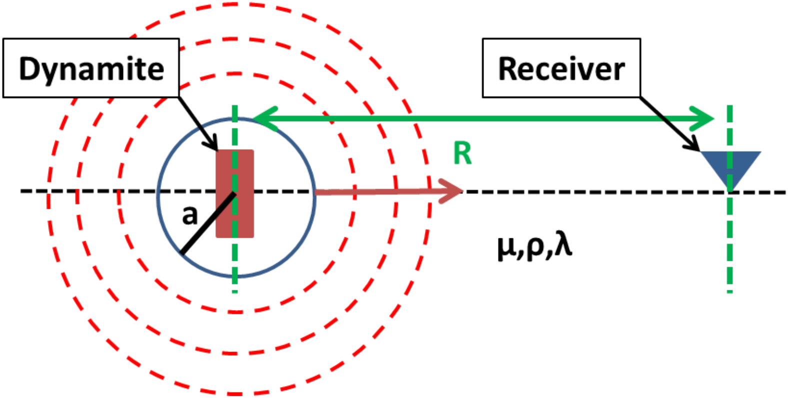
Introduction

- Dynamite is a cheap, commonly used means of elastic wave production in exploration seismology
- Understanding the nature of dynamite explosions and the resulting waves can result in vast improvements in surveys that utilize dynamite
- The nature of propagating waves near the source of a dynamite explosion is a poorly understood phenomenon in Physics due to the nonlinear nature of the subsurface in close proximity to the explosion
- There are numerous mathematical models that have been developed which attempt to account for this nonlinear behavior and predict wave behavior in the far-field
- In this study, we investigate the viability of of the SHCM in predicting the behavior of compressional waves emitted from dynamite

The Sharpe Hollow Cavity Model

Theorem

An explosive pressure source can be modeled by a hollow cavity that is being acted on from the inside by a uniformly distributed pressure pulse.



The Sharpe Hollow Cavity Model

- According to Sharpe, the area inside the cavity represents the region in which emitted waves do not behave linearly
- Compressional waves are assumed to emanate directly from the outside wall of the cavity in a spherical form
- The size of the cavity is directly proportional to the charge size

Derivation of the Sharpe Hollow Cavity Model

By exploiting the spherical symmetry of the proposed problem, the wave equation can be written as:

$$\frac{\partial^2 (r\Phi)}{\partial t^2} = v^2 \frac{\partial^2 (r\Phi)}{\partial r^2}. \tag{1}$$

The solution, Φ, in this case must be both divergent and decreasing with time in order to represent a spherical waveform. Sharpe proposed the following:

$$\Phi = \frac{1}{r} e^{-int} \tag{2}$$

where t is time, and r is the distance from the source.

Using this solution for Φ and solving for the displacement, ie, $u = \partial\Phi/\partial r$, results in the displacement equation:

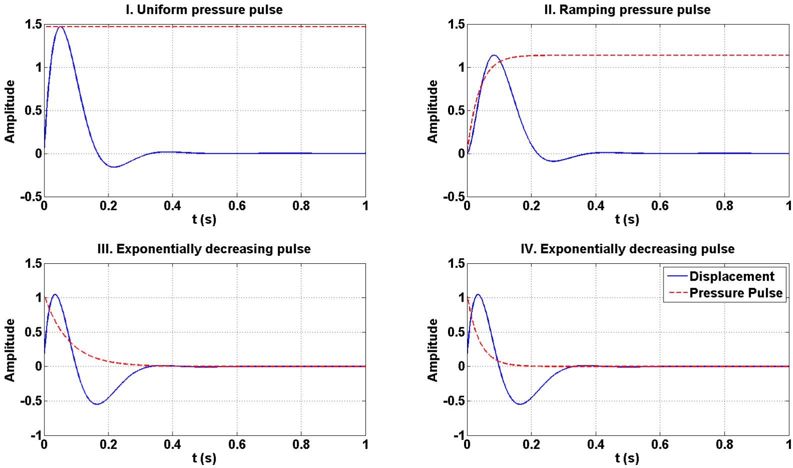
$$u = \frac{a^2 p_o}{2\sqrt{2}\mu r} e^{-\omega t/\sqrt{2}} \sin \omega t, \tag{3}$$

where r is the distance from the center of the source, a is the cavity radius, μ is the medium rigidity, p_o is a uniform pressure pulse, and ω is the angular frequency of the oscillating solution represented by

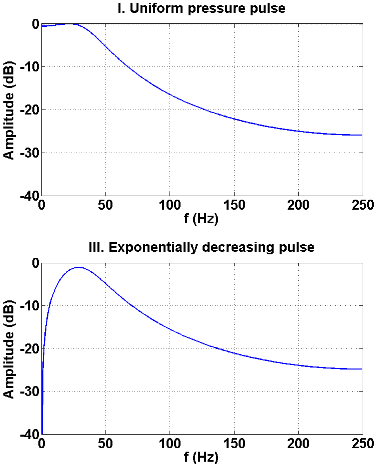
$$\omega = \frac{2\sqrt{2}v}{3a}, \tag{4}$$

where v is the velocity of the p-waves emitted by the source. Assigning different pressure pulses to the cavity can be accomplished via convolution of the desired pressure pulse with that of the displacement shown above.

Pressure pulse forms



Pressure pulse forms



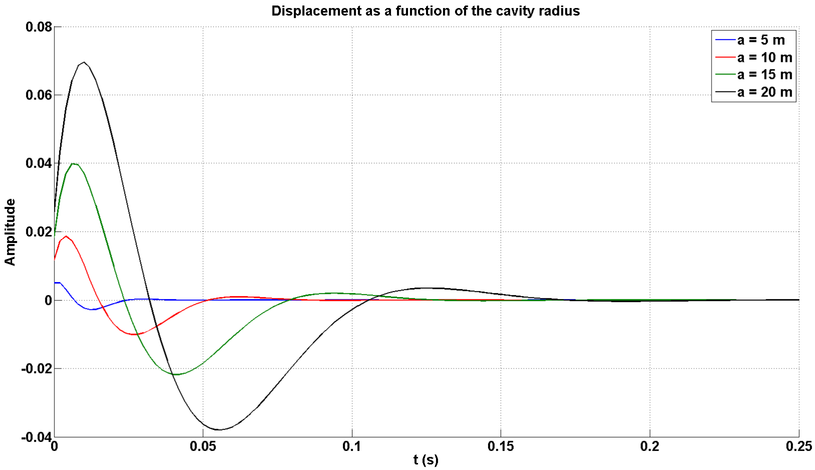
Predictions of the SHCM

$$u = \frac{a^2 p_o}{2\sqrt{2}\mu r} e^{-\omega t/\sqrt{2}} \sin \omega t, \tag{5}$$

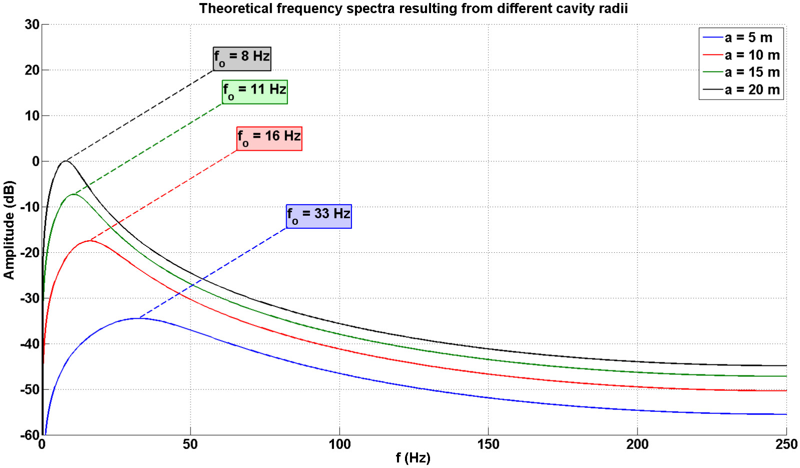
$$\omega = \frac{2\sqrt{2}v}{3a}, \tag{6}$$

- Amplitude response should INCREASE with larger charge sizes
- Dominant frequency should DECREASE with increased charge size
- Amplitude should DECREASE with increased rigidity
- A low frequency roll off should be present if in fact a decreasing exponential best represents a dynamite explosion

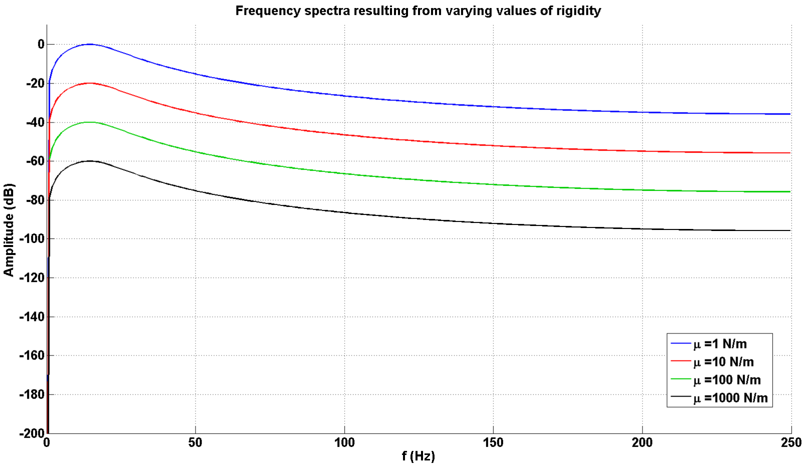
The effect of cavity radius on displacement



The effect of cavity radius on the frequency spectra



The effect of medium rigidity on the frequency spectra



Charge size and cavity radius

In previous studies, we found that charge size is related to cavity radius via a cubic relationship such that

$$m = ca^3 \tag{7}$$

where m is the charge size in kilograms, a is the cavity radius in meters, and c is the constant in kilograms per meter cubed that links them. If we assume the dominant frequency is given by

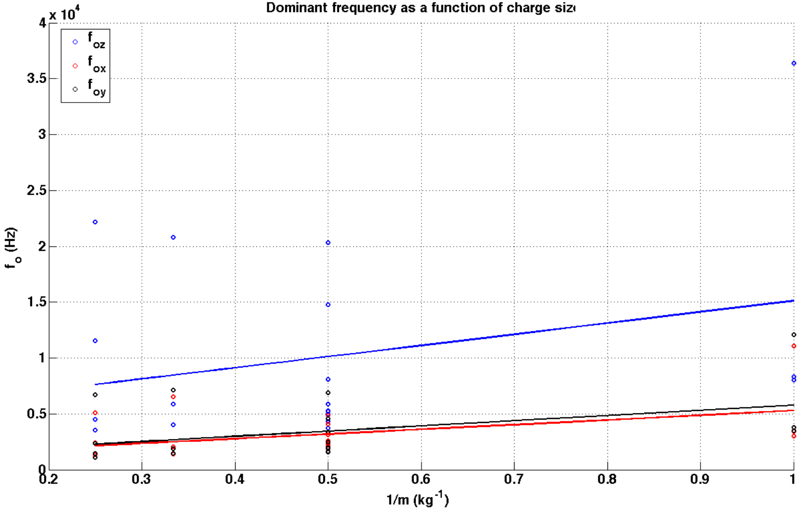
$$f_o = \frac{\omega}{2\pi} = \frac{\sqrt{2}v}{3a} \tag{8}$$

Then we can use Equation 7 with Equation 8 to obtain the following:

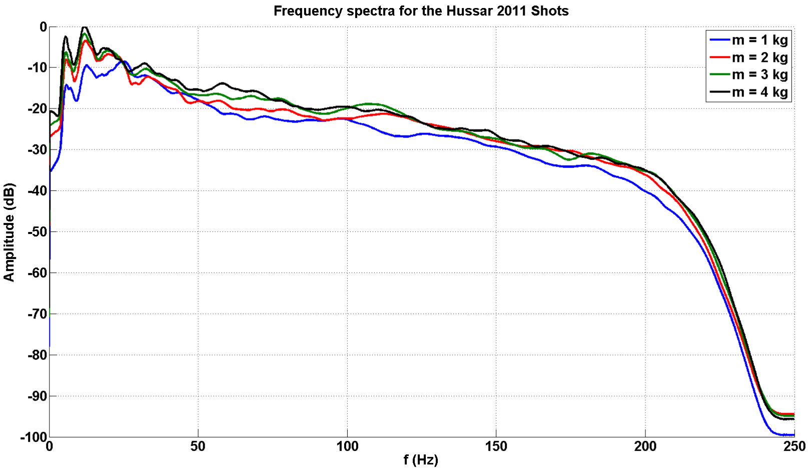
$$f_o{}^3 = c\left(\frac{\sqrt{2}v}{3}\right)^3 \frac{1}{m} \tag{9}$$

where v is the wave velocity in meters per second. From this, we should be able to estimate c from our data.

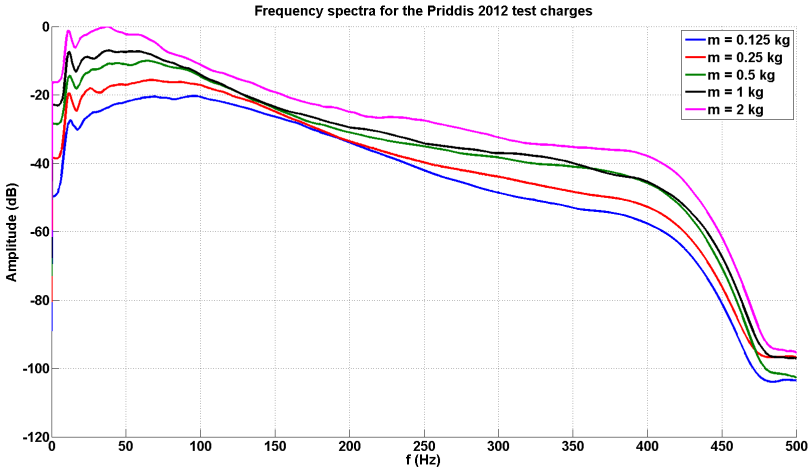
Estimating the value of c in Hussar



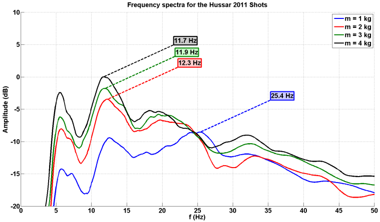
Data from the Hussar 2011 experiment

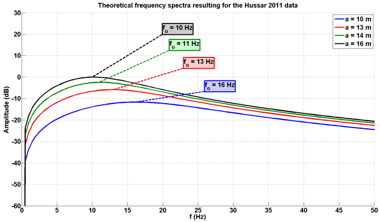


Data from the Priddis 2012 experiment



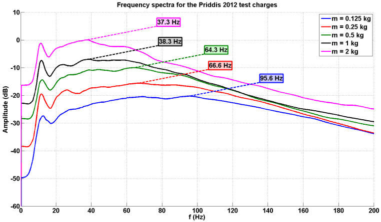
Hussar comparison with the SHCM

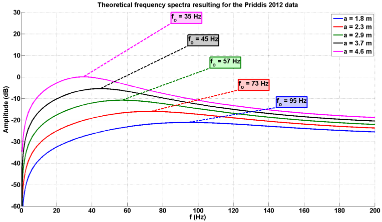




$c = 0.001\ kg/m^3$				
	Measured Data		*Theoretical*	
m (kg)	**fo (Hz)**	**A (dB)**	**fo (Hz)**	**A (dB)**
1	25.4	-8	16	-11
2	12.3	-4	13	-7
3	11.9	-3	11	-3
4	11.7	0	10	0

Priddis comparison with the SHCM





$c = 0.015\ kg/m^3$				
	Measured Data		*Theoretical*	
m (kg)	**fo (Hz)**	**A (dB)**	**fo (Hz)**	**A (dB)**
0.125	95.6	-20	95	-20
0.25	66.6	-17	73	-17
0.5	64.3	-10	57	-10
1	38.3	-8	43	-7
2	37.3	0	35	0

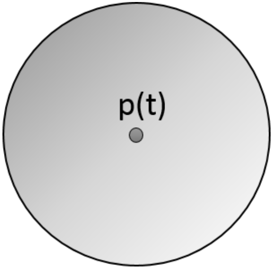
Assumptions in the SHCM

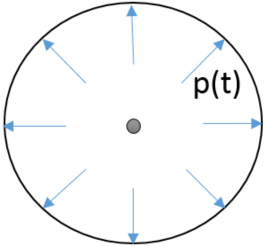
The SHCM makes several assumptions which could lead to potential problems when trying to develop an accurate model. These include:

- The medium is perfectly elastic, ie, the Lame parameters are equal to one another
- Only compressional waves are emitted from the surface of the cavity
- Pressure is uniformly distributed over the interior of the cavity, and thus the response to the applied pressure is the same across the surface of the cavity

These assumptions are extremely inaccurate in situations commonly present in real surveys, it would therefore be worthwhile to develop a method for correcting for these assumptions.

The Sharpe Approximation

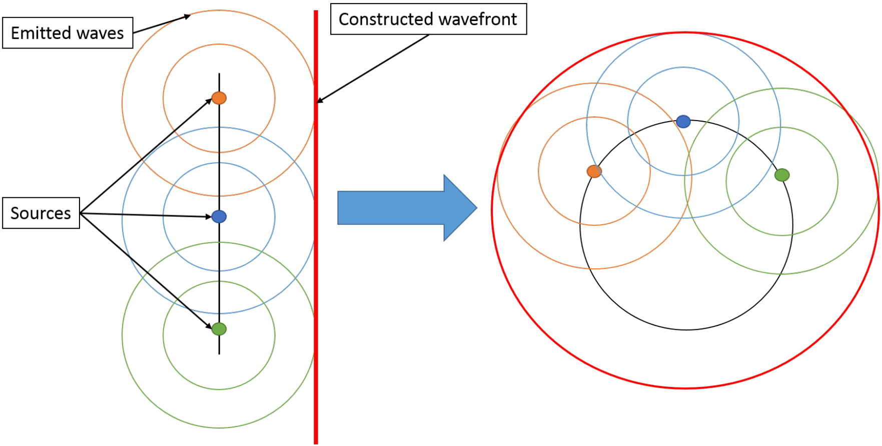




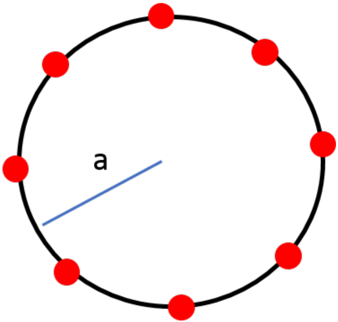
Waves in the shaded region do not behave elastically, p(t) originates from the source.

Shaded region is replaced by a hollow cavity, where p(t) acts uniformly over the inside.

Constructing Wavefronts

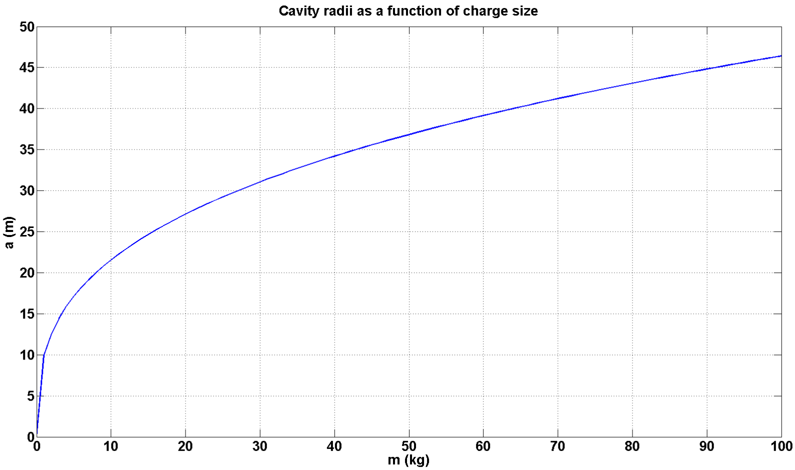


Circle Array

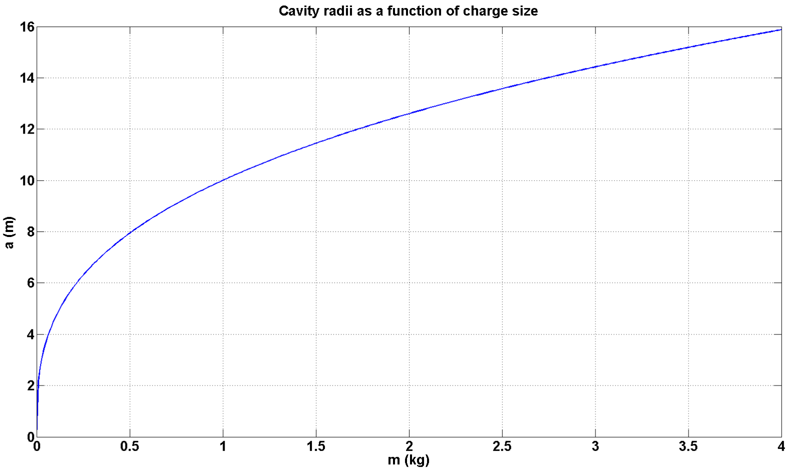


- Create arbitrary wave fronts by controlling the source wavelet
- This gives us more control over the pulse emitted from the surface of the cavity
- May allow us to account for subsurface variability and some of the inaccuracies in the model presently

Charge Size and Cavity Radius



Charge Size and Cavity Radius



Conclusions

- The SHCM is a viable model for predicting that nature of waves emitted from a dynamite explosion
- We need to determine a more accurate link between charge size and cavity radius to move forward with this model
- We may be able to improve on this model using numerical rather than analytical techniques since we're not constrained by certain assumptions solving this problem numerically

Acknowledgements

- I would like to thank this time to thank all of the CREWES sponsors for their contributions to our project: without you nothing here would be possible
- I would also like to thank my supervisor Dr. Gary Margrave for his continued guidance and supervision
- I would also like to thank Kevin Hall, Helen Isaac, and Malcolm Bertram for their assistance with the data analysis
- I would also like to thank Peter Manning for his assistance with this project
- I would finally like to acknowledge all of the CREWES staff and students