A review of fracture models for azimuthal anisotropy studies

by

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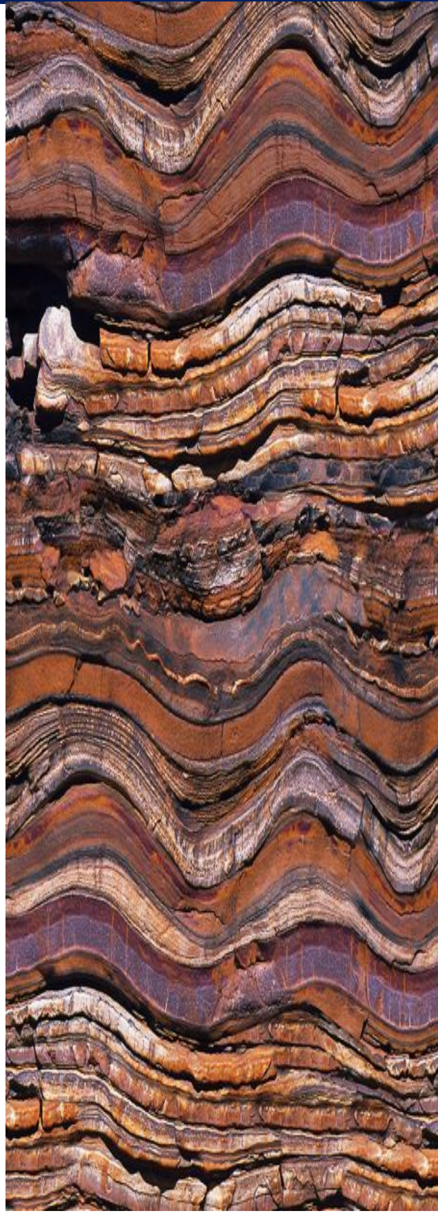


Introduction

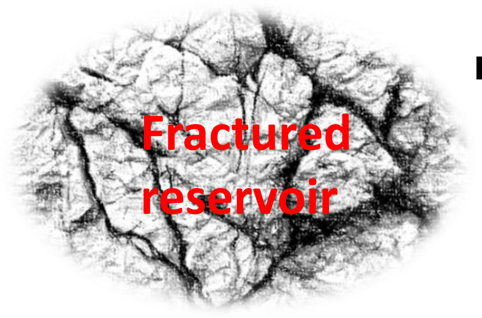
- Fractures are everywhere
- Unlike faults, their sub-seismic length makes it even more difficult to image directly
- This creates a need to develop effective media theories for characterizing reservoir fractures
- The increasing reliance on effective medium theories begs the need for understanding the validity, the limit of applicability and assessment of their usefulness for reservoir fracture studies
- With this in mind, We will compare two popular seismological theories of Hudson and Schoenberg

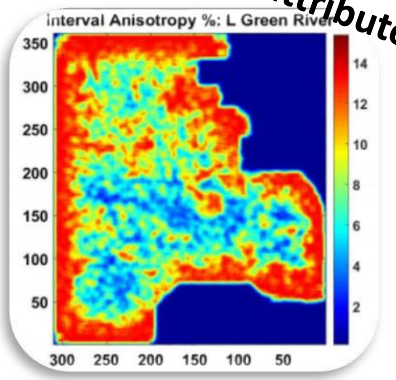
Definition of parameters

Rock Physics ⟷ Seismic Method

fracture properties- α *and* e

α, aspect ratio (crack shape)
e , crack density (0<e<0.2)

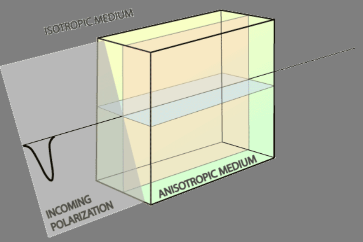




- ✓ eg. azimuth-dependent NMO velocity
- ✓ anisotropic AVO gradient
- ✓ Interval velocity and traveltime delays
- ✓ Fracture Orientation

- ✓ fracture properties- Δ_N and Δ_T
- ✓ $0 < \Delta_N$ and $\Delta_T < 1$
- ✓ Δ_N and Δ_T=0; no fracture
- ✓ Δ_N and Δ_T=1; extreme fracturing

Thomsen's anisotropy
ε^{v} , $\delta^{(v)}$, $\gamma^{(v)}$
and $\eta^{(v)}$



Fracture models

- They are based on continuum hypothesis ($\lambda \gg d$)

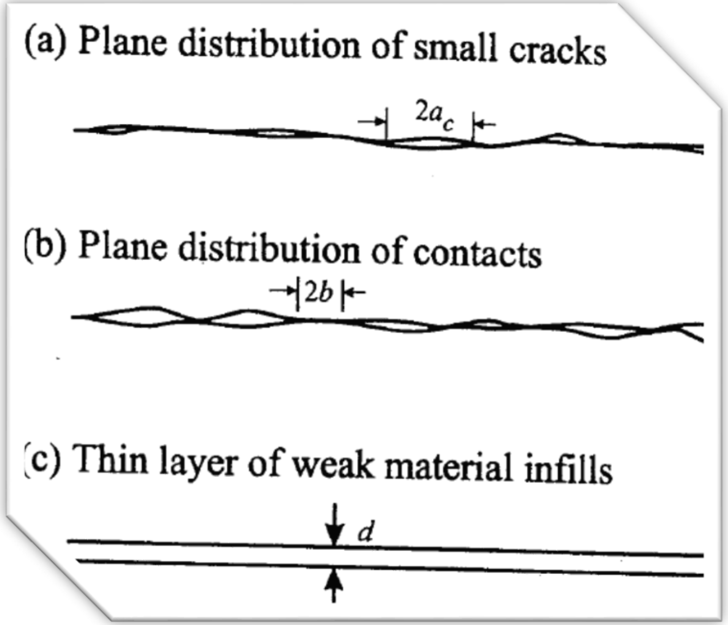
continuum theory

$(\boldsymbol{\lambda} \gg \boldsymbol{d})$

Inhomogeneous medium

homogeneous medium

Justification: wave equation is simplified and seismic wavelength (λ) is much greater than the scale of material (d) under probe



Representations of fracture models (Liu et. al., 2000)

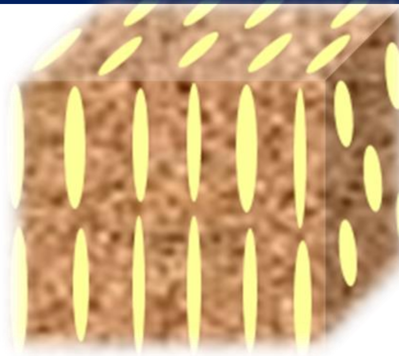
- ✓ *Hudson's microcrack model and*
- ✓ *Schoenberg's parallel fracture model.*
- ✓ *Self-Consistent model*
- ✓ *Kuster-Toksoz's model*
- ✓ *Differential Effective model*

Hudson microcrack theory

$$c_e = c^{(0)} + ec^{(1)} + e^2c^{(2)} + O(e^3)$$

- c_e: *Effective stiffness*
- $c^{(0)}$: *Isotropic stiffness tensor of host rock*
- $ec^{(1)}$: *1st order perturbation describe single scattering (isolated cracks)*
- $e^2c^{(2)}$: *2nd order perturbation accounts for crack-crack interactions*

The linear term dominates at sufficiently small e



$$c^{(1)} = -\frac{1}{\mu} * \begin{bmatrix} \lambda_b(\lambda_b+2\mu_b)^2U_{33} & \lambda_b(\lambda_b+2\mu_b)U_{33} & \lambda_b(\lambda_b+2\mu_b)U_{33} & 0 & 0 & 0 \\ \lambda_b(\lambda_b+2\mu_b)U_{33} & {\lambda_b}^2U_{33} & {\lambda_b}^2U_{33} & 0 & 0 & 0 \\ \lambda_b(\lambda_b+2\mu_b)U_{33} & {\lambda_b}^2U_{33} & {\lambda_b}^2U_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & {\mu_b}^2U_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & {\mu_b}^2U_{11} \end{bmatrix}$$

- e is crack density
- λ_b and μ_b are the lame parameters of the host rock

$$c^{(2)} = \frac{q}{15} * \begin{bmatrix} X\,{U_{33}}^2 & \lambda_b{U_{33}}^2 & \lambda_b e{U_{33}}^2 & 0 & 0 & 0 \\ \lambda_b{U_{33}}^2 & Me{U_{33}}^2 & Me{U_{33}}^2 & 0 & 0 & 0 \\ \lambda_b{U_{33}}^2 & M{U_{33}}^2 & M{U_{33}}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E{U_{11}}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & E{U_{11}}^2 \end{bmatrix}$$

- U_{11} and U_{33} are the dimensionless quantities that depends on the BC's of the crack face, infill material and crack direction
- q,X,M and E depend of λ_b and μ_b (not shown)

Schoenberg’s parallel fracture model

$$\boldsymbol{s} = \boldsymbol{s_b} + \boldsymbol{s_f}$$

$$[\sigma_{11}] = [\sigma_{22}] = [\sigma_{33}] = 0$$

$$[u_1] = h(K_N\sigma_{11} + K_{NH}\sigma_{12} + K_{NV}\sigma_{13})$$
$$[u_2] = h(K_{NH}\sigma_{11} + K_H\sigma_{12} + K_{VH}\sigma_{13})$$
$$[u_3] = h(K_{NV}\sigma_{11} + K_{VH}\sigma_{12} + K_V\sigma_{13}),$$

$$s_f = \begin{bmatrix} K_N & 0 & 0 & 0 & K_N & K_N \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ K_{NV} & 0 & 0 & 0 & K_V & K_{VH} \\ K_{NH} & 0 & 0 & 0 & K_{VH} & K_H \end{bmatrix}$$

- A special case for rotationally invariant fractures

$$K_{NV} = K_{NH} = K_{VH} = 0,$$
$$K_V = K_H$$

As a result, the fracture compliances matrix s_f reduces to

$$s_f = \begin{bmatrix} K_N & 0 & 0 & 0 & K_N & K_N \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_T & 0 \\ 0 & 0 & 0 & 0 & 0 & K_T \end{bmatrix}$$

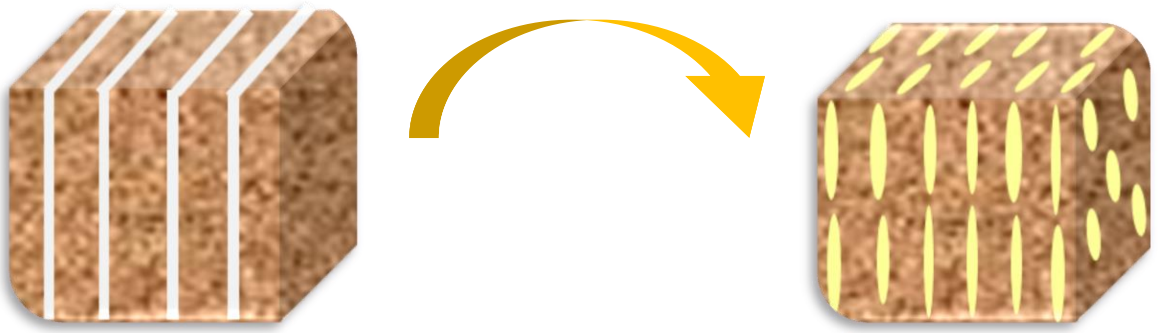
$$\boldsymbol{c} = \boldsymbol{s}^{-1} = \boldsymbol{c_b} + \boldsymbol{c_f}$$

$$\boldsymbol{c_{fhti}} = -\frac{1}{\mu} * \begin{bmatrix} M\Delta_N & \lambda_b\Delta_N & \lambda_b\Delta_N & 0 & 0 & 0 \\ \lambda_b\Delta_N & \lambda_b^2\Delta_N/M & \lambda_b^2\Delta_N/M & 0 & 0 & 0 \\ \lambda_b\Delta_N & \lambda_b^2\Delta_N/M & \lambda_b^2\Delta_N/M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_b\Delta_T & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_b\Delta_T \end{bmatrix}$$

$$\boldsymbol{\Delta_N} = {}^{MK_N}/_{1+MK_N},$$
$$\boldsymbol{\Delta_T} = {}^{\mu K_N}/_{1+\mu K_N}$$
$$\boldsymbol{K_N} = \boldsymbol{s_{f,11}}$$
$$\boldsymbol{K_T} = \boldsymbol{s_{f,55}} = \boldsymbol{s_{f,66}}$$

- ✓ $0 \le \Delta_N$ and $\Delta_T \le 1$
- ✓ Δ_N and Δ_T = 0; no fracturing
- ✓ Δ_N and Δ_T = 1; high degree of fracturing
- ✓ $c_{f,44}$ is not influenced by presence of fracture

Comparison of Hudson and Schoenberg



Schoenberg and Douma (1988) pointed out that the effective stiffnesses of both Schoenberg's LST theory and Hudson's model have the same structure and become identical if the fracture weaknesses satisfy the following relations:

$$\Delta_N = {}^{(\lambda_b+2\mu_b)U_{33}e}/_{\mu_b},$$

$$\Delta_T = U_{11}e$$

$$\Delta_N = \frac{4e}{3g(1-g)\left[1+\frac{1}{\pi g(1-g)}\left(\frac{K_f+4/3\mu_f}{\mu_b}\right)\left(\frac{a}{c}\right)\right]}$$

$$\Delta_T = \frac{16e}{3(3-2g)\left[1+\frac{1}{\pi(3-2g)}\left(\frac{\mu_f}{\mu_b}\right)\left(\frac{a}{c}\right)\right]}$$

$$g = ({}^{Vs}/_{Vp})^2$$

For dry cracks, K_f and μ_f=0

$$\Delta_N = \frac{4e}{3g(1-g)}$$

$$\Delta_T = \frac{16e}{3(3-2g)}$$

If the cracks are filled with fluid, μ_f=0, but $K_f \neq 0$,

$$\Delta_N = 0,$$

$$\Delta_T = \frac{16e}{3(3-2g)}$$

$$\epsilon^{(V)} = -2g(1-g)\Delta_N,$$

$$\delta^{(V)} = -2g[(1-2g)\Delta_N + \Delta_T],$$

$$\gamma^{(V)} = -\frac{\Delta_T}{2},$$

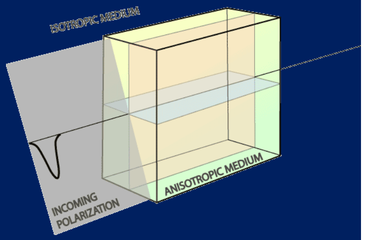
$$\eta^{(V)} = 2g(\Delta_T - g\Delta_N).$$

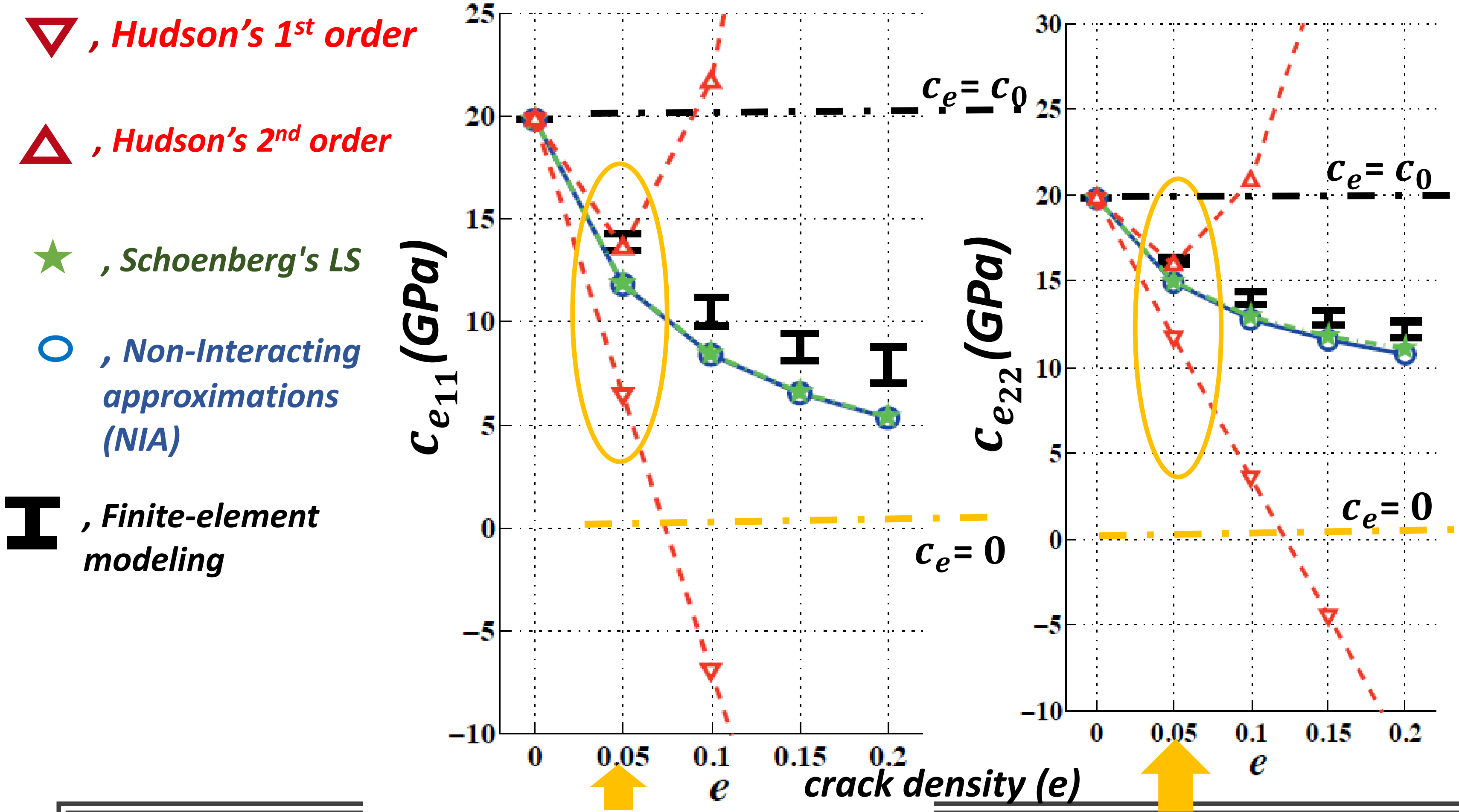
$$\epsilon^{(V)} = -\frac{8}{3}e,$$

$$\delta^{(V)} = -\frac{8}{3}e\left[1+\frac{g(1-2g)}{(3-2g)(1-g)}\right],$$

$$\gamma^{(V)} = -\frac{8e}{3(3-2g)},$$

$$\eta^{(V)} = \frac{8}{3}e\left[\frac{g(1-2g)}{(3-2g)(1-g)}\right].$$



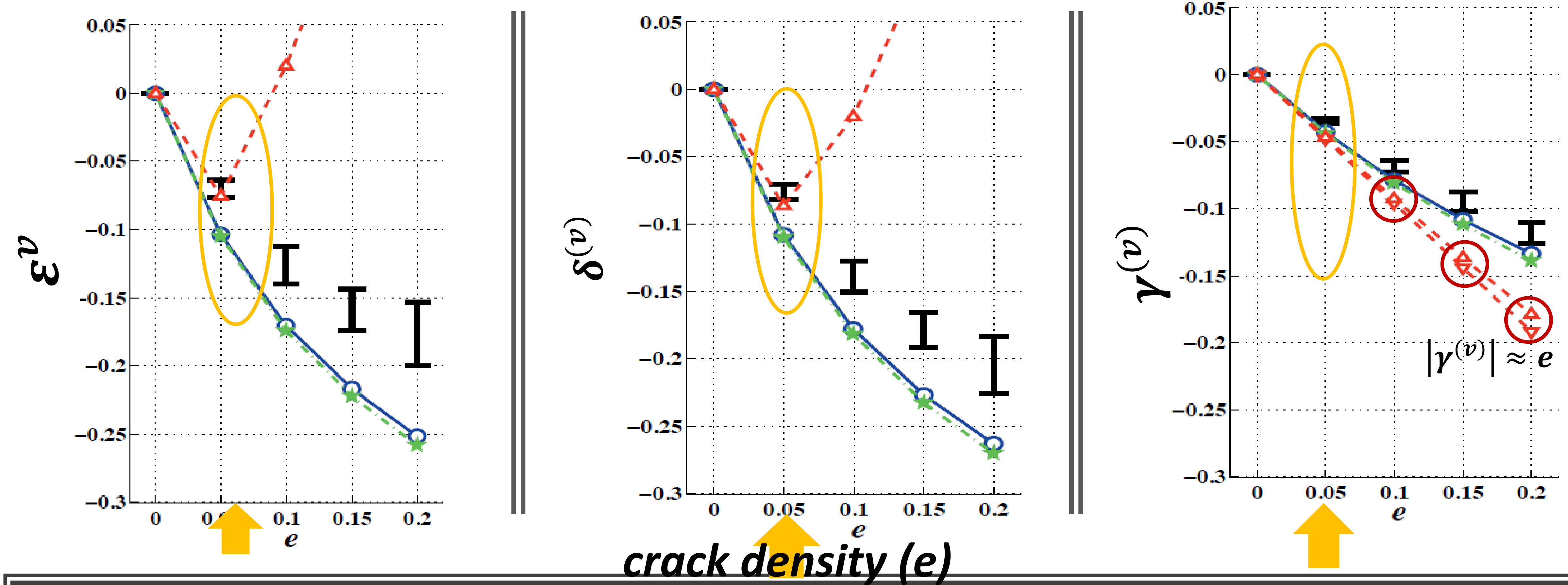


Hudson's theory problematic for large Poisson values(Vs/Vp very small) when $c_{e_{11}}$ *and* $c_{e_{22}}$ *< 0, Physically Implausible as this violates elasticity stability condition*

The quadratic term in 2nd order Hudson yields positive coefficients of fracture stiffness which makes $c_{e_{11}}$ *and* $c_{e_{22}}$ *to begin to increase at some value of crack density exhibiting unphysical behavior*

Schoenberg result has close alliance with NIA and numerical modeling

Critiquing Hudson's theory (Grechka and Kachanov, 2006b)

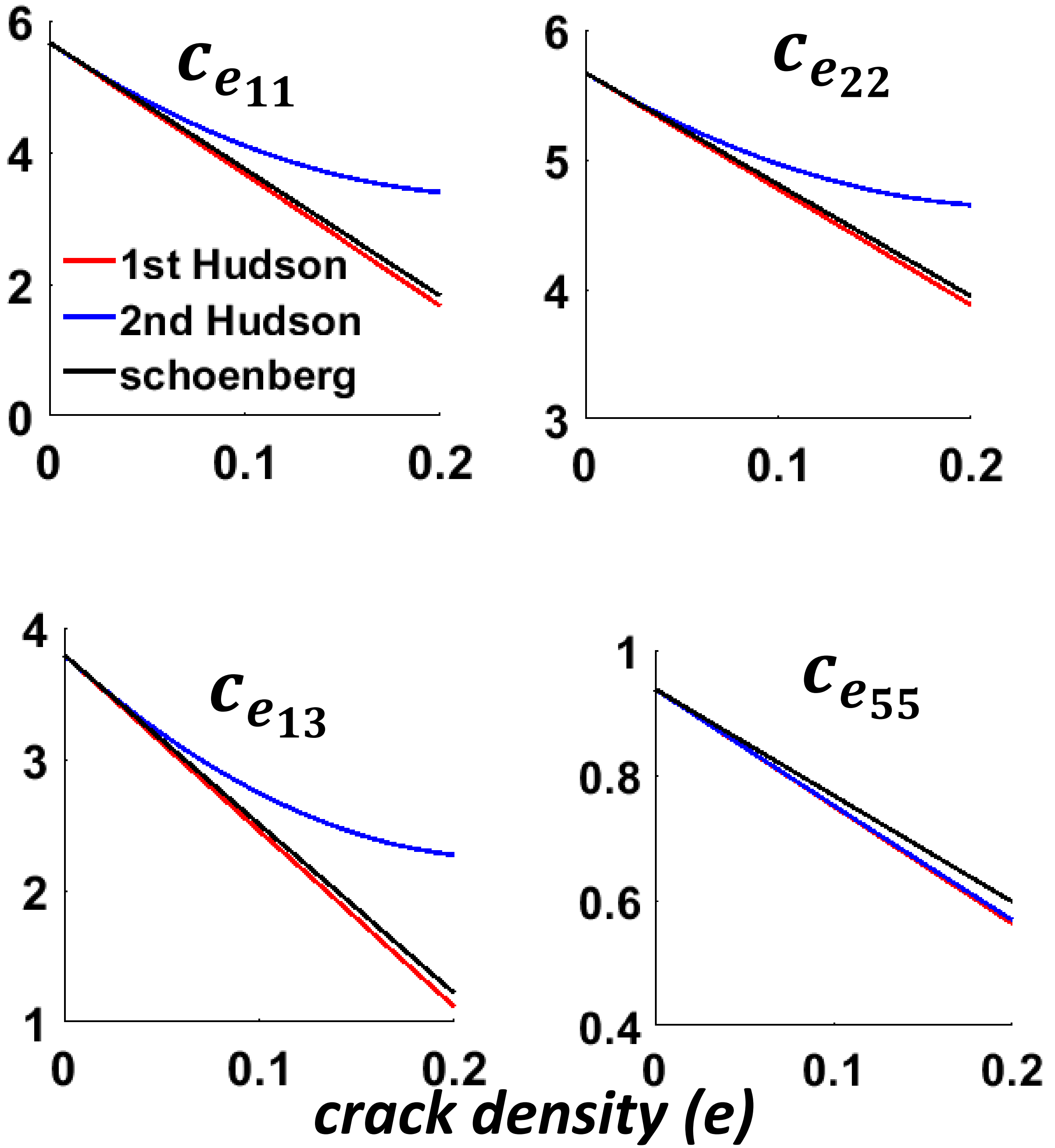


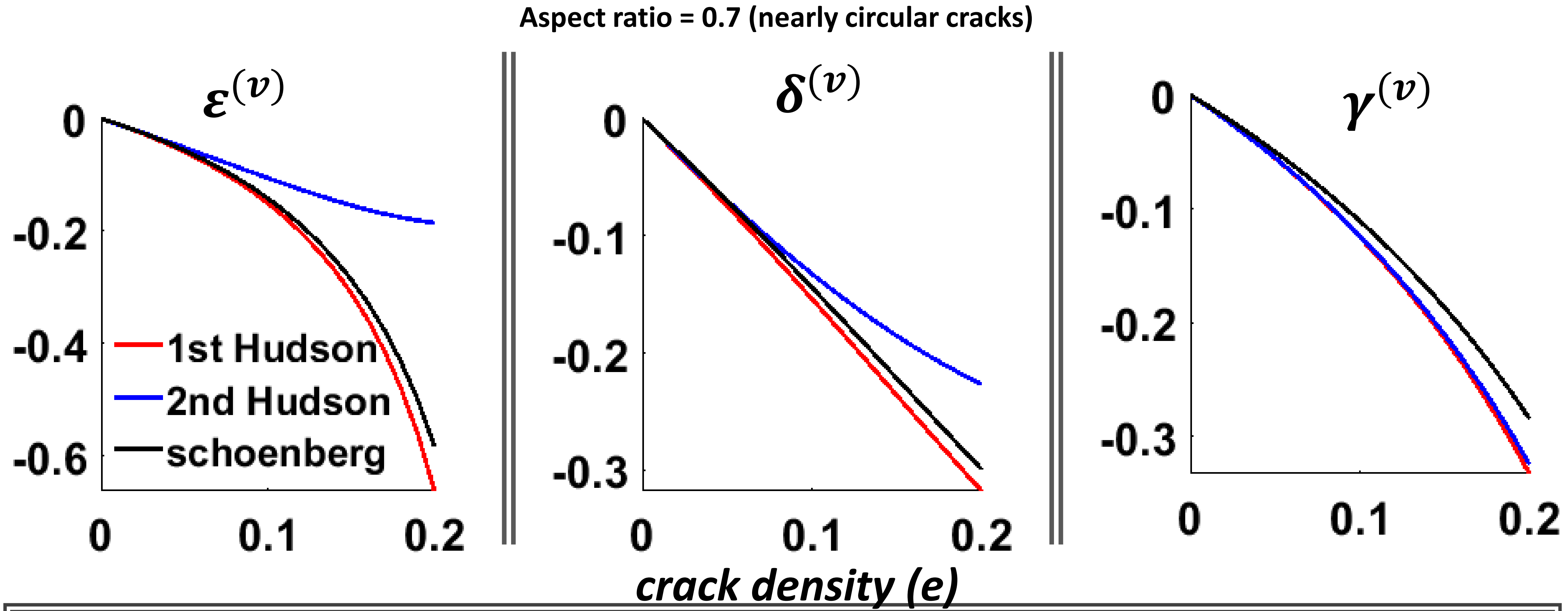
Comparison of Hudson and Schoenberg (Grechka and Kachanov, 2006b)

Sensitivity of stiffness parameters to crack density

Aspect ratio = 0.7 (circular cracks)

Aspect ratio = 0.7 (nearly circular cracks)

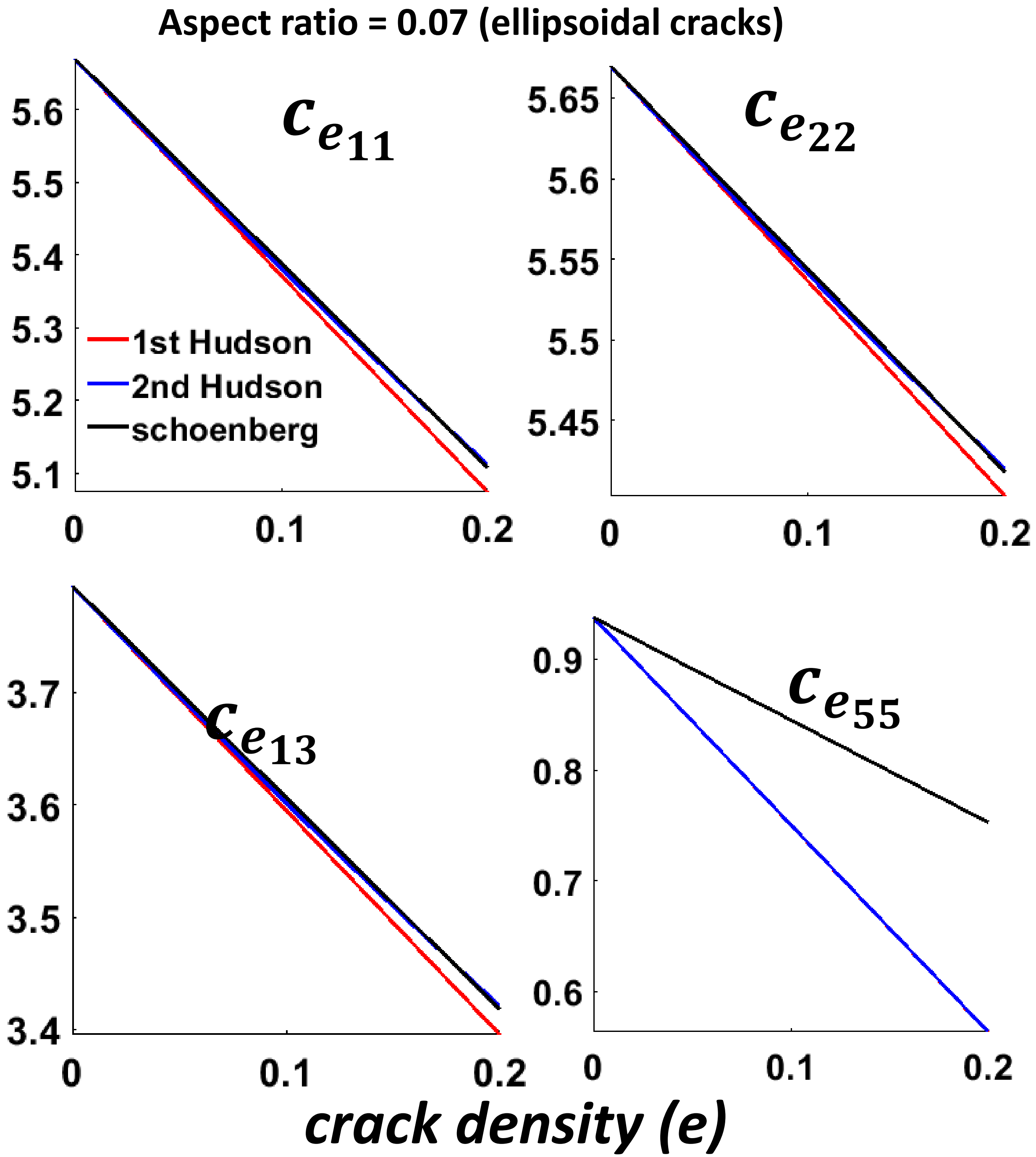




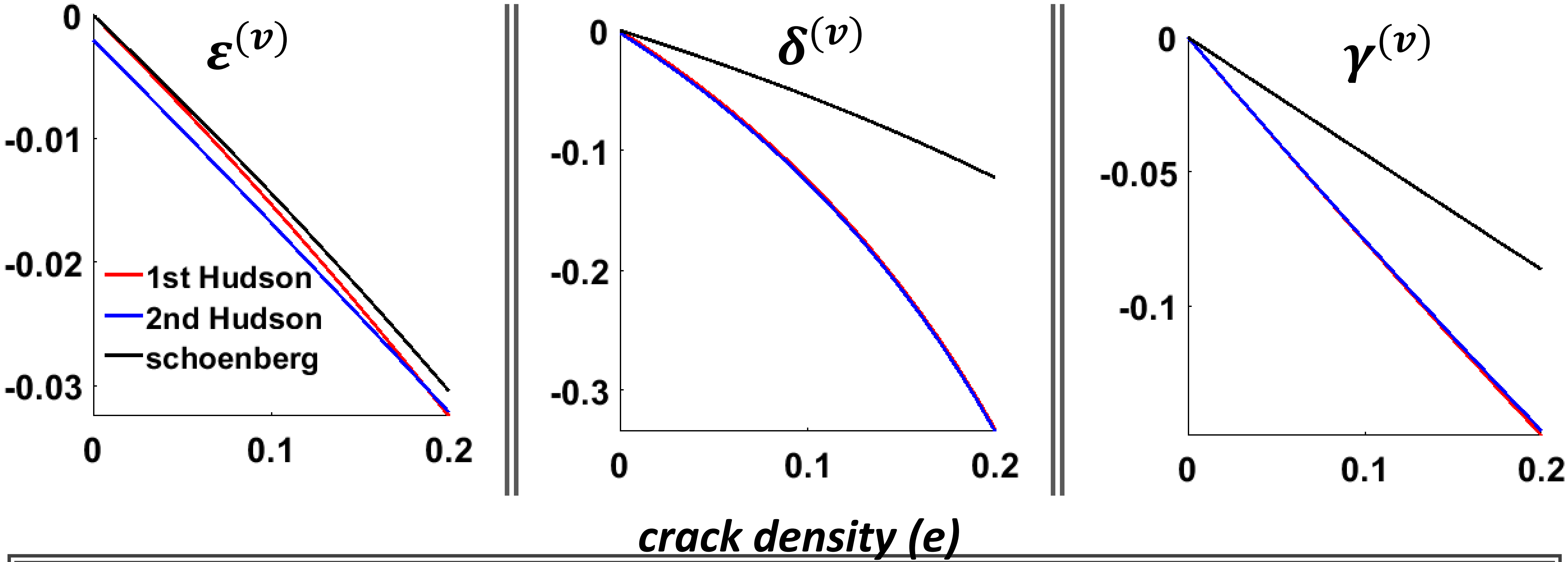
Sensitivity of anisotropy parameters to crack density

Sensitivity of stiffness parameters to crack density

Aspect ratio = 0.07 (ellipsoidal cracks)

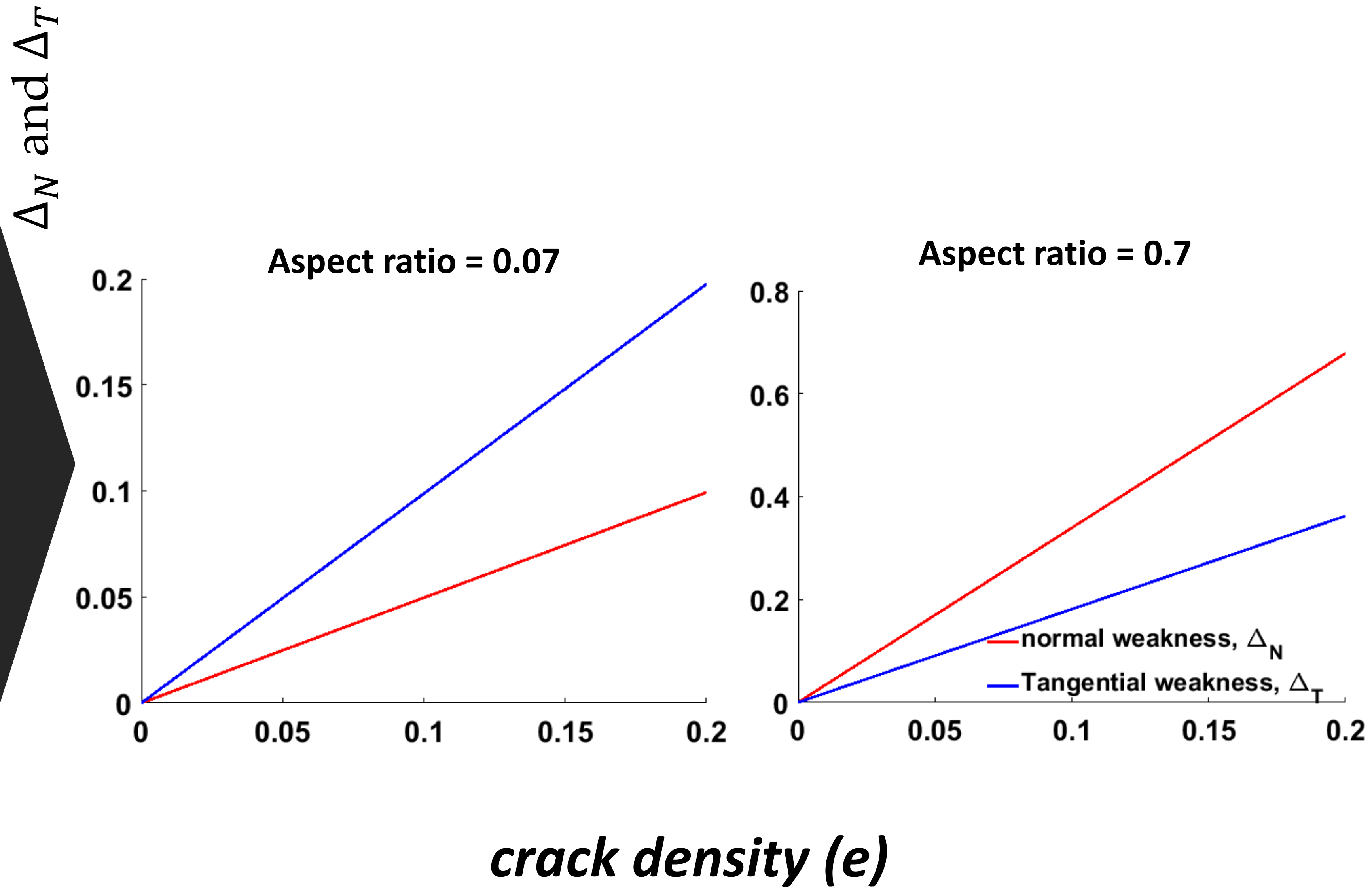


Aspect ratio = 0.07 (ellipsoidal cracks)

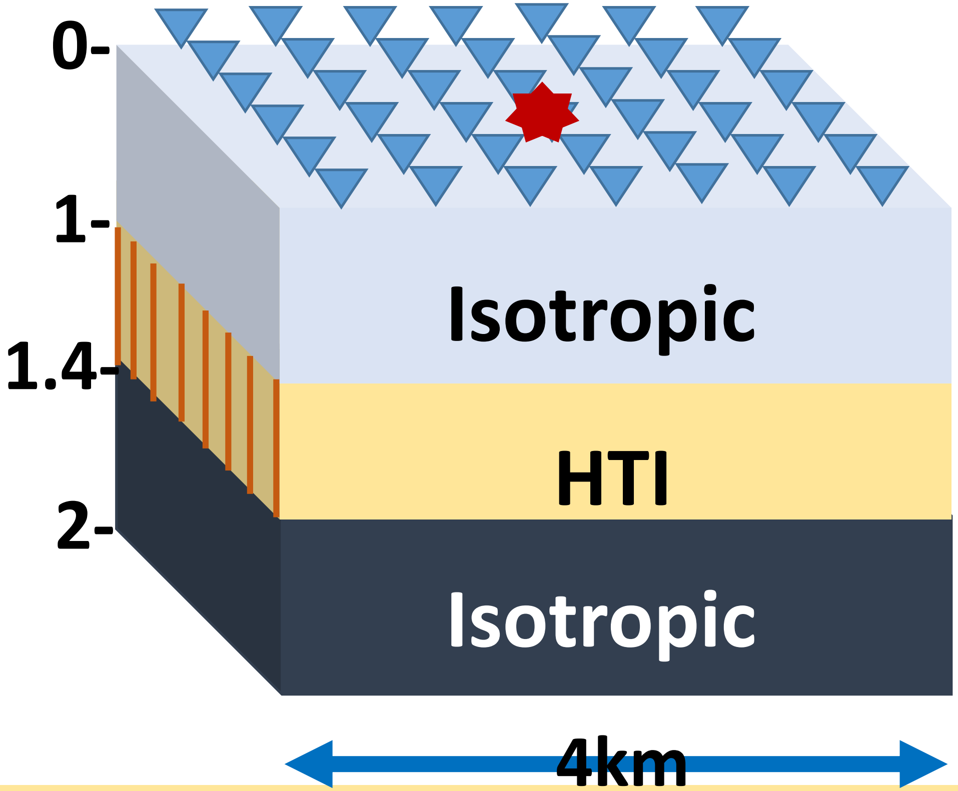


Sensitivity of anisotropy parameters to crack density

Sensitivity of fracture weaknesses to crack density



FD modeling



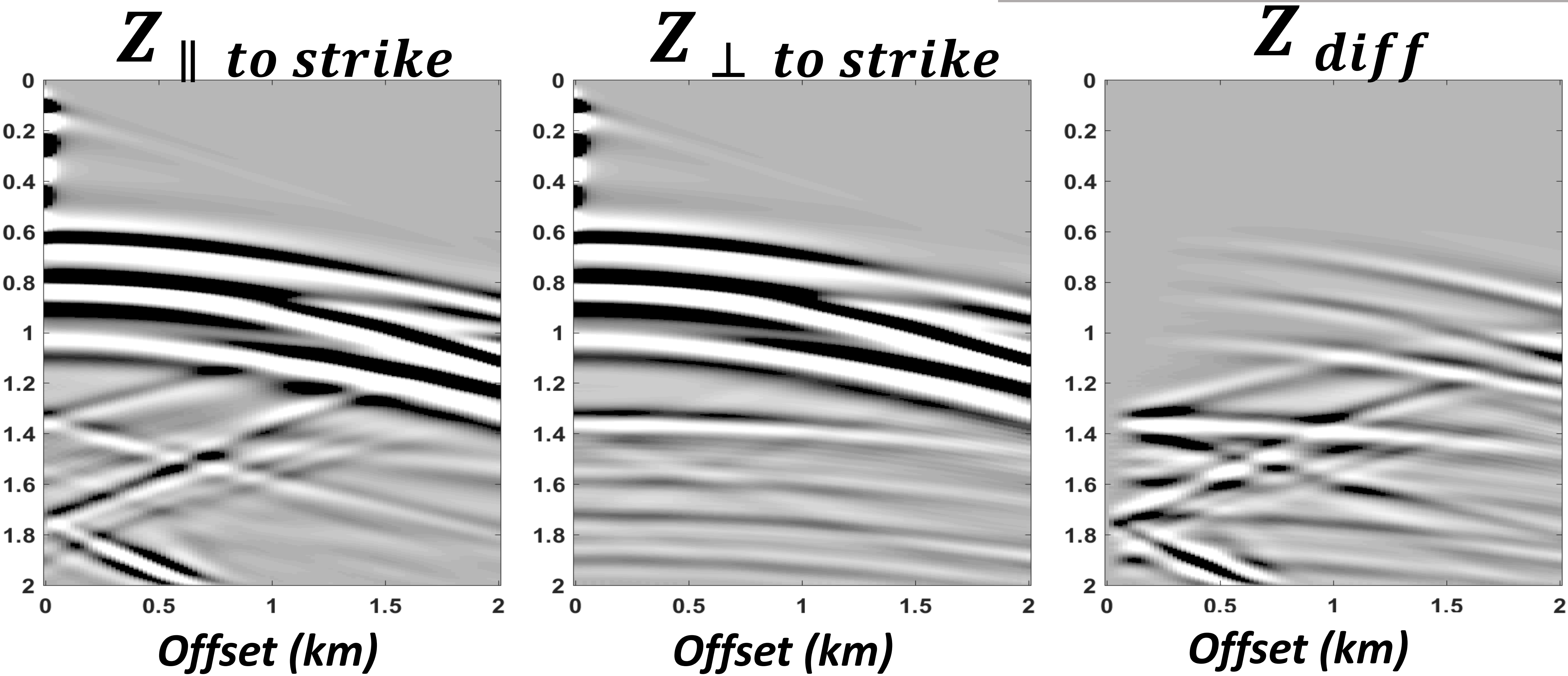
	Homogeneous equivalent model from Schoenberg and Muir (1989) theory
Layer 1	$Vp = 3500, Vs = 2140, \quad \rho = 2200$
HTI Layer	$Vp_0 = 4438,$ $Vs_0 = 2746,$ $\rho^e = 2401,$ $\epsilon^e = -.0034,$ $\gamma^e = -.0607,$ $\delta^e = -.0545$
Layer 3	$Vp = 5000, Vs = 3300, \quad \rho = 2900$

Acquisition

- 3D-3C acquisition WAZ
- Orthogonal design
- Finite difference
- Explosive P source.
- 40m source & receiver depth
- Source frequency is 15hz

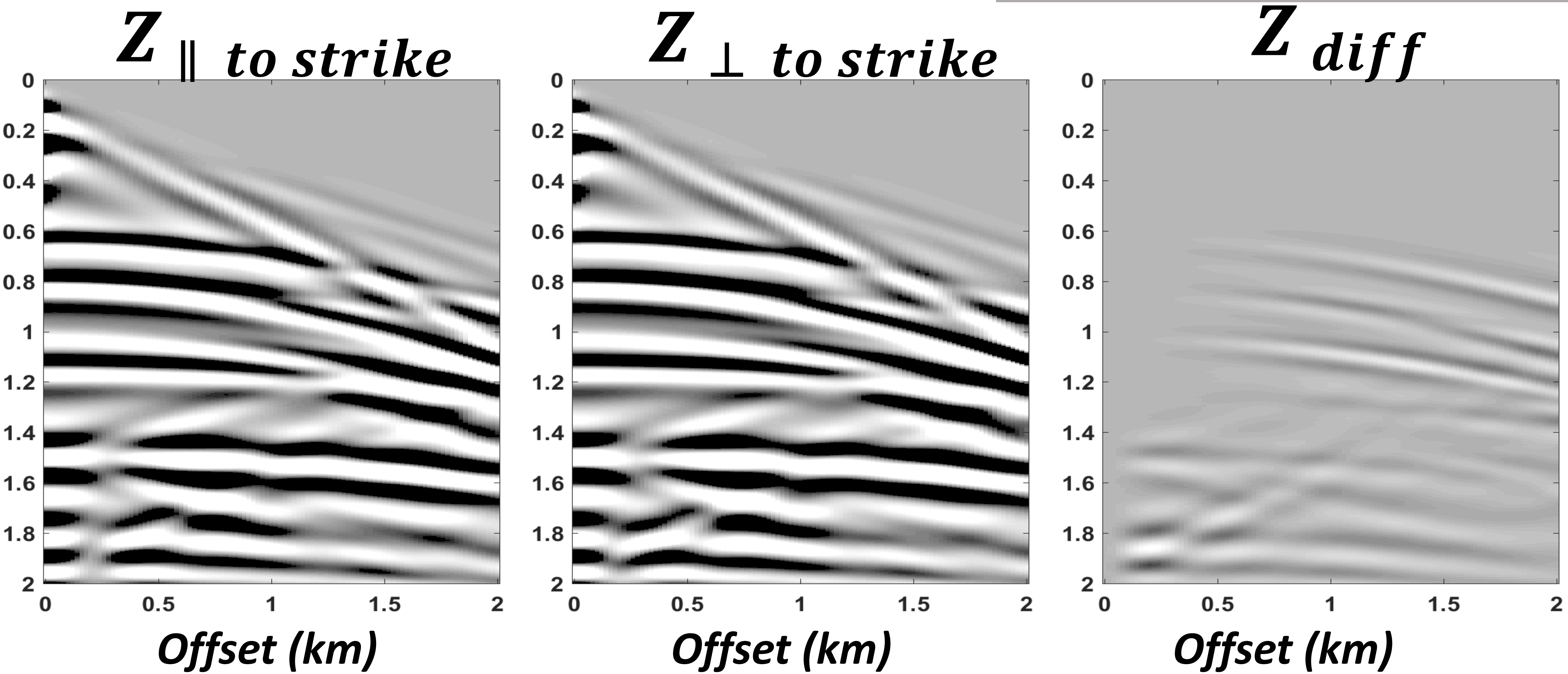
Example: FD modeling

Elastic modeling



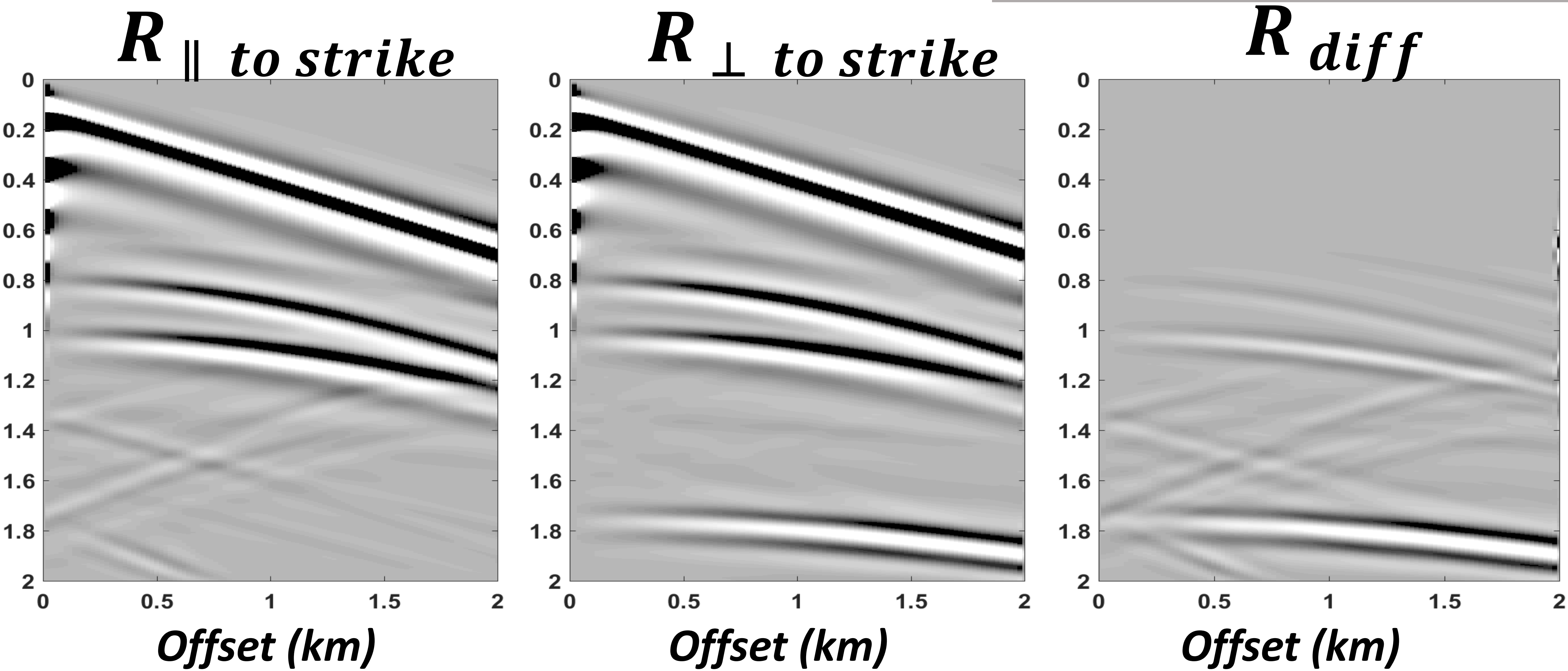
Example: FD modeling

Equivalent modeling



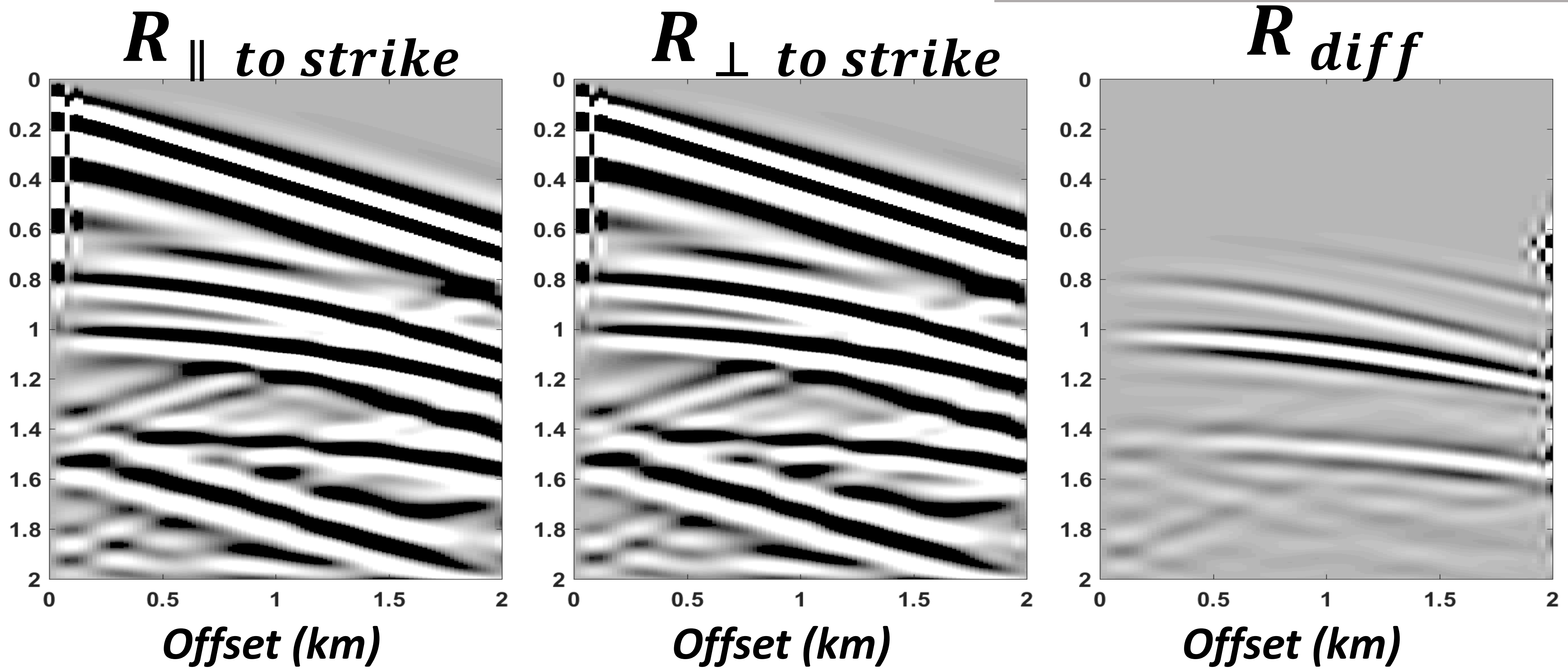
Example: FD modeling

Elastic modeling



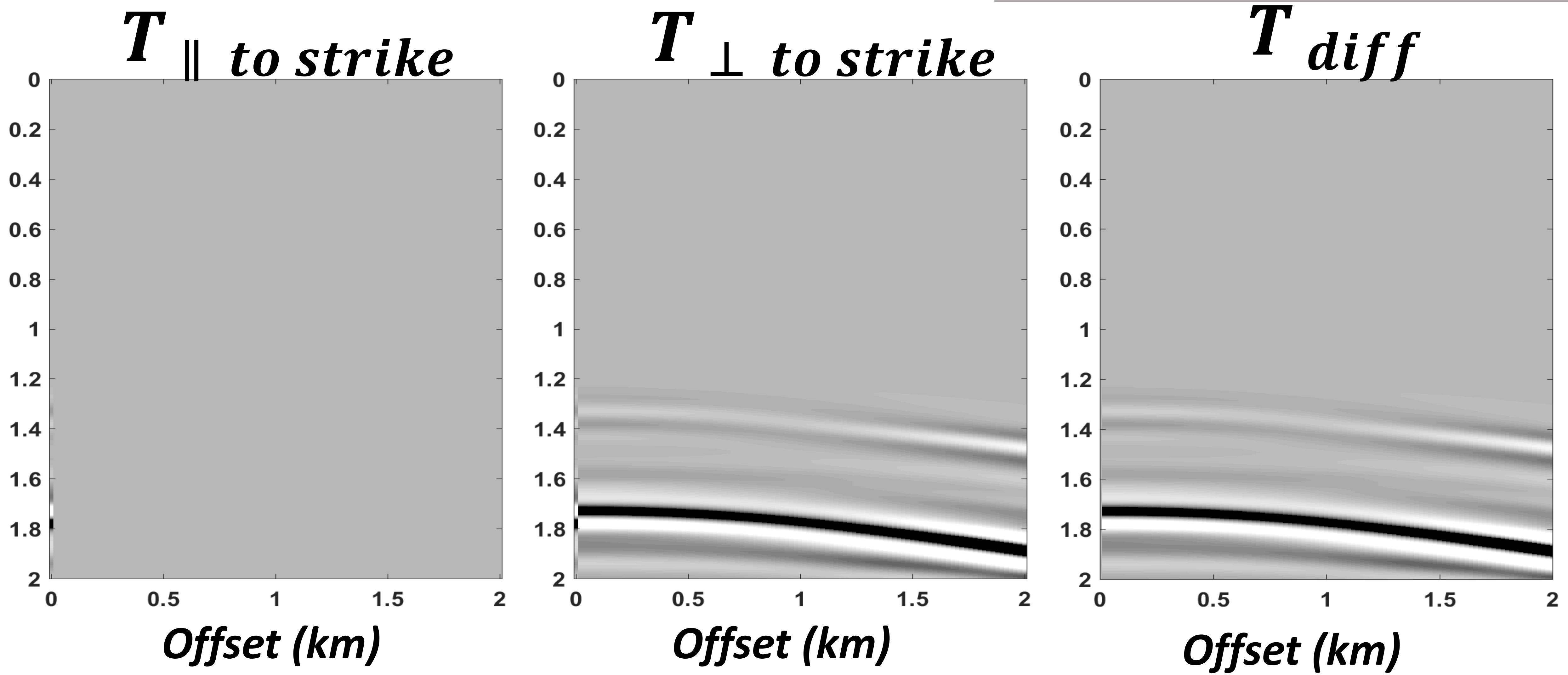
Example: FD modeling

Equivalent modeling



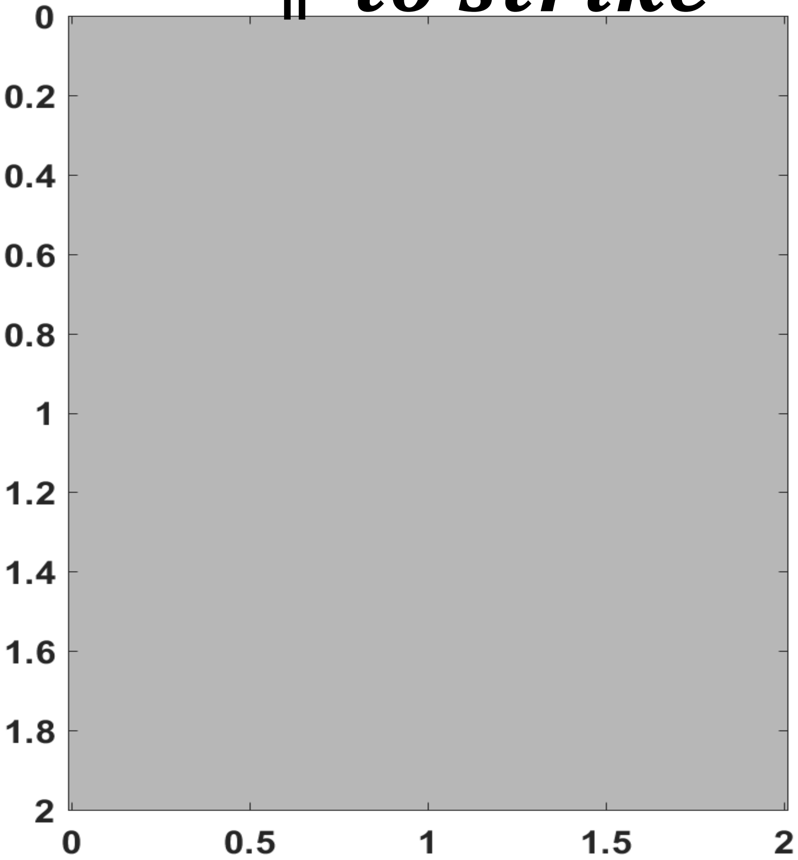
Example: FD modeling

Elastic modeling



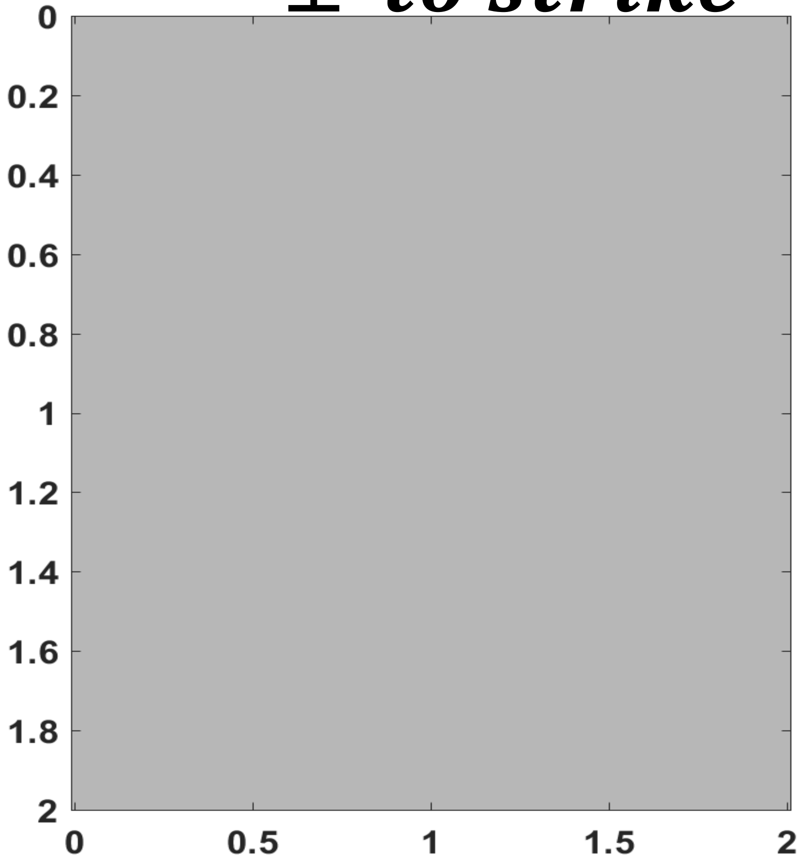
Equivalent modeling

$T_{\parallel}$ *to strike*



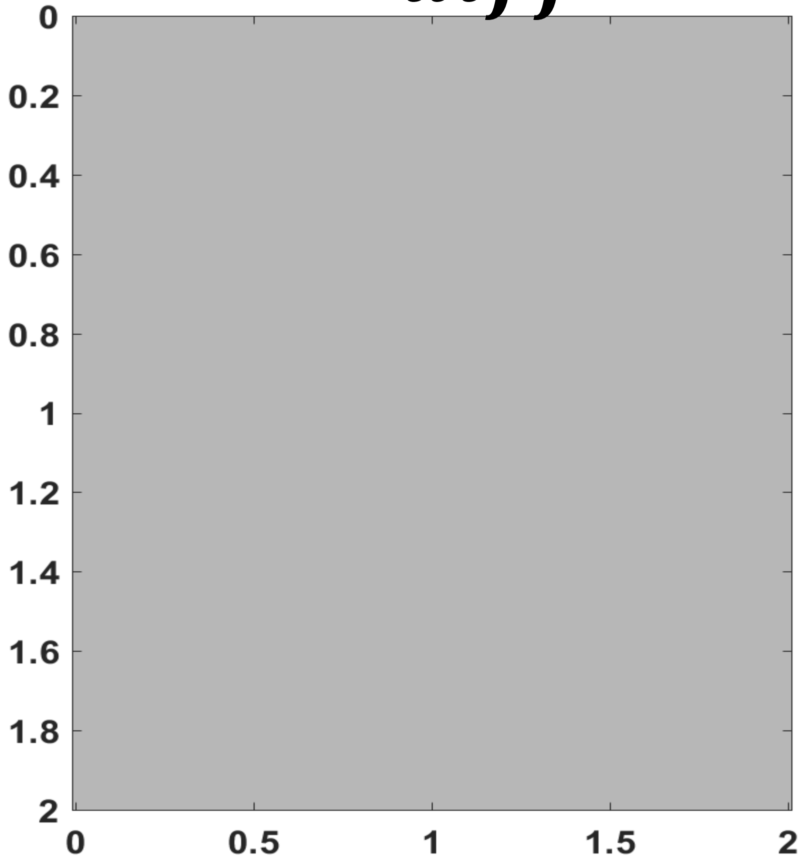
Offset (km)

$T_{\perp}$ *to strike*



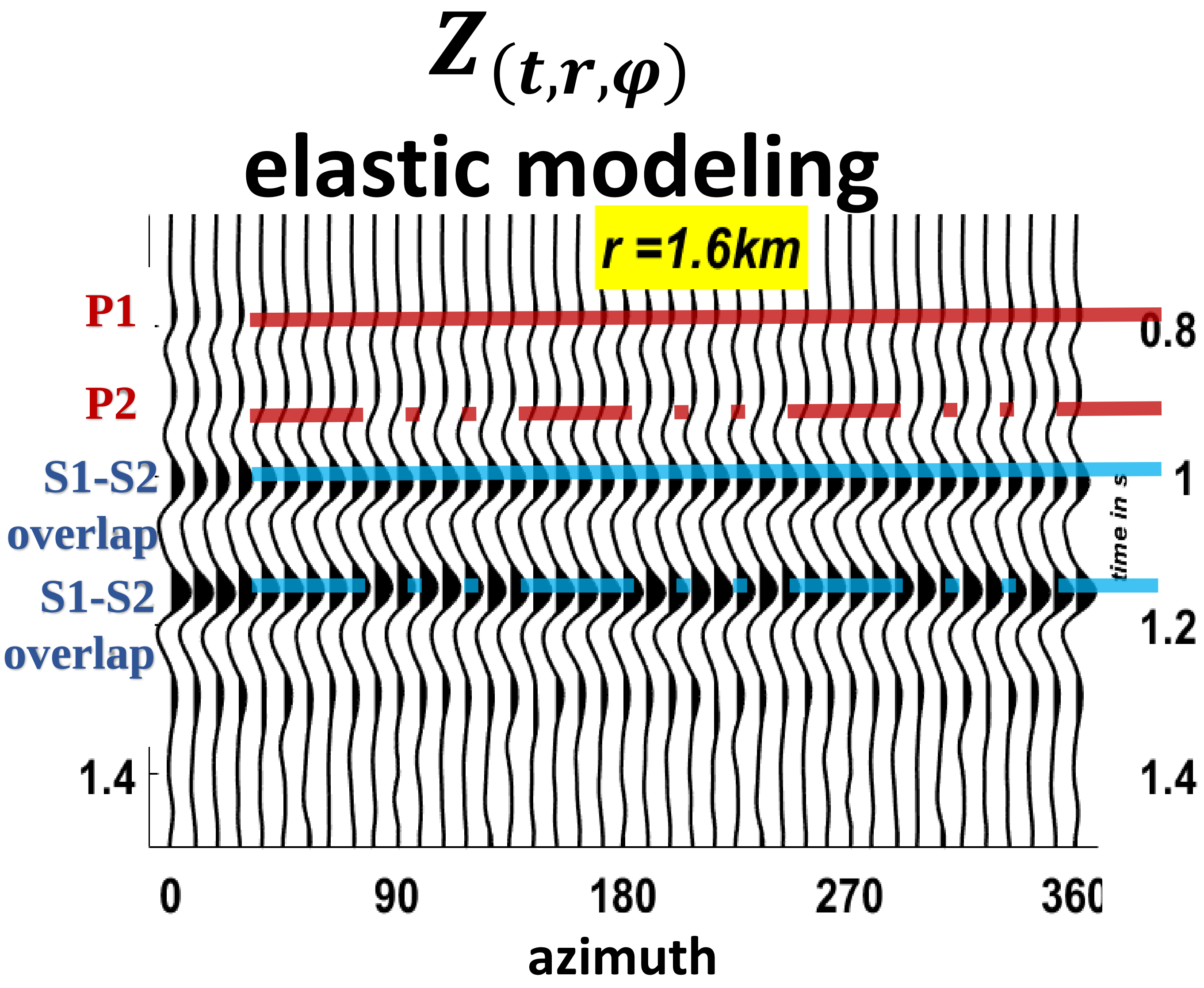
Offset (km)

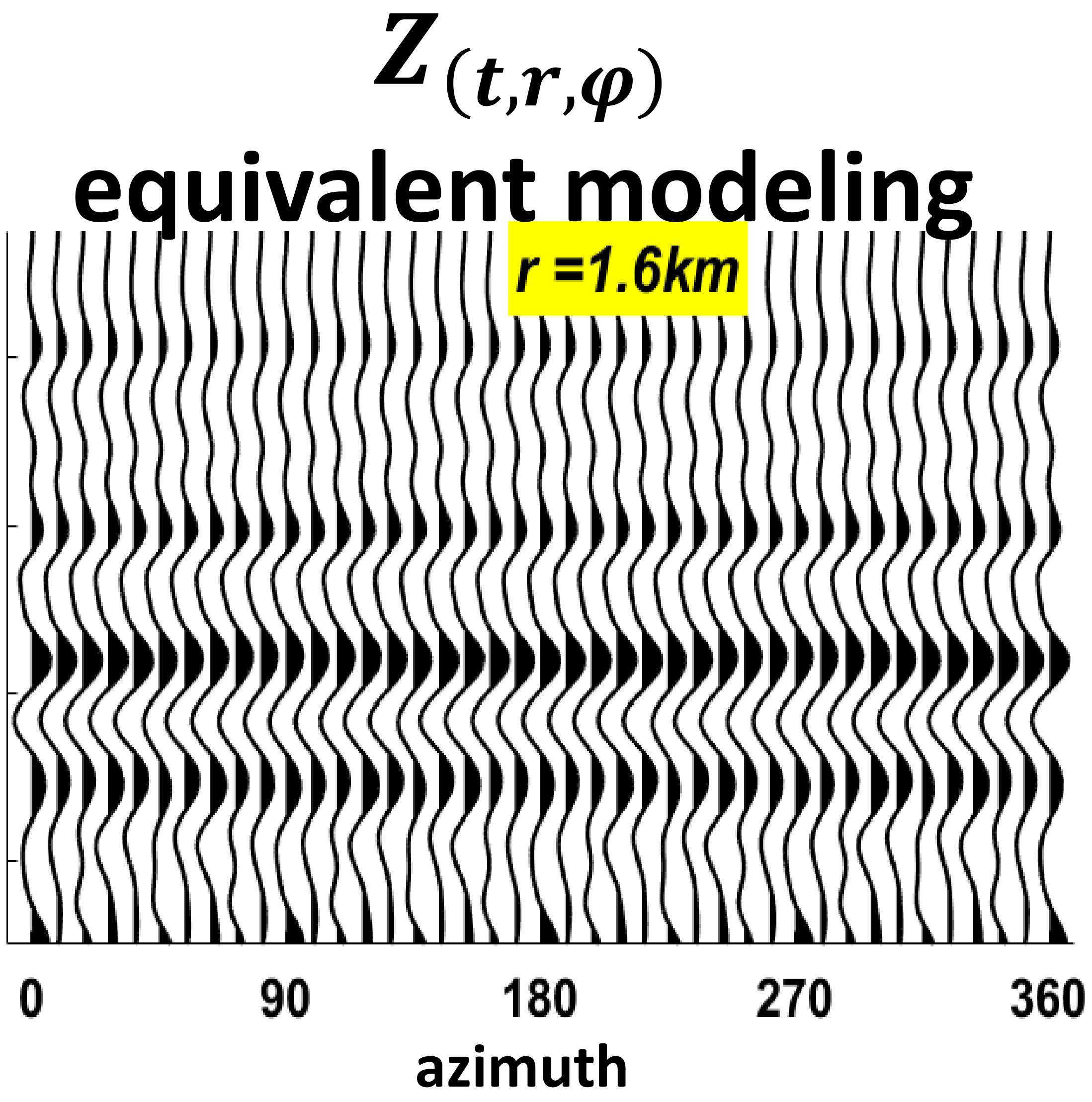
T_{diff}



Offset (km)

Example: Constant-offset azimuthal scans





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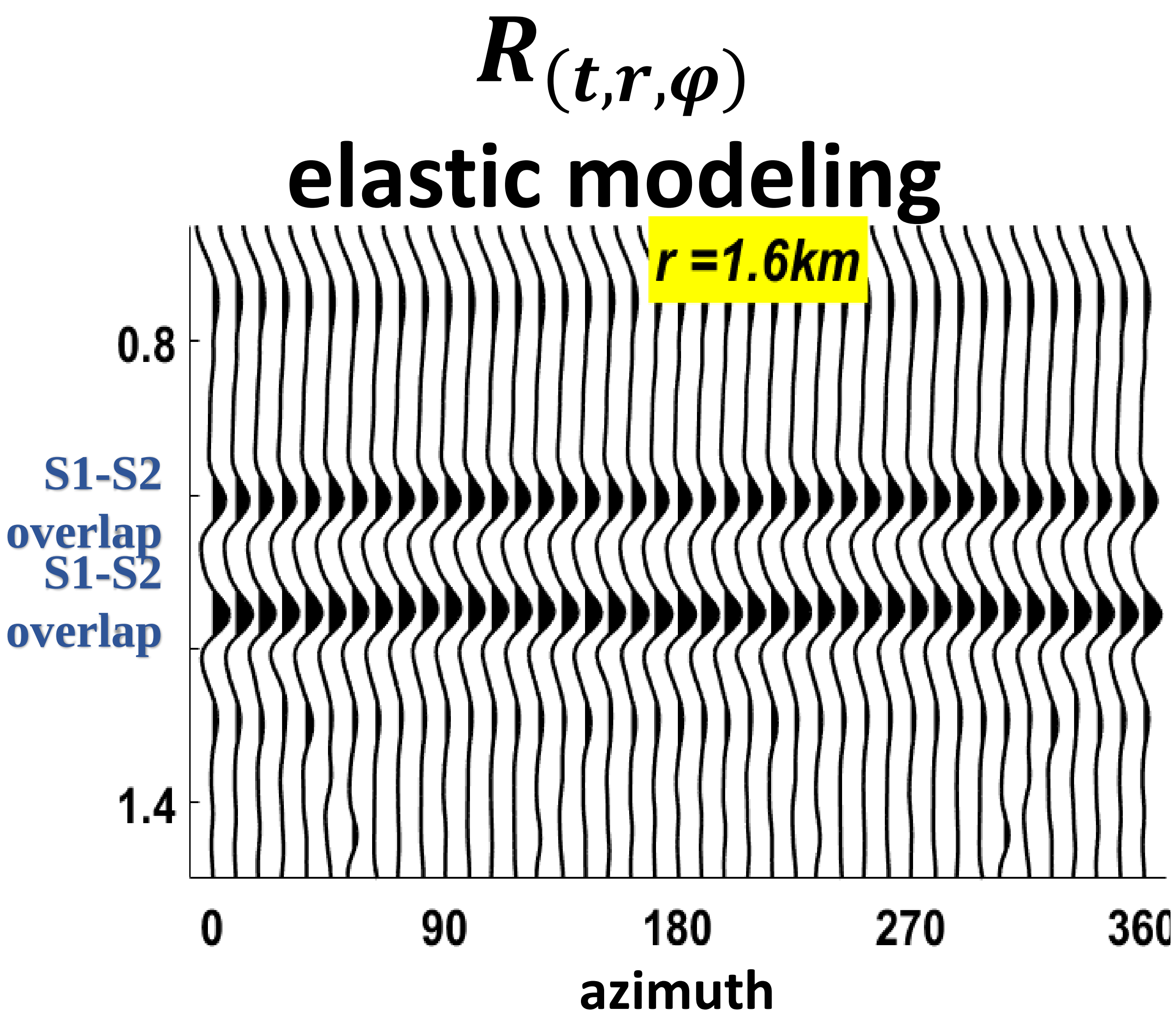
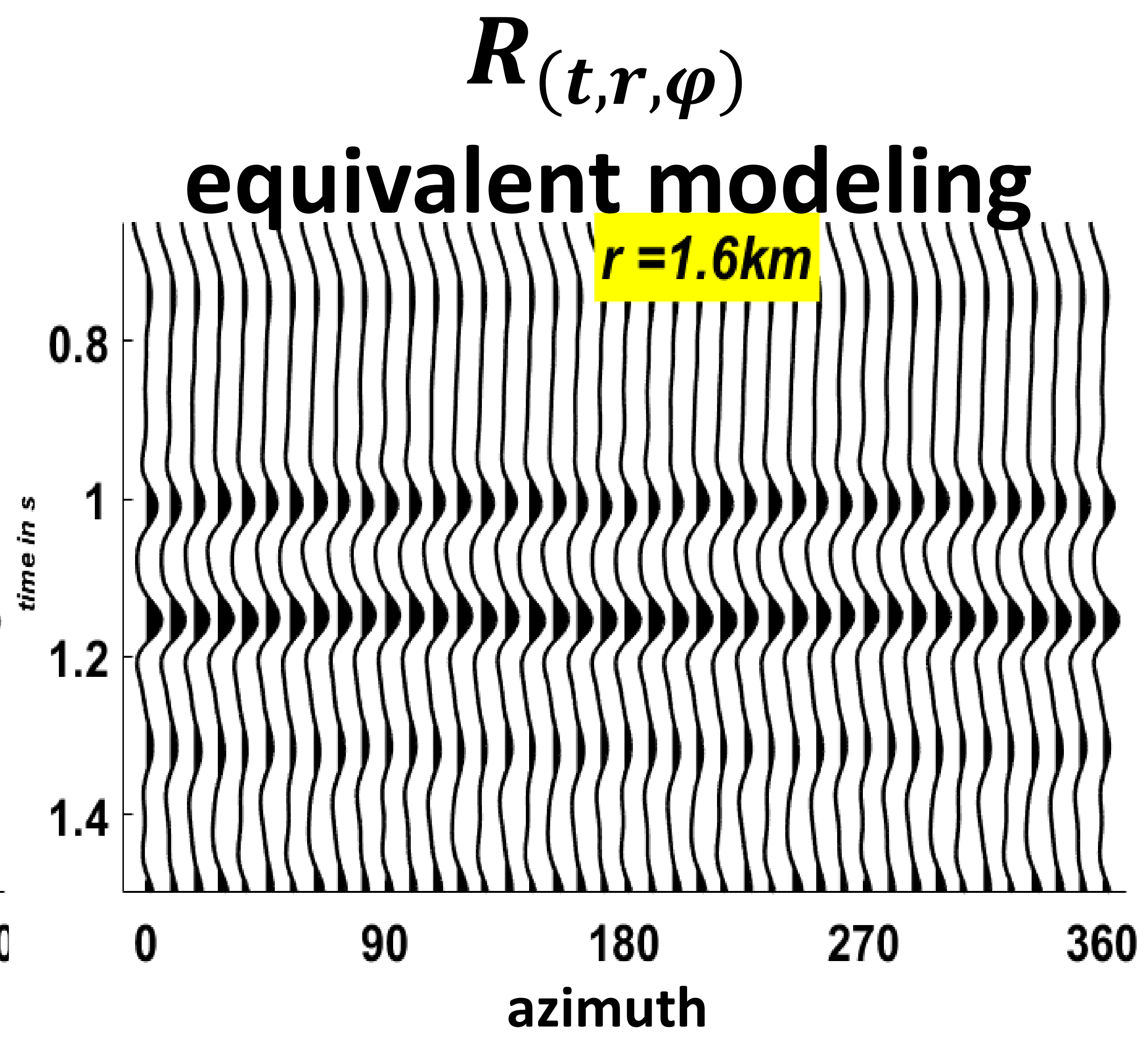
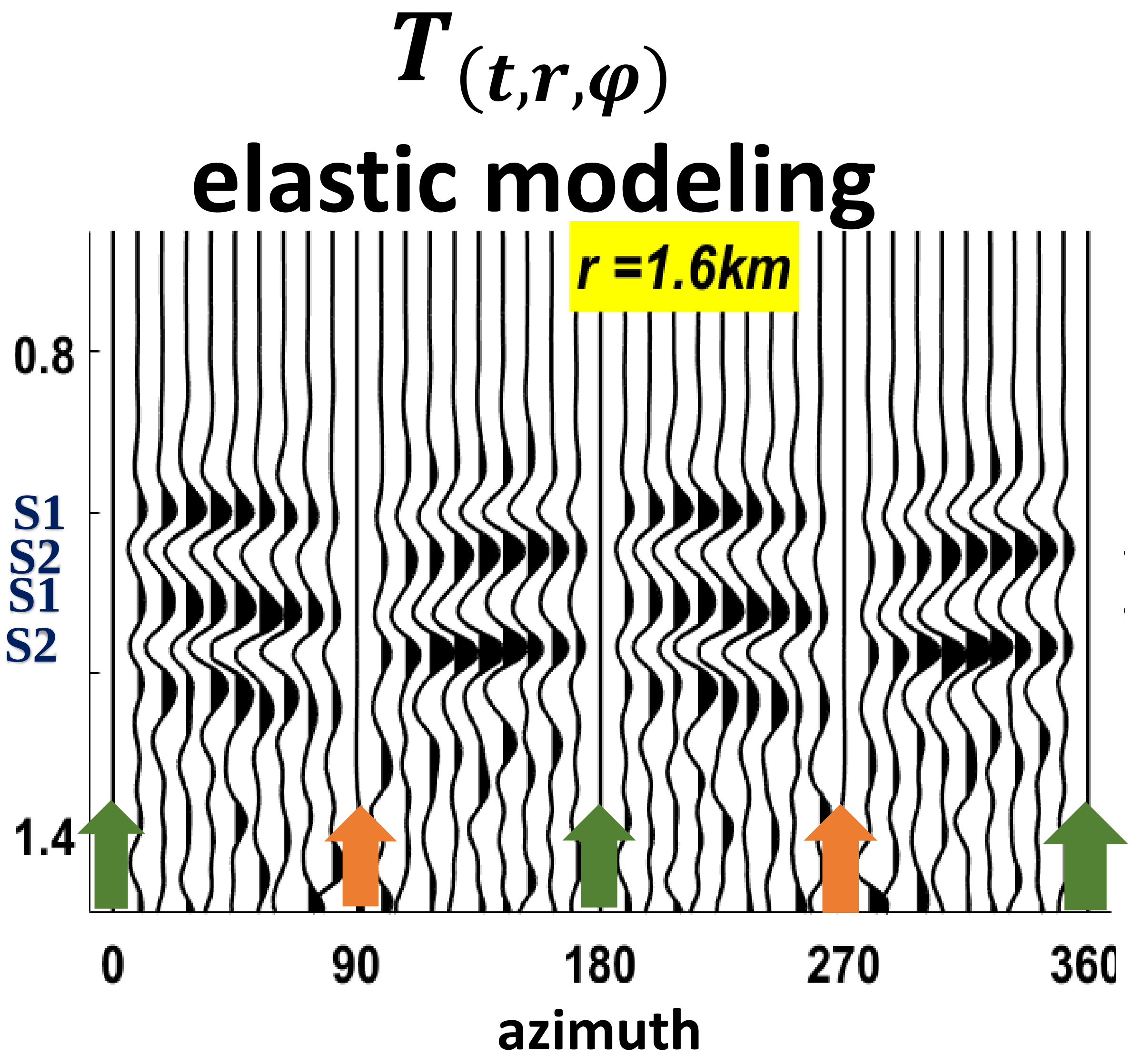
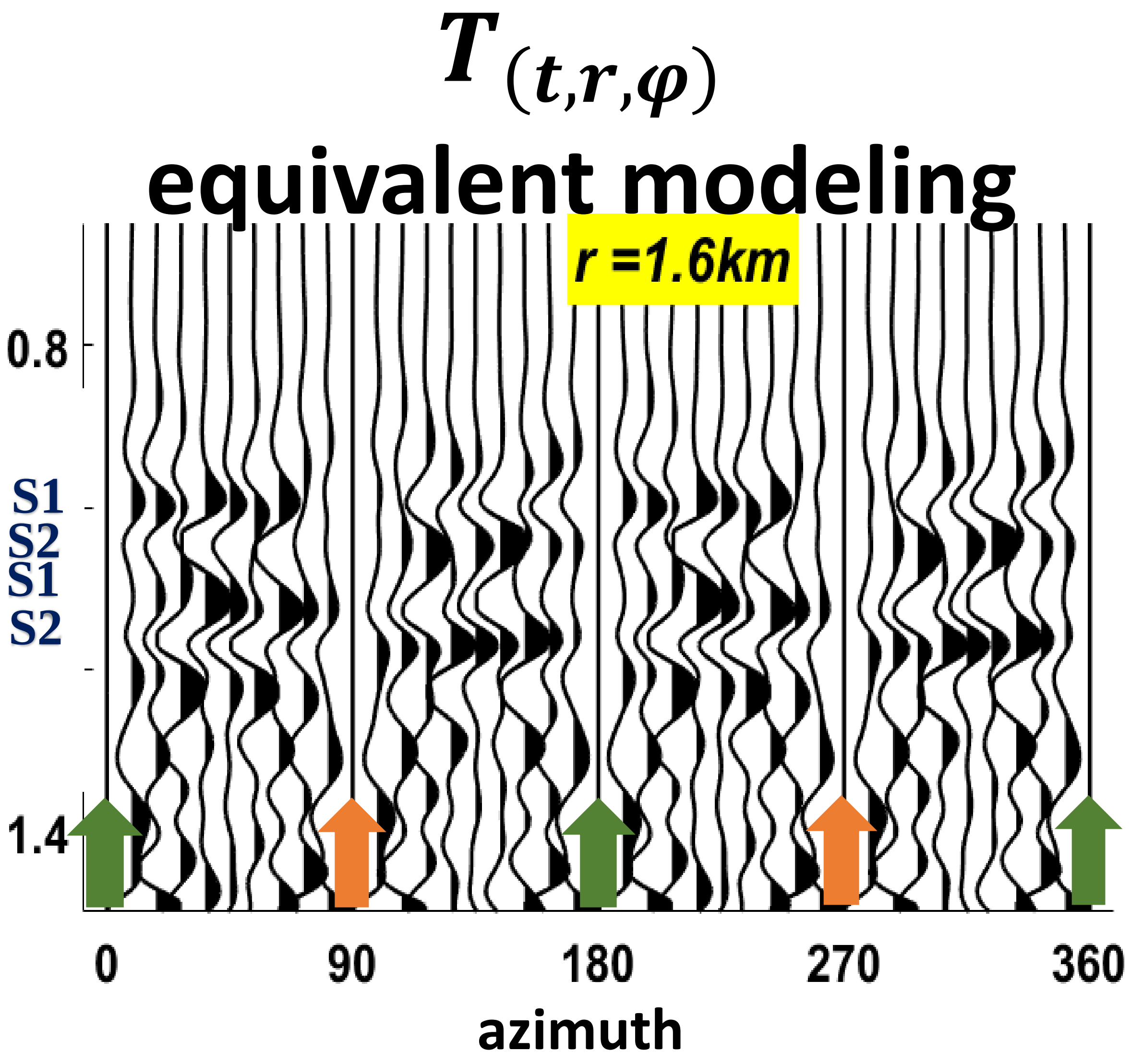


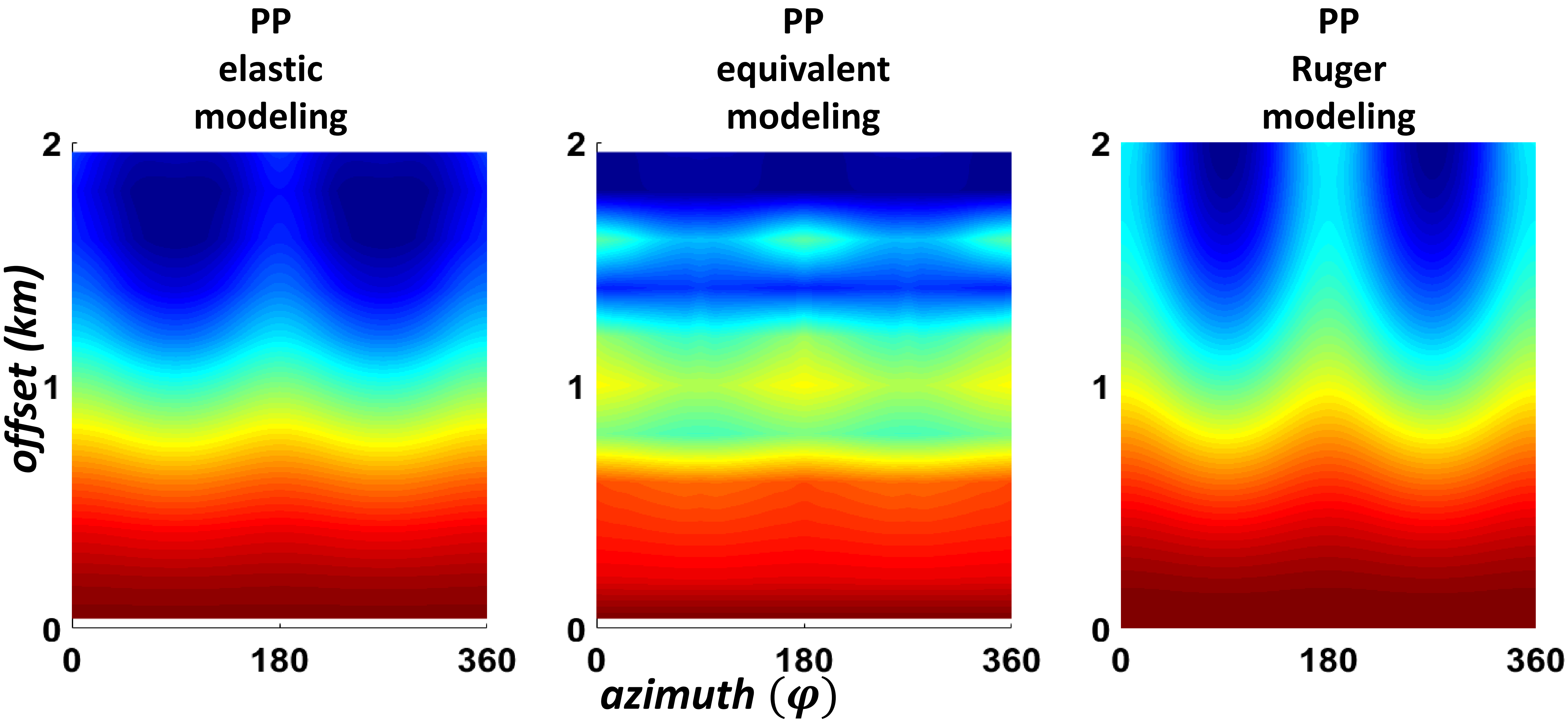


Constant-offset azimuthal scans





Example: Offset-Azimuth analysis: Top of HTI



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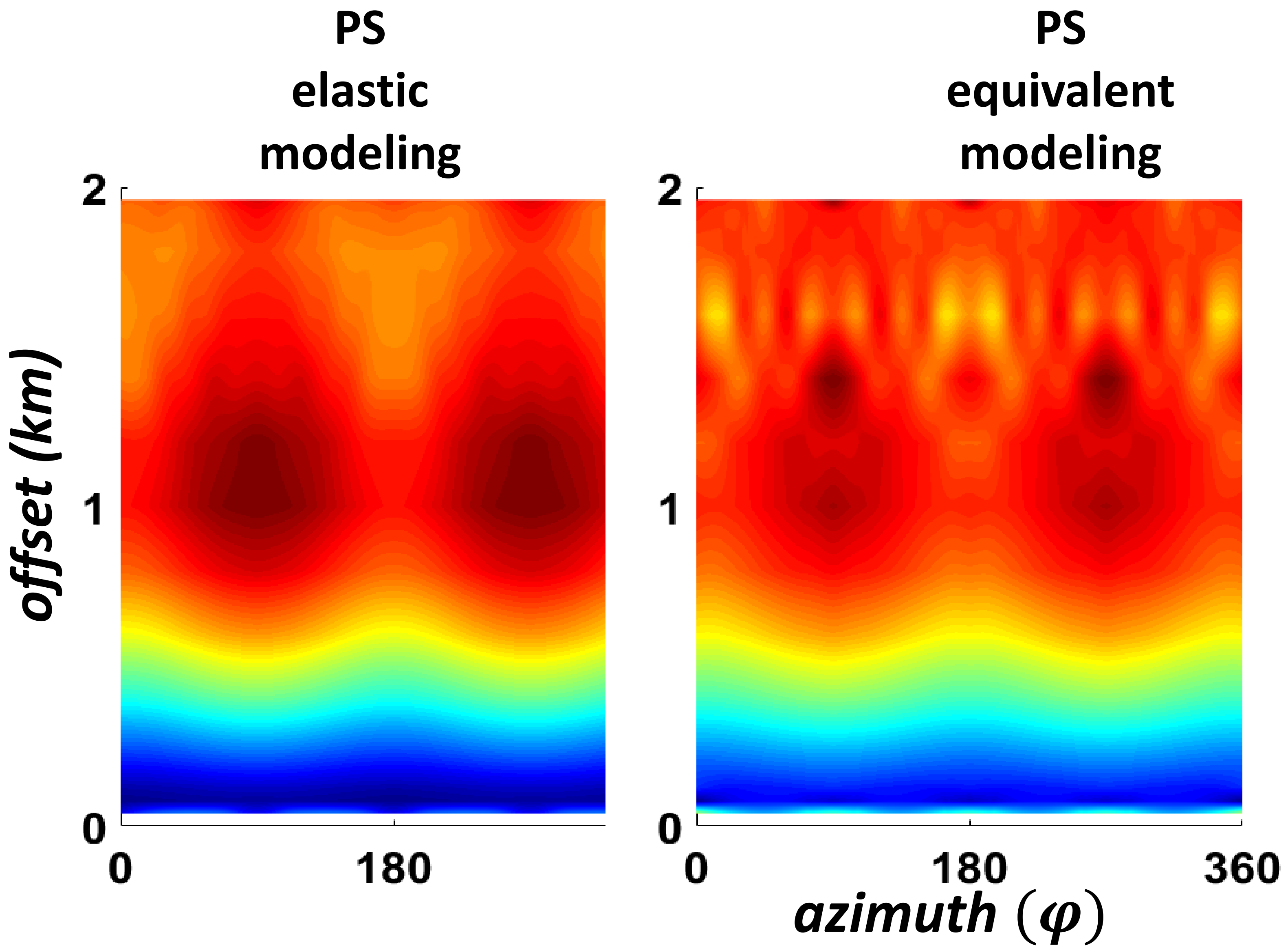
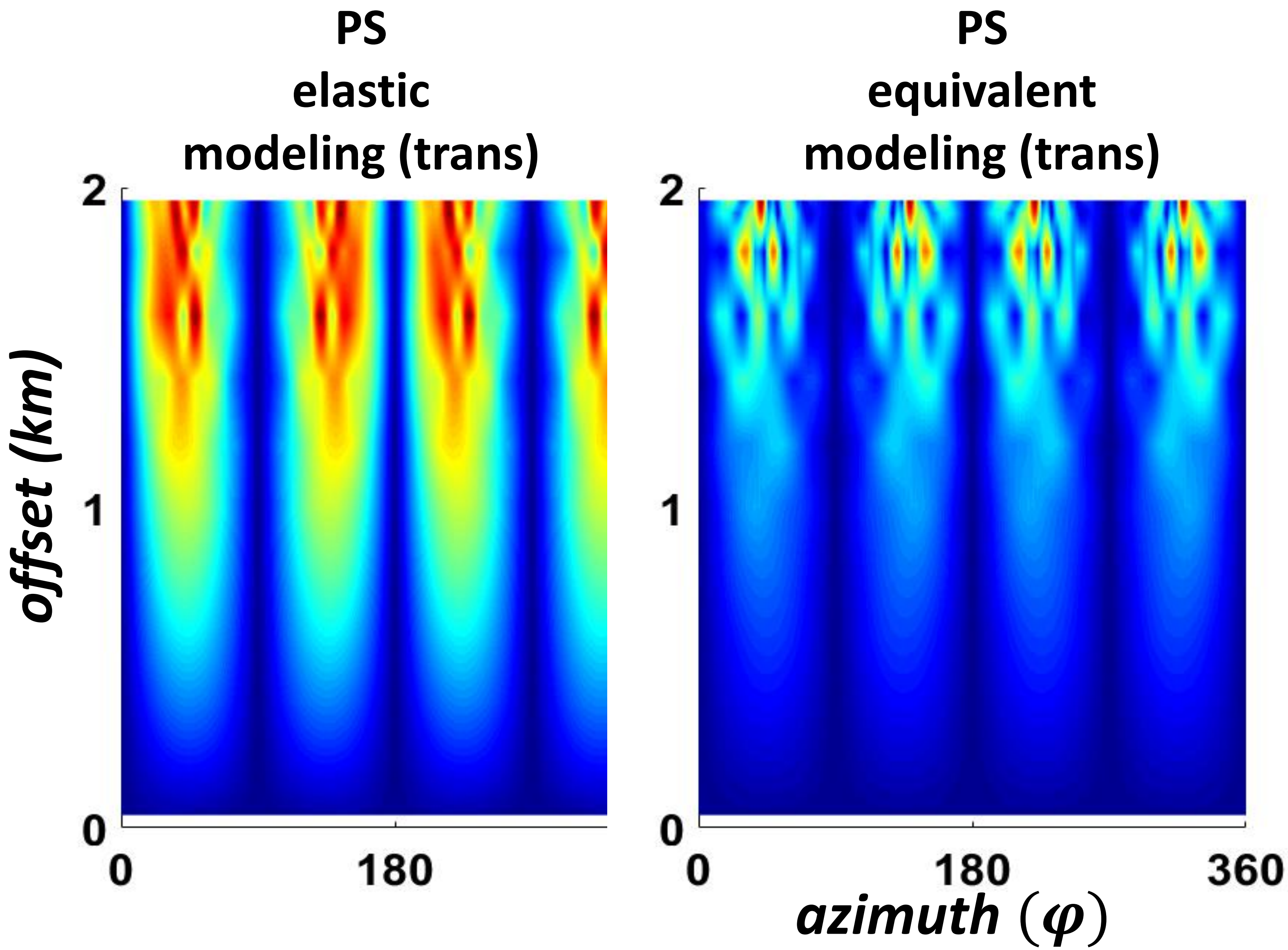


Example: Offset-Azimuth analysis: Top of HTI



Conclusion

- Fracture models provide a link between anisotropic properties and fracture properties of fractured reservoir.

- We have studied the equivalence between Hudson's microcrack model and Schoenberg's linear slip theory however knowledge of how to estimate aspect ratio and crack density is crucial in order to successfully relate these two models.

- We have shown that the equivalent Schoenberg Linear slip theory formulated by Schoenberg and Douma is closer to the 1^{st} order Hudson's theory; However, Grechka and Kachanov studies show that at certain crack density (0.05 in his paper) Hudson's model gave unphysical results.

- For small aspect ratios, however, Hudson's first and second order theories are close.

Conclusion

- TIGER finite difference modeling result comparison between elastic and Schoenberg's equivalent modeling for the same reservoir show that the Schoenberg linear slip theory is reliable.

- Overall we conclude that the linear slip theory which is much closer to the numerical modeling is superior to Hudson's first and second order schemes.

- The next immediate work will be to look in Grechka and Kachanov papers for clues on how to better under Hudson's model especially for thinly fractured medium and carry out similar analysis.

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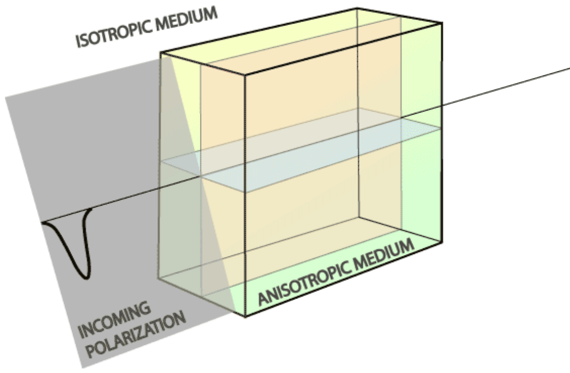

Thank you for listening