Elastic wave-vector decomposition using wave vector rotation in anisotropic media

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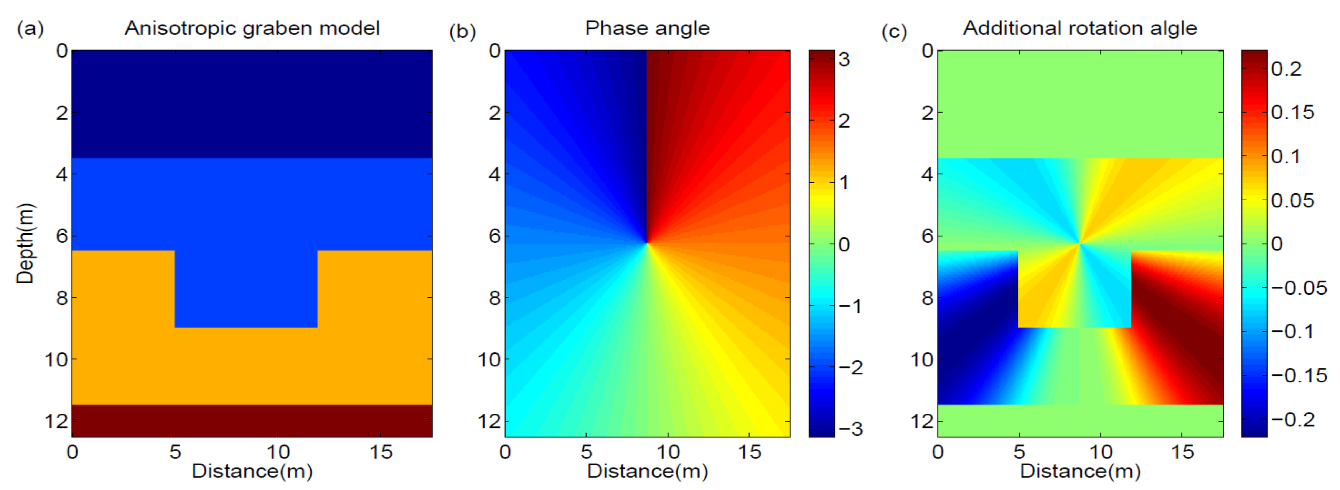
Abstract

Elastic wave propagating in anisotropic media can be treated as a polarization deviation of wave vector in terms of isotropic media. The deviation angle between the wave normal and qP-wave's polarization direction can be estimated based on the phase angle and elastic constants(or Thomsen parameters). The anisotropic polarization vectors' components are thus determined according to the rotation matrix calculated by the deviation angle. The anisotropic wavefield-separation operators are constructed at each point using the calculated polarization vectors. Finally, the vector wavefield decomposition based on the Helmholtz theory are then used to decompose coupled qP- and SV- modes.



ELASTIC WAVE VECTOR DECOMPOSITION

Given any vector field U(x,y,z), it can be decomposed into a curl-free part U^p and U^S. In Fourier domain, the wavefield can be described as

$$\widehat{\nabla}\cdot\hat{\boldsymbol{U}} = \widehat{\nabla}\cdot\hat{\boldsymbol{U}}^P \qquad \widehat{\nabla\times}\hat{\boldsymbol{U}} = \widehat{\nabla\times}\hat{\boldsymbol{U}}^S$$

Zhang and McMechan (2010) proposed the solution for the three Fourier components of P-wavefield in isotropic media as

$$\hat{\boldsymbol{U}^P} = \boldsymbol{K}(\boldsymbol{K}\cdot\hat{\boldsymbol{U}}), \qquad \hat{\boldsymbol{U}^P} = \boldsymbol{A^P}(\boldsymbol{A}^P\cdot\hat{\boldsymbol{U}}),$$

$$\hat{\boldsymbol{U}^S} = -\boldsymbol{K}\times(\boldsymbol{K}\times\hat{\boldsymbol{U}}). \qquad \hat{\boldsymbol{U}^S} = -\boldsymbol{A^P}\times(\boldsymbol{A}^P\times\hat{\boldsymbol{U}}).$$

P- and S- wave polarization vectors are obtained by Christoffel equation in anisotropic medium (Slawinski, 2003).

In this paper, according to the wavenumber and polarization angle as well as azimuthal angle

$$(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) = (K_x, K_y, K_z)$$

When anisotropy is present, $K \rightarrow A^P$, with an additional rotation angle

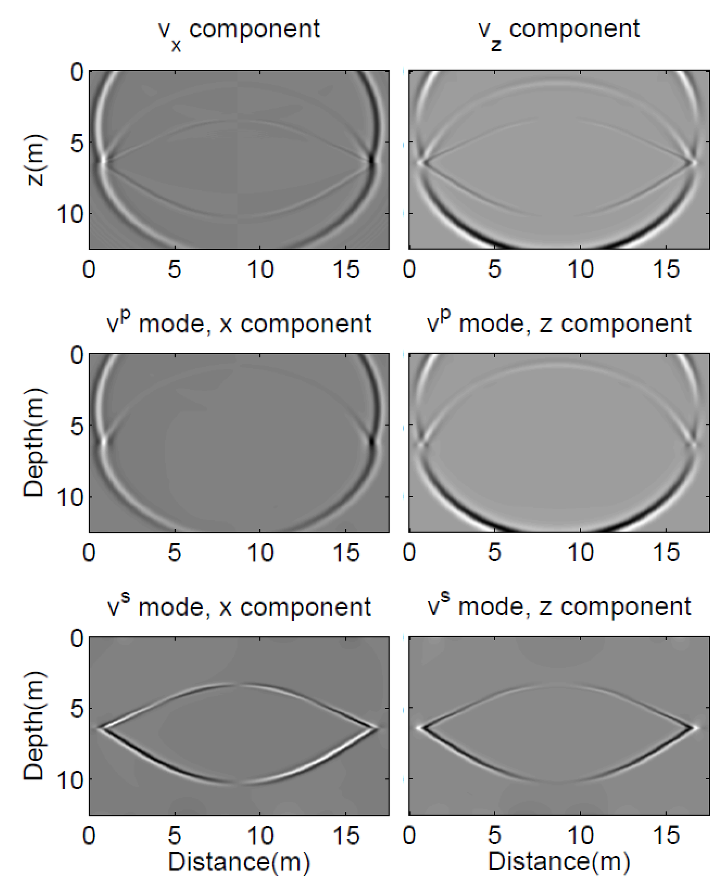
$$\Delta\theta = B[\delta + 2(\varepsilon - \delta)\sin^2\theta]\sin 2\theta$$

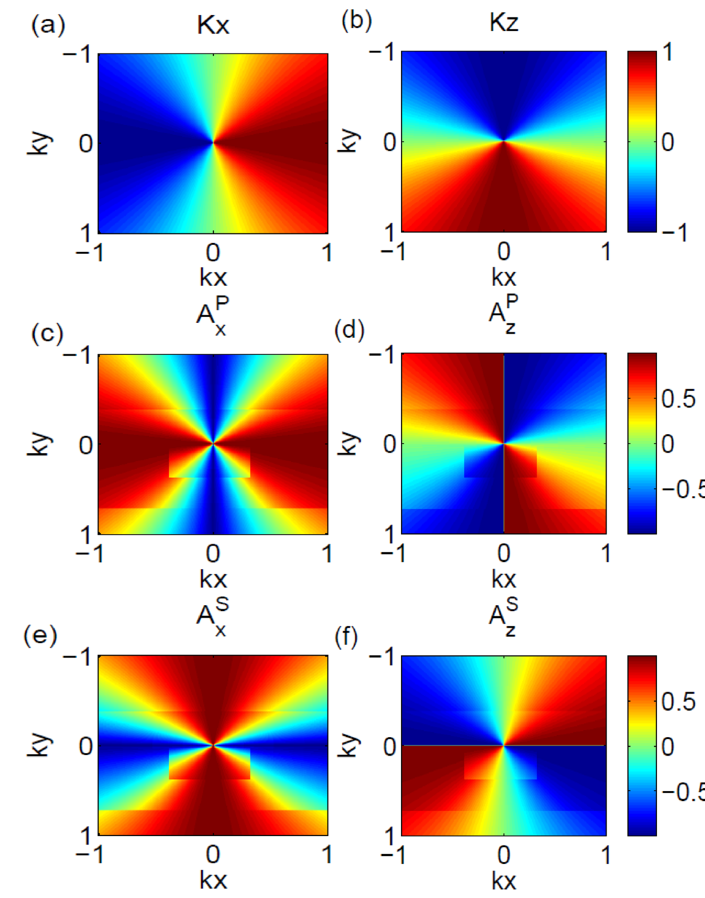
$$\hat{U_x^{qP}} = (\sin(\theta+\Delta\theta))^2\hat{U}_x + (\sin(\theta+\Delta\theta)\cos(\theta+\Delta\theta))\hat{U}_z,$$

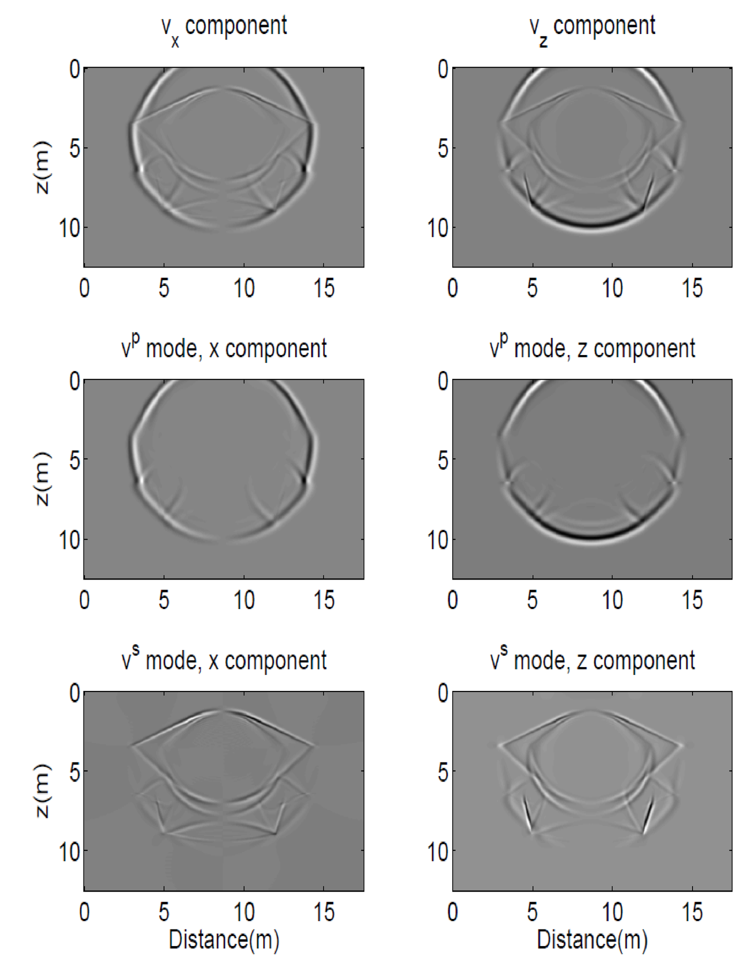
$$\hat{U_z^{qP}} = \cos^2(\theta+\Delta\theta)\hat{U}_z + (\sin(\theta+\Delta\theta)\cos(\theta+\Delta\theta))\hat{U}_x,$$

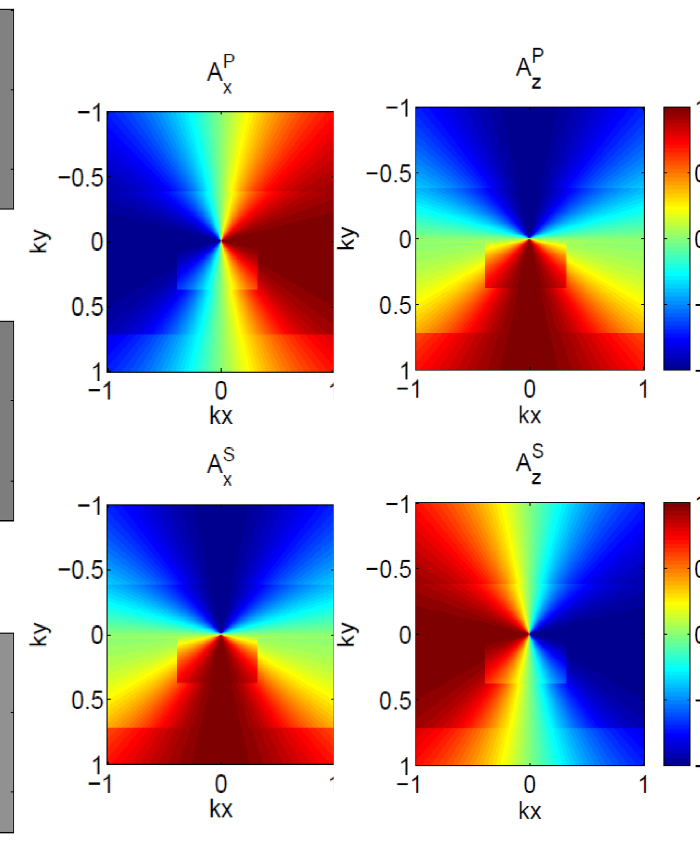
$$\hat{U_x^{qSV}} = (\cos^2(\theta+\Delta\theta))\hat{U}_x - (\sin(\theta+\Delta\theta)\cos(\theta+\Delta\theta))\hat{U}_z,$$

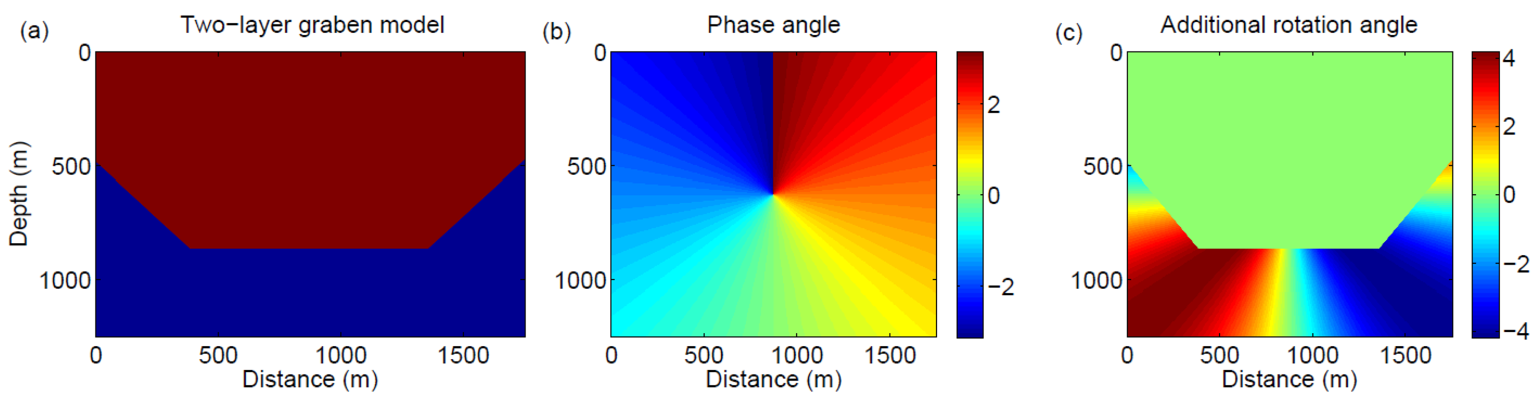
$$\hat{U_z^{qSV}} = \sin^2(\theta+\Delta\theta)\hat{U}_z - (\sin(\theta+\Delta\theta)\cos(\theta+\Delta\theta))\hat{U}_x.$$

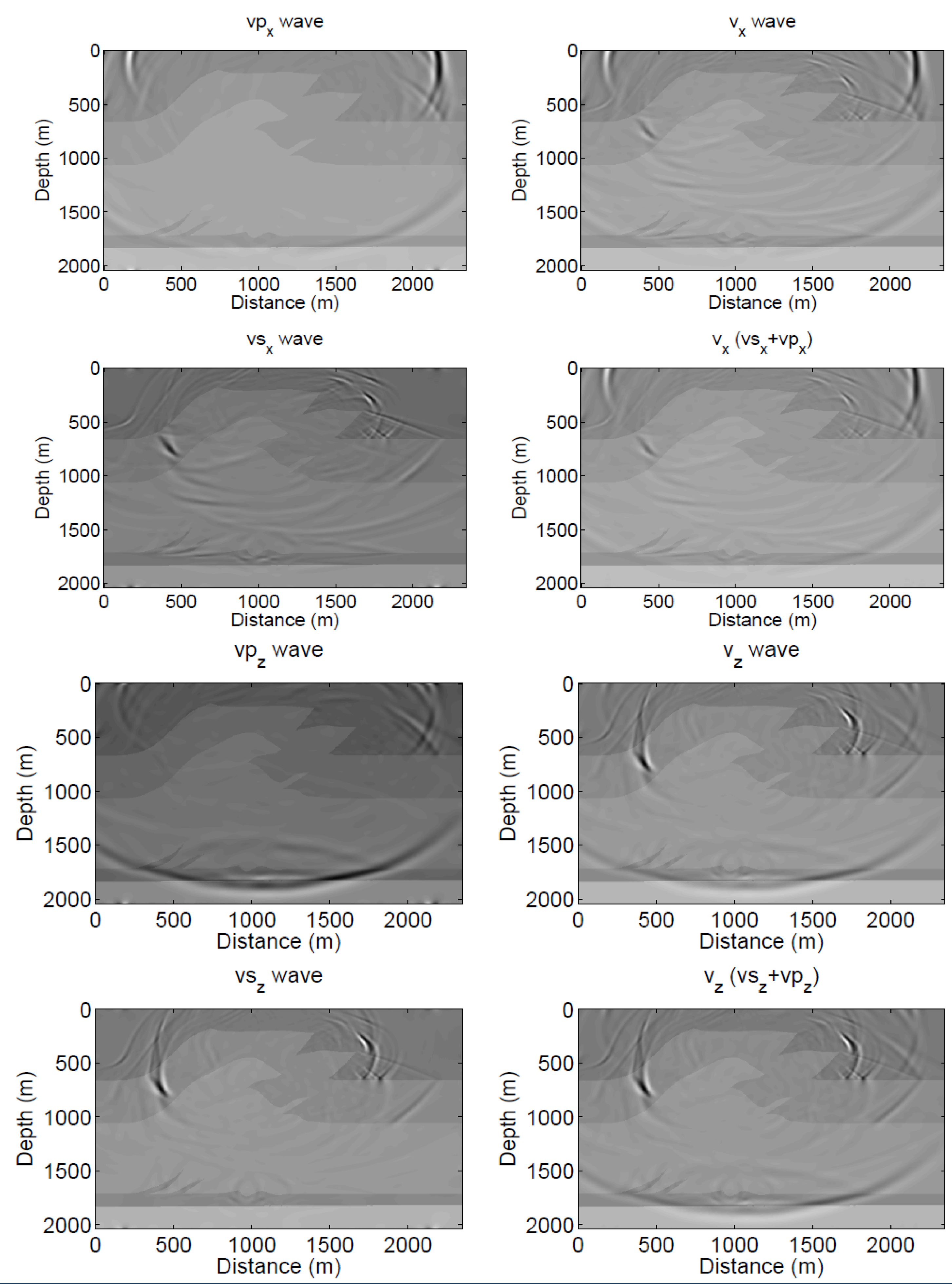


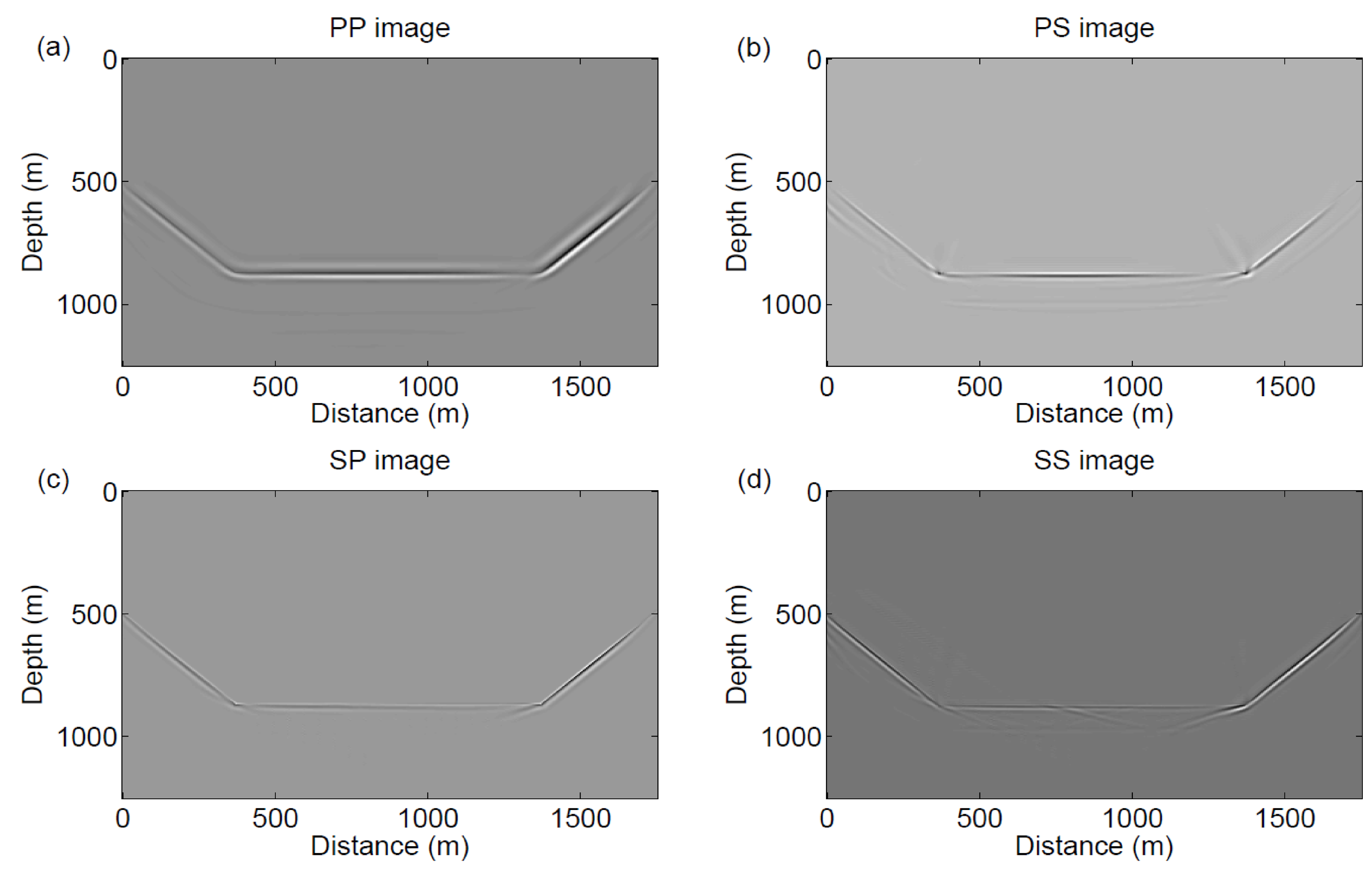












Conclusions

A 3D elastic wave field decomposition scheme for anisotropic media is proposed in this paper, in which the anisotropic polarization vectors' components are determined according to the rotation matrix calculated by the deviation angle. This new scheme solves the uncertainty issue of the polarization vector direction. The decomposition example of a 2D synthetic anisotropic model as well as the anisotropic thrust fault model prove the validity of this new scheme. Finally, the decomposed components are used in ERTM to get the. PP-, PS-, SP- and SS imaging results, the imaging results correlate to the model quite well, which further validates our decomposition method.

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