Literature review and discussions of inverse scattering series on internal multiple prediction

Jian Sun*, Kristopher Innanen
sun1@ucalgary.ca

Abstract

Internal multiple attenuation is an increasingly high priority in seismic data analysis in the wake of increased sensitivity of primary amplitudes in quantitative interpretation, due to more information intend to be squeezed from seismic data. Removal of internal multiple is still a big challenge even though several various methods has been proposed. Inverse scattering series internal multiple attenuation algorithm, with great potential, developed by Weglein and collaborators in the 1990s, indicated that all internal multiples can be estimated by combining those sub-events satisfying a certain schema, which is the lower-higher-lower criterion. Many considerable discussions of internal multiple attenuation have been made based on inverse scattering series algorithm. In this paper, start with forward scattering series, we comprehensive review inverse scattering series internal multiple attenuation algorithm both in theoretical and its applications.

Algorithms in variant domains

By resetting the inverse scattering series and equating like orders:

$$\begin{aligned}
\boldsymbol{b}_1 &= \boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{\phi}_0,\\
\boldsymbol{0} &= \boldsymbol{G}_0\boldsymbol{V}_2\boldsymbol{\phi}_0 + \boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{\phi}_0,\\
\boldsymbol{0} &= \boldsymbol{G}_0\boldsymbol{V}_3\boldsymbol{\phi}_0 + \boldsymbol{G}_0\boldsymbol{V}_2\boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{\phi}_0 + \boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{G}_0\boldsymbol{V}_2\boldsymbol{\phi}_0 + \boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{G}_0\boldsymbol{V}_1\boldsymbol{\phi}_0,\\
&\vdots
\end{aligned}$$

where, $\boldsymbol{b}_1 = \mathrm{i}2\nu_s(\boldsymbol{G}-\boldsymbol{G}_0) = \mathrm{i}2\nu_s\boldsymbol{D}$ is the weighted scattered wave field of point sources, $\phi_0(x_g, z_g, k_s, z_s, \omega) = e^{\mathrm{i}(k_s x_g + \nu_s|z_g - z_s|)}$ is superposition of Green function.

The internal multiple prediction algorithm in pseudo-depth domain was proposed by Weglein (1997):

$$\begin{aligned}
b_3(k_g, k_s, \omega) = &-\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} \mathrm{d}k_1\mathrm{d}k_2 e^{\mathrm{i}\nu_1(z_s-z_g)}e^{\mathrm{i}\nu_2(z_g-z_s)}\int_{-\infty}^{+\infty}\mathrm{d}z_1 e^{\mathrm{i}(\nu_1+\nu_g)z_1}b_1(k_g,k_1,z_1)\\
&\times\int_{-\infty}^{z_1-\epsilon}\mathrm{d}z_2 e^{-\mathrm{i}(\nu_2+\nu_1)z_2}b_1(k_1,k_2,z_2)\int_{z_2+\epsilon}^{+\infty}\mathrm{d}z_3 e^{\mathrm{i}(\nu_s+\nu_2)z_3}b_1(k_2,k_s,z_3)
\end{aligned}$$

The plane wave algorithm was first mentioned by Coates and Weglein (1996), and then implemented by Sun and Innanen (2015):

$$\begin{aligned}
b_3(p_g, p_s, \omega) = &-\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} \mathrm{d}p_1\mathrm{d}p_2 e^{\mathrm{i}q_1(\tau_s-\tau_g)}e^{\mathrm{i}q_2(\tau_g-\tau_s)}\int_{-\infty}^{+\infty}\mathrm{d}\tau_1 e^{\mathrm{i}\omega\tau_1}b_1(p_g,p_1,\tau_1)\\
&\times\int_{-\infty}^{\tau_1-\epsilon}\mathrm{d}\tau_2 e^{-\mathrm{i}\omega\tau_2}b_1(p_1,p_2,\tau_2)\int_{\tau_2+\epsilon}^{+\infty}\mathrm{d}\tau_3 e^{\mathrm{i}\omega\tau_3}b_1(p_2,p_s,\tau_3)
\end{aligned}$$

Also was presented (Sun and Innanen, 2015) in (p,z) domain:

$$\begin{aligned}
b_3(p_g, p_s, \omega) = &-\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} \mathrm{d}p_1\mathrm{d}p_2 e^{\mathrm{i}\nu_1(z_s-z_g)}e^{\mathrm{i}\nu_2(z_g-z_s)}\int_{-\infty}^{+\infty}\mathrm{d}z_1 e^{\mathrm{i}(\nu_1+\nu_g)z_1}b_1(p_g,p_1,z_1)\\
&\times\int_{-\infty}^{z_1-\epsilon}\mathrm{d}z_2 e^{-\mathrm{i}(\nu_2+\nu_1)z_2}b_1(p_1,p_2,z_2)\int_{z_2+\epsilon}^{+\infty}\mathrm{d}z_3 e^{\mathrm{i}(\nu_s+\nu_2)z_3}b_1(p_2,p_s,z_3)
\end{aligned}$$

To incorporating a non-stationary search parameter, Innanen (2016) transformed the algorithm into time-offset domain:

$$IM_3(x,t) = \int \mathrm{d}x_1 \int \mathrm{d}t_1 d(x-x_1, t_1-t)\int \mathrm{d}x_2 \int_{\alpha(t,t_1)}^{\beta(t)} \mathrm{d}t_2 d(x_1-x_2, t_1-t_2)d(x_2,t_2)$$

where $\alpha(t,t_1) = t_1 - (t-\epsilon_2)$ and $\beta(t) = t - \epsilon_1$.

However, consider time consuming, time-offset domain was replaced by time-wavenumber domain:

$$B_{IM3}(k_g,t) = \int_{-\infty}^{+\infty}\mathrm{d}t_1 D(k_g, t_1 - t)\int_{\alpha(t,t_1)}^{\beta(t)}\mathrm{d}t_2 D(k_g, t_1-t_2)D(k_g,t_2)$$

Search parameter and implementations

To investigate the effects of search parameter and implementati -on domains, the prediction are performed in (k_g, z), (k_g, t), (p_g, z), and (p_g, τ)domains, respectively. Figure 1 shows the velocity model and synthetic data created by finite difference method with four absorbing boundaries, which means only primaries and multiples left.

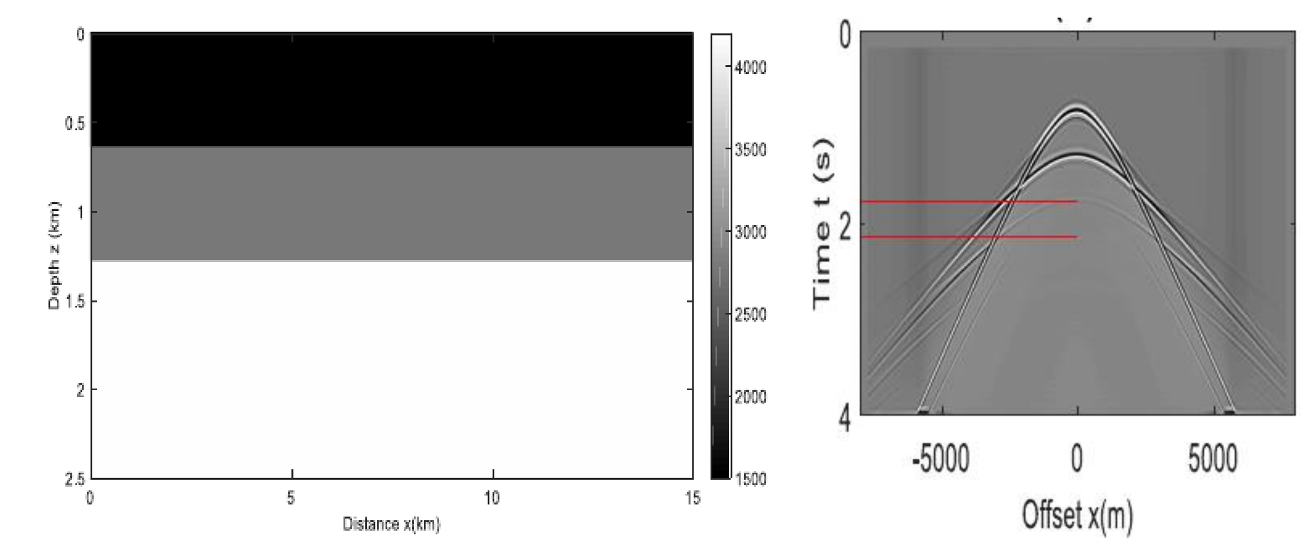

FIG.1. Left : velocity model: 1500m/s, 2800m/s, 4500m/s. Right: synthetic shot gather. 1st and 2nd order IMs indicated in red.

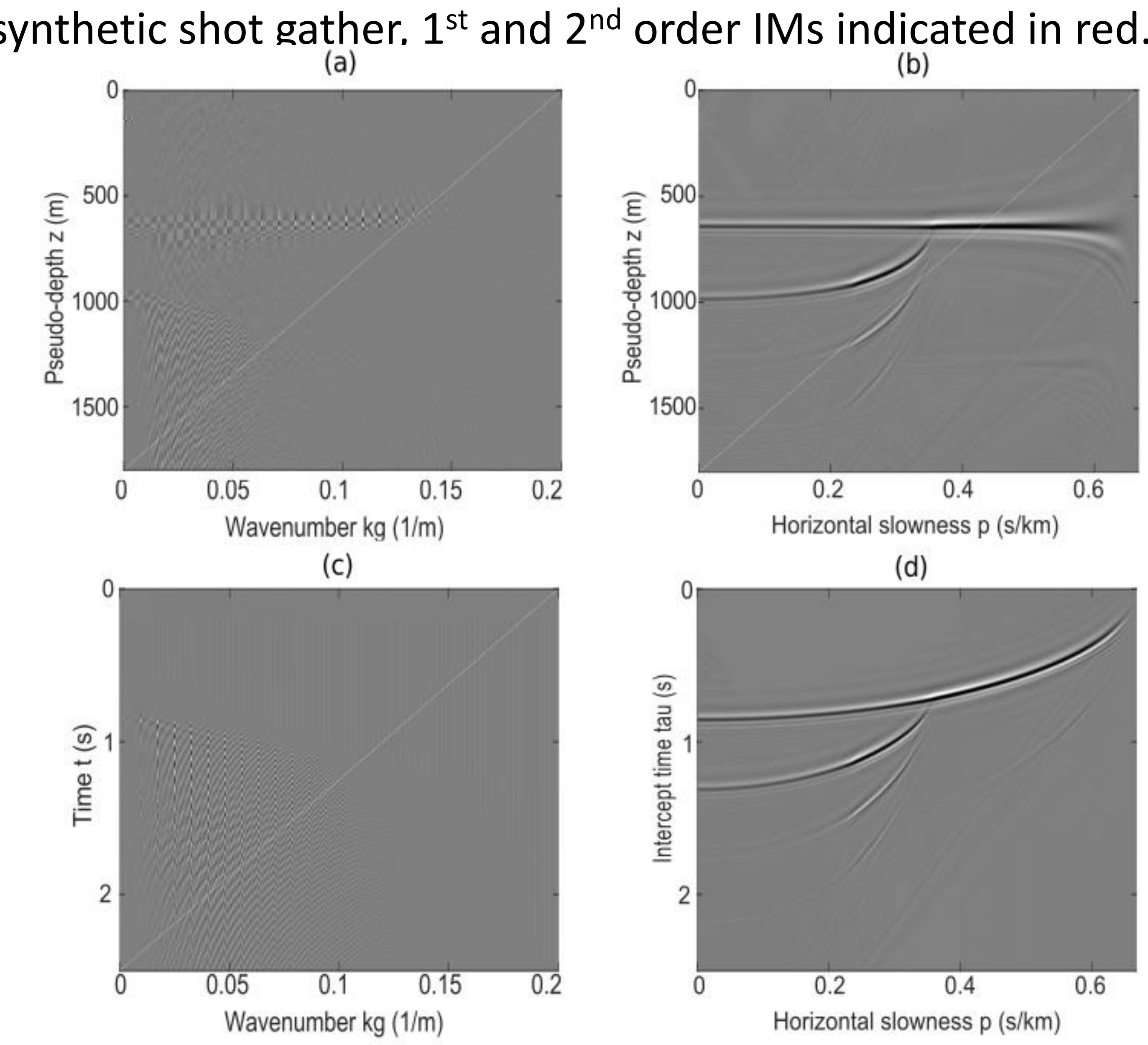

FIG2. The distribution of inputs in variant domains. (a) $b_1(k_g, z)$, (b) $b_1(p_g, z)$, (c) $b_1(k_g, t)$, (d) $b_1(p_g, \tau)$.

Compared to (p_g, z) and (p_g, τ) domains, the amplitude dispe -rsion in (k_g, z) and (k_g, t) domains will introduce high angle artifacts if an fixed search parameter is applied. To mitigate these artifacts the epsilon has to be a function of wavenumber. A simple version of wavenumber dependent search parameter was proposed by Innanen and Pan (2015), shown in Figure 3. Table 1 shows the limits of integrals for the implementation in relevant domains.

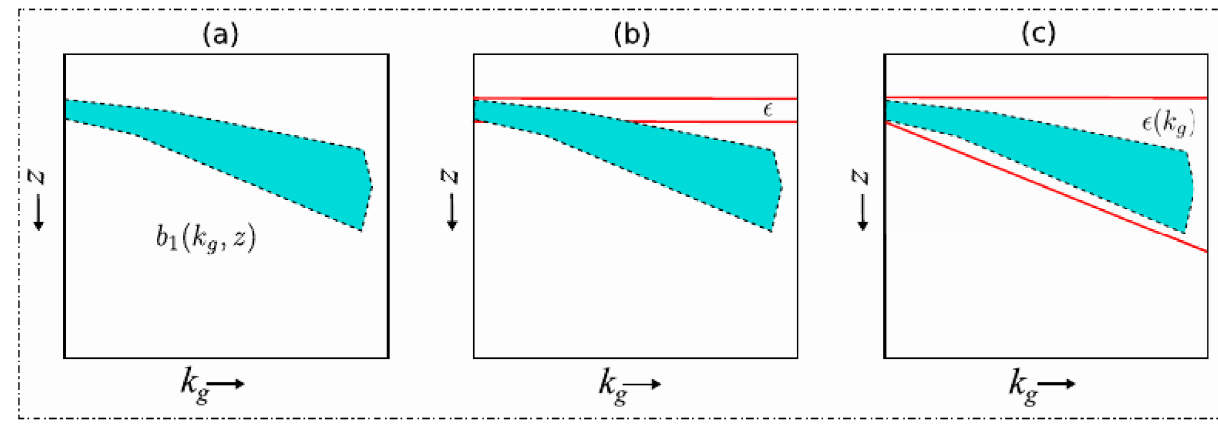

FIG.3. Mechanism of variant search parameter in (k_g, z) domain (Innanen and Pan, 2015).

Domains	Limits of integrals applied
(k_g, z)	$\epsilon = 0.3km$
(p_g, z)	$\epsilon = 0.3km$
(k_g, t)	$\alpha(t, t_1, \epsilon(k_g))$ and $\beta(t, \epsilon(k_g))$
(p_g, τ)	$\epsilon = 0.3s$

Table 1. The corresponded limits of integrals for implementing IMs prediction in variant domains.

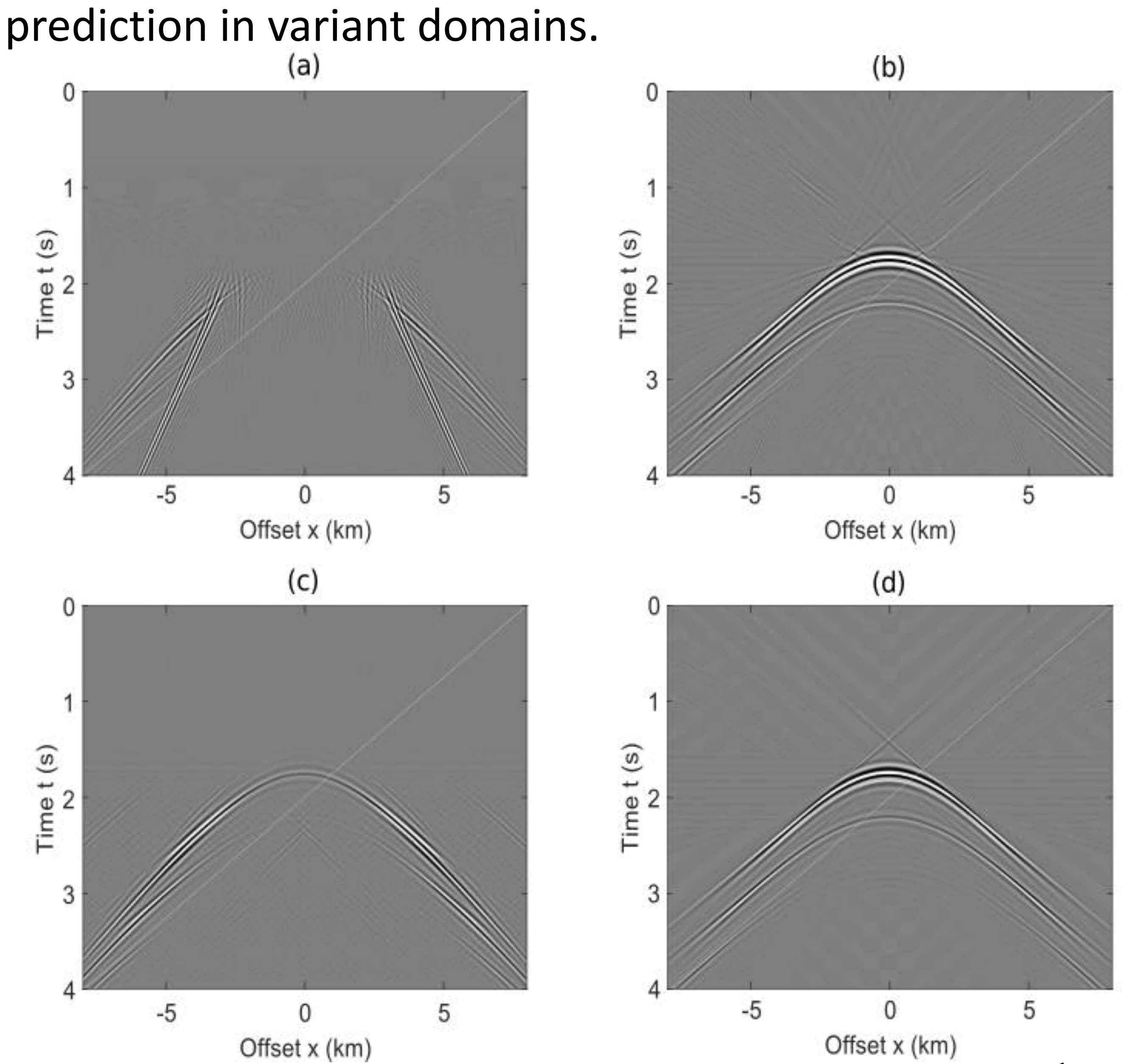

FIG.4. The predicted IMs by performing algorithm in (a) (k_g, z), (b) (p_g, z), (c) (k_g, t), (d) (p_g, τ) domains, respectively.

Conclusion and Acknowledgements

When consider efficiency and accuracy, we advocate performing IMs prediction in the plane wave domain.

We thank the sponsors of CREWES for continued support.