Pre-Stack F-K Median Filtering

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ABSTRACT

Conventional P-wave recording makes use of receiver group arrays to attenuate the low-frequency, low-velocity ground roll. Additionally, a group of N geophones increases the signal-to-noise ratio by a factor of $N^{1/2}$ times that recorded on a single geophone. In the case of converted P-SV wave data, pre-stack filtering should not only reduce the amplitudes of surface waves but also improve the signal-to-noise ratio by removing unwanted random noise and noise spikes. The median filter using f-k weights removes unwanted noise glitches, attenuates the often aliased surface waves, and improves the signal-to-noise ratio. A synthetic shot gather demonstrates these features of a weighted median filter relative to the more conventional frequency-wavenumber type filters. A 2-component (vertical and radial) line shot by the University of Calgary field school is processed with both types of pre-stack filters. The f-k weighted median removes aliased noise because it is a time domain operator and not subject to frequency domain wrap-around. In addition, random noise rejection is superior because the non-linear median weighted filter removes the unwanted noise but the conventional f-k filter smears within its pass band. Noise glitches in a shot record remain as the f-k filter response after conventional f-k filtering but are completely removed by a weighted median filter.

INTRODUCTION

Conventional P-wave recording uses multi-geophone arrays to enhance the signal-to-noise ratio and reduce the often large amplitude, low velocity, low frequency ground roll waveform on a seismic record. Current P-SV converted wave acquisition generally uses a single 3-component geophone at each station location, with most P-SV energy on the radial channel.

This fundamental difference in acquisition leads to three important questions. Can noise on the converted wave record be sufficiently removed providing comparable seismic sections to those obtained in conventional recording? In areas of significant, often aliased ground roll the frequency and apparent velocity are within the same range as the shear waves (Knopoff, 1952). Is there a method of removing the often large amplitude surface waves from the P-SV record without degrading the reflected signal? Will recording the full wavefield enable elimination of noise and hence improve the resolving power of a seismic section?

Brown (1990) implies one of the future benefits of recording the full wavefield is the potential of improvement in image quality. An example using a filter based on the polarization analysis of the ground roll on the vertical and radial shear channels is demonstrated by Shieh and Herrmann(1989).

In this paper we are primarily concerned with the first two questions and have not yet considered the advantages of the weighted median filter to the full wavefield data set. The objectives are to improve the P-SV converted wave section by reducing random noise, removing noise glitches, and attenuating unwanted aliased dipping noise.

THEORY

Two statistical averages used to filter digital images and seismic data are the mean and the median (Claerbout, 1985). Median filters are more robust because extraneous data errors such as noise spikes or glitches are not included in the average (Claerbout and Muir, 1973).

If it is desirable to bias the output to some series of coefficients the weighted median concept can be used. This is done by repeating the more important data values by the multiplicity given by the absolute value of the weighting coefficients, sorting this data series and selecting the middle value. Stewart (1985) applied this technique to an NMO corrected shot gather and used the time domain coefficients of an f-k fan filter as the weights. This enables dip rejection and/or enhancement coupled with the desired properties of the median filter (glitch rejection, edge preservation).

Spatially aliased noise on a pre-stack gather occurs when the spatial wavenumber/temporal frequency ratio or dip wraps around to the opposite dip in the f-k domain. Referring to Figure 1 event A is not spatially aliased although if it was not temporally anti-aliased filtered it could reappear at A^* as shown. Event B is spatially aliased and reappears at a negative dip as B^*. In the time domain spatial aliasing can be recognized by dip reversal. 2-D time domain filtering may not suffer from this problem as it arises because of the Fourier transformation of the data (Hatton et al, 1986).

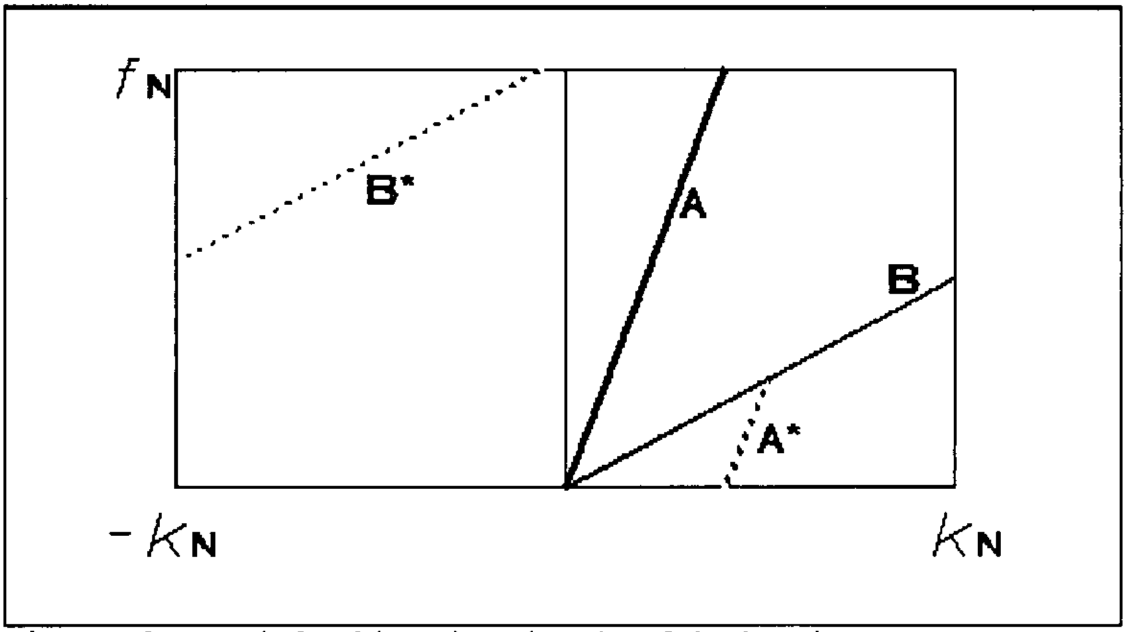

Figure 1 Spatial aliassing in the f-k domain.

The mean value of a sequence is a statistical measure commonly applied to the digital processing of seismic data. We can define the minimization of the sum of the squared differences x_{mean} of a data series x_i where N is the number of samples in the filter window as:

$$x_{mean}: \min \sum_{i=1}^{N} (x_{mean} - x_i)^2 \quad (1)$$

This can be minimized by setting the partial derivative of the square of the sum of differences equal to zero:

$$\sum_{i=1}^{N} 2\,(x_{mean} - x_i) = 0 \quad (2)$$

or

$$x_{mean} = \frac{1}{N} \sum_{i=1}^{N} x_i \quad (3)$$

In simple median filters all data values within the window have the same influence on the resulting output analogous to an equally weighted running average filter.

We can minimize the sum of absolute values to obtain the median (Claerbout, 1985) as:

$$x_{median}:\min\sum_{i=1}^{N}|x_{median}-x_i| \tag{4}$$

Again this can be minimized by setting the partial derivative of this function equal to zero

$$\sum_{i=1}^{N} sgn(x_{median}-x_i)=0 \tag{5}$$

where the sgn function is +1 when the difference is positive and -1 when the difference is negative. Equation 5 formally defines x_{median} so that it is greater than x_i for N/2 terms and less for the other N/2 terms. Note that if N is an even number equation 4 suggests that x_{median} be between the middle two values of x_i.

Similarly we can define the weighted median function as:

$$x_w:\min\sum_i |w_i||x_w-x_i| \tag{6}$$

where x_w is the weighted median value. This reduces to the previous definition if all of the weighting factors are equal to 1. Including a weight of 2 means duplicating the same data value twice before selecting the middle value. Also note that the output median value is always equal to an actual input data value even if the weights are non-integer. Based on the above definitions negative weights are not included into this estimation process. Stewart and Schieck (1989), demonstrate how the negative filter coefficients (band-pass, f-k) commonly used in digital signal processing can be applied using the weighted median definitions. The data values are multiplied by the sign of the associated filter coefficients and weighted according to the absolute value of the filter coefficients:

$$x_w:\min\sum_i |f_i||(sgnf_i)*x_i-x_w| \tag{7}$$

Again this function is minimized by setting the partial derivative equal to zero

$$\sum_{i=1}^{N} |f_i| \frac{\partial}{\partial x_w} |(sgnf_i) * x_i - x_w| = 0 \qquad (8)$$

$$\sum_{i=1}^{N} |f_i| \frac{\partial |\gamma_i|}{\partial x_w}, where \gamma_i = (sgnf_i) * x_i - x_w$$

$$\sum_{i=1}^{N} |f_i| \frac{\partial}{\partial x_w} (sgn\gamma_i) * \gamma_i = \sum_{i=1}^{N} |f_i| \gamma_i, with \frac{\partial sgn\gamma_i}{\partial x_w} = 0$$

$$\sum_{i=1}^{N} |f_i| [(sgnf_i) * x_i - x_w] = 0 \qquad (9)$$

In applying the weighted median process with negative coefficients f_i the data values are multiplied by the polarity of the coefficients and augmented by the absolute value of these weights. This augmented array is then sorted and the middle value is selected. Note also from equation 9 that the output value although corresponding to an input value of the augmented array could actually be reversed in polarity. This is analogous to a mean process in which the data values are multiplied by the filter coefficients and summed to obtain an output value.

For example, consider the filter weights $f_i = (-2,1,1,3)$ and a series of data values $x_i = (-1,3,1000,-2,1,4,3,...)$. At the first sample location the absolute values of the filter weights are attached to the data and the first sample is reversed in polarity as follows:

$$x_i = 1,\ 3, 1000,\ -2$$
$$f_i = 2,\ 1,\ \ 1,\ \ 3$$

then the data is sorted in pairs carrying the attached filter weights.

$$x_i = -2,\ 1,\ 3, 1000$$
$$f_i = \ 3,\ 2,\ 1,\ \ 1$$

The total sum of the filter coefficients is 7 so the middle value occurs where the sum of the weights equals 4 or $x_w = 1$. The resulting weighted median output series would then be $x_{w+2} = (1,1,1,3,...)$. Similarly, by multiplying the weights by their corresponding data values and summing at successive indices the mean process yields the output sequence $x_{w+2} = (999,995,-1989,18,...)$.

METHODS

The computational time for a mean process is proportional to N, the number of points at each spatial application of the operator. However the time required to order a list of numbers is NlnN (Wirth, 1986). For example a 2-D operator such as will be demonstrated in this paper requires 13 traces by 15 time samples or 195 data points to be augmented by filter coefficients and sorted at each data point. This amounts to roughly 5 times the computational effort for a weighted median process relative to the mean. Sorting algorithms can be significantly improved if the data is partially sorted. In this way the computational effort can be reduced to 3N suggesting the f-k weighted median as applied in this paper will take only 3 times more time than the more conventional mean f-k filters commonly in seismic processing.

One method of supplying a partially sorted array to the sorting algorithm is to use the sorted 2-D data box of the previous time sample and replace only the new locations with the data values at the current time location (refer Fig. 2). Two additional considerations must be addressed to use this roll-along method. First the data values that were previously reversed due to negative filter coefficients that are now positive (or visa versa) must be flipped in polarity before sorting. Second, all of the filter coefficients previously attached to the data must also be moved down in time. This is easily achieved be attaching only the address of the filter coefficients and decrementing this address by the spatial width of the 2-D operator (ie. -13 traces). The rank of the sorted array will indicate where within the sorted data array each index of the original 2-D data matrix is actually located thus enabling direct replacement of the oldest time slice with the new data values into the previously sorted array.

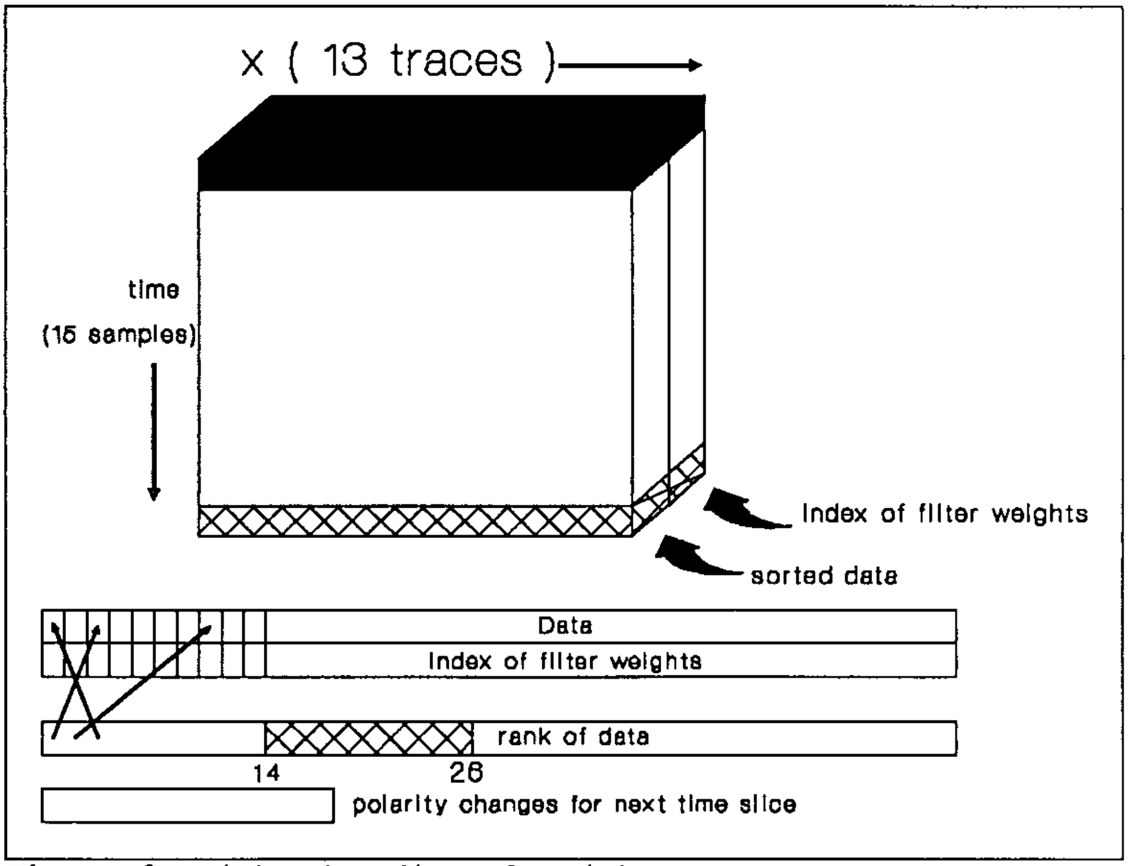

Figure 2 Weighted median Algorithm

RESULTS

To test the aliased dip reject characteristics, random noise reduction and the de-glitching aspects of the f-k weighted median filter a synthetic shot gather was generated (Fig. 3). This consists of 11 primary events derived from a velocity function representative of South-central Alberta, band limited to 10-70 Hz. Three large amplitude noise glitches of the same frequency bandwidth located below the last primary are included to test the de-glitching capabilites of the mean and median filters. Also, two aliased dipping events which might be representative of non-dispersive ground roll (8-15 Hz, 70 ms/trace) and a near surface multiple refraction (14-70 Hz, 16 ms/trace) are inserted in the synthetic. The two dipping events are spatially aliased as seen in the f-k transform plots both wrap-around to negative dips as previously discussed in figure 1.

This synthetic is f-k filtered with a dip reject of +/- 4 ms/trace (Fig. 4). The glitches are not removed and remain as the time domain operator of the f-k filter. The higher frequency, 16 ms/trace aliased dip appears as a negative dip 50 to 70 Hz event and

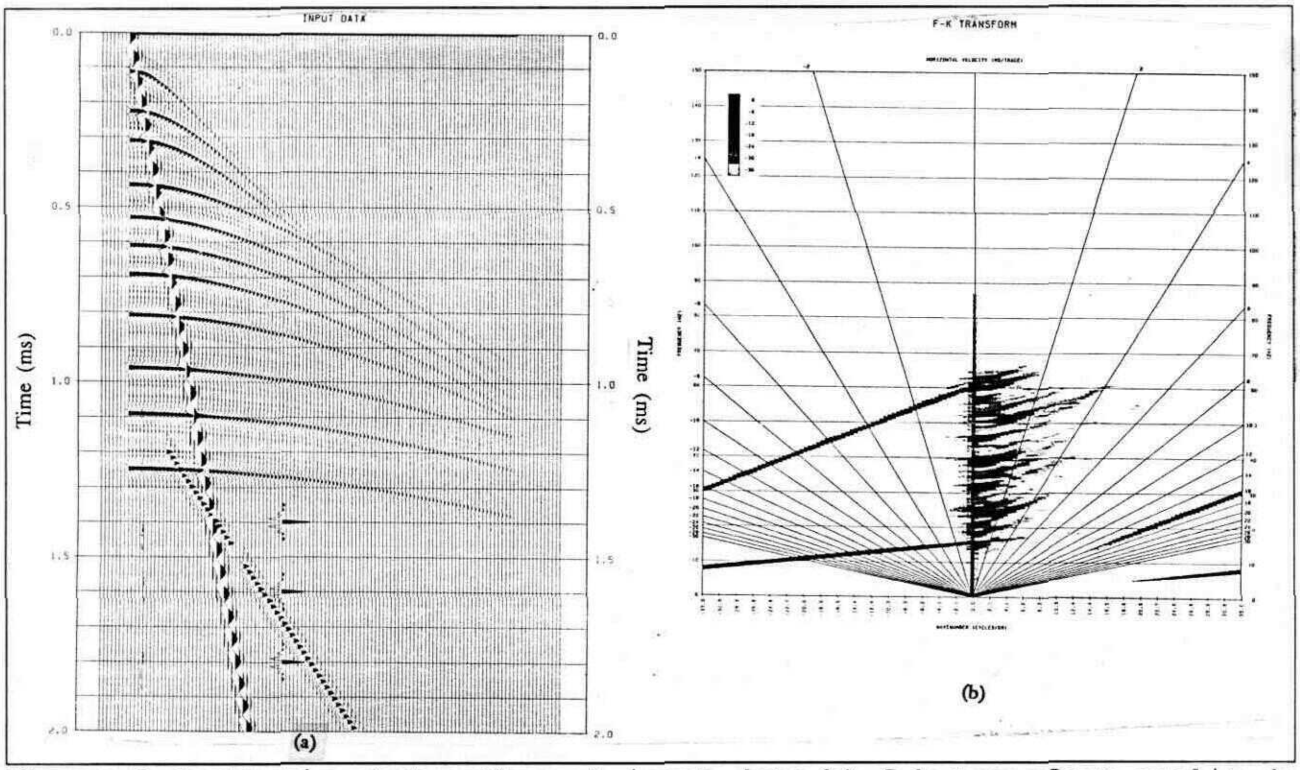

Figure 3 Synthetic shot gather a) input data b) f-k transform amplitude spectrum

the lower frequency ground roll is not completely removed. The primary events are virtually identical to the input shot gather in Figure 3. Random noise, band limited to 15-70 Hz, is added with an RMS magnitude 1/3 that of the primary events (Fig. 5). Figure 6 is the output after applying the +/- 4 ms/trace f-k filter previously applied as in Figure 3. Random noise is reduced only by the smear of the time domain operator and still remains within the pass band of the spatial filter. Figure 7 is the result of applying the same spatial operator truncated to a 13 trace by 15 sample 2-D matrix and used as weights in a weighted median process. The primaries remain relatively untouched yet the aliased dips are completely removed. Random noise is not smeared as can be seen in the absolute rejection of the large amplitude noise glitches.

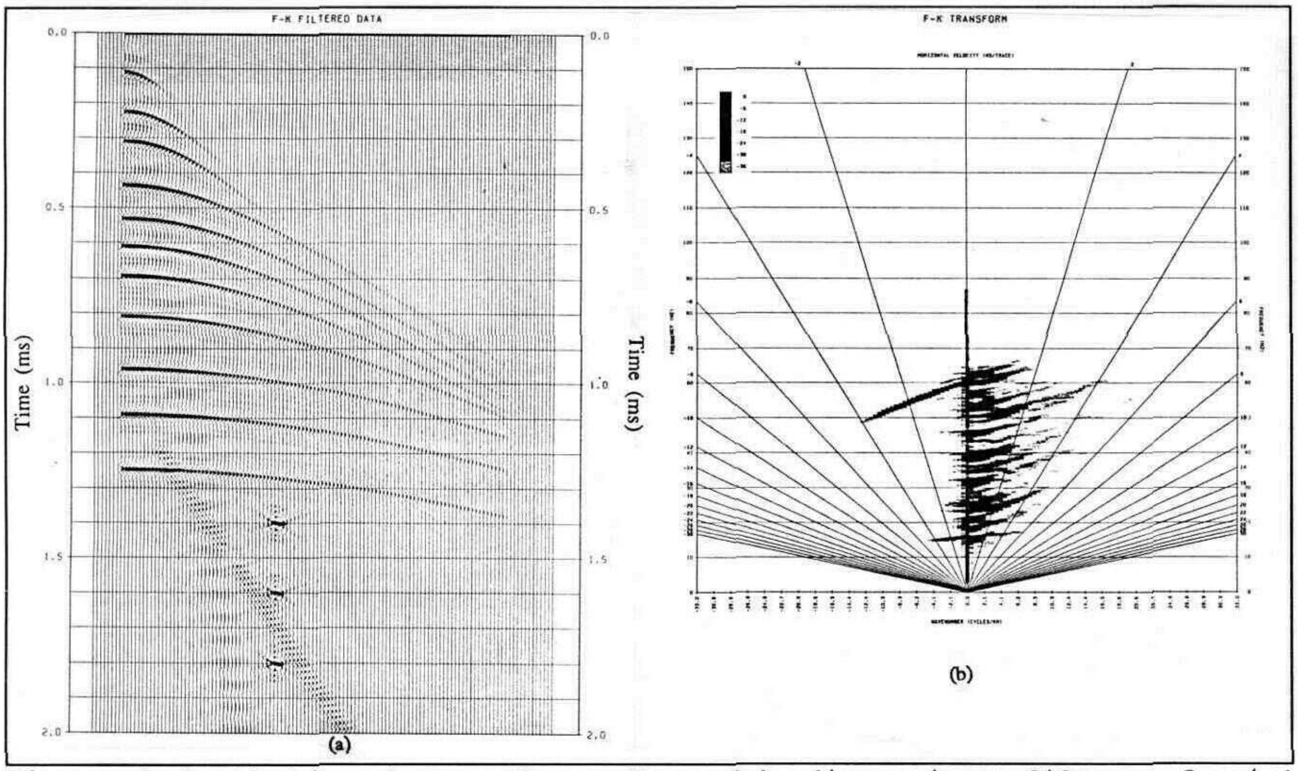

Figure 4 Synthetic shot gather after f-k dip reject filter of +/-4 ms/trace. a) time domain shot gather b) f-k transform

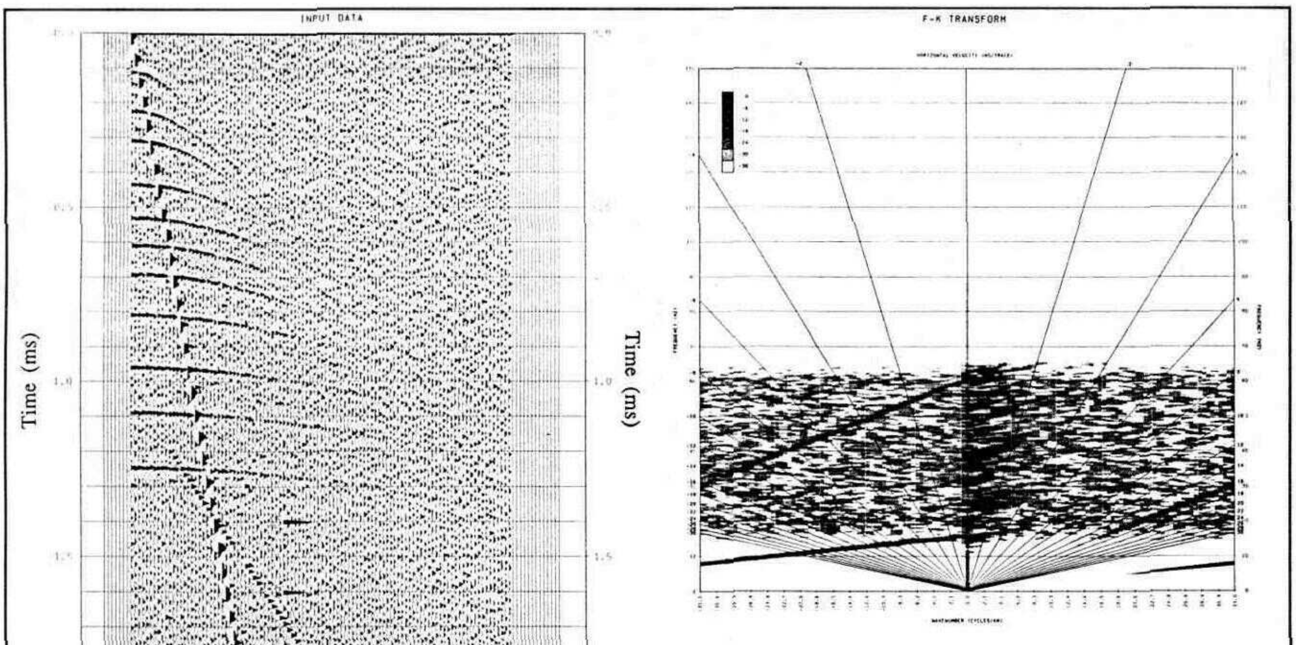

Figure 5 Synthetic shot gather after band limited random noise added a) time domain shot gather b) f-k transform

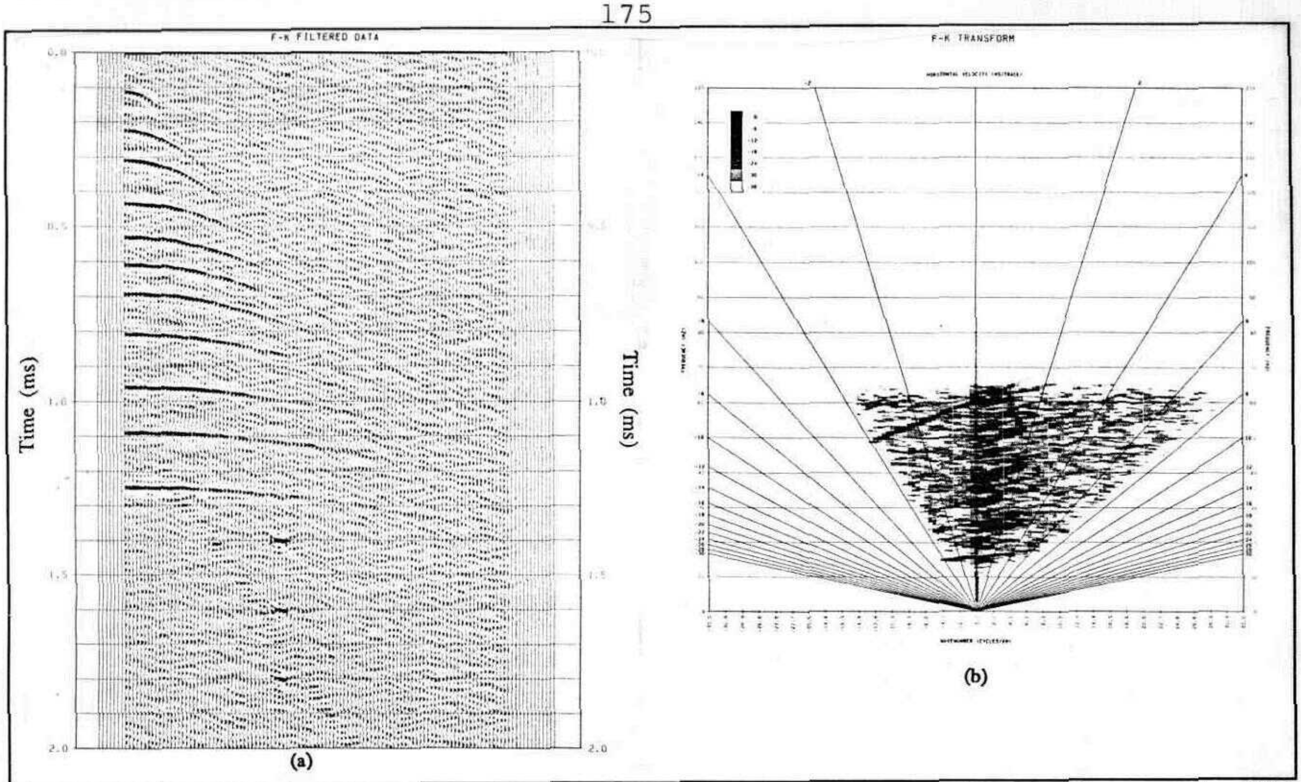

Figure 6 Synthetic shot gather wih random noise added after f-k dip reject filter of +/-4 ms/trace. a) time domain b) f-k transform

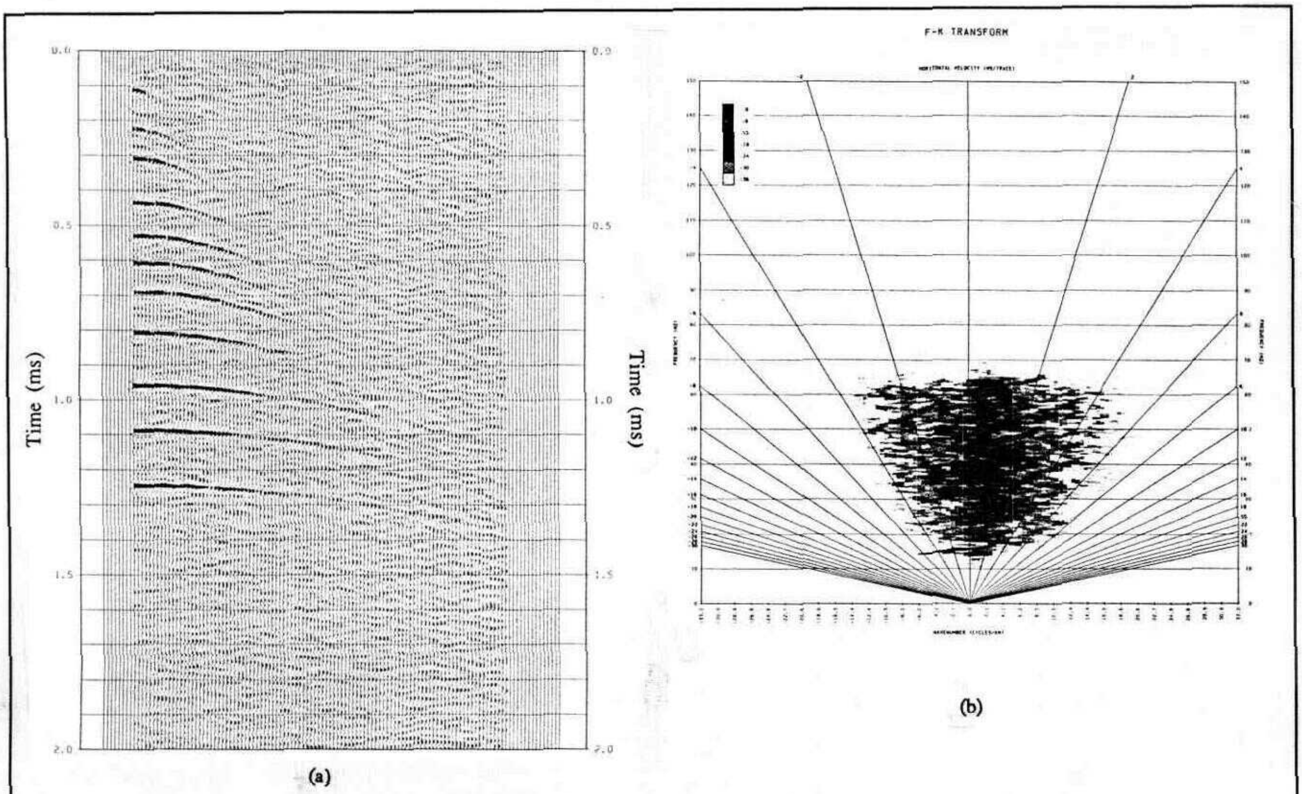

Figure 7 Synthetic shot gather with random noise added after f-k weighted median with +/- 4 ms/trace reject a) time domain b) f-k transform

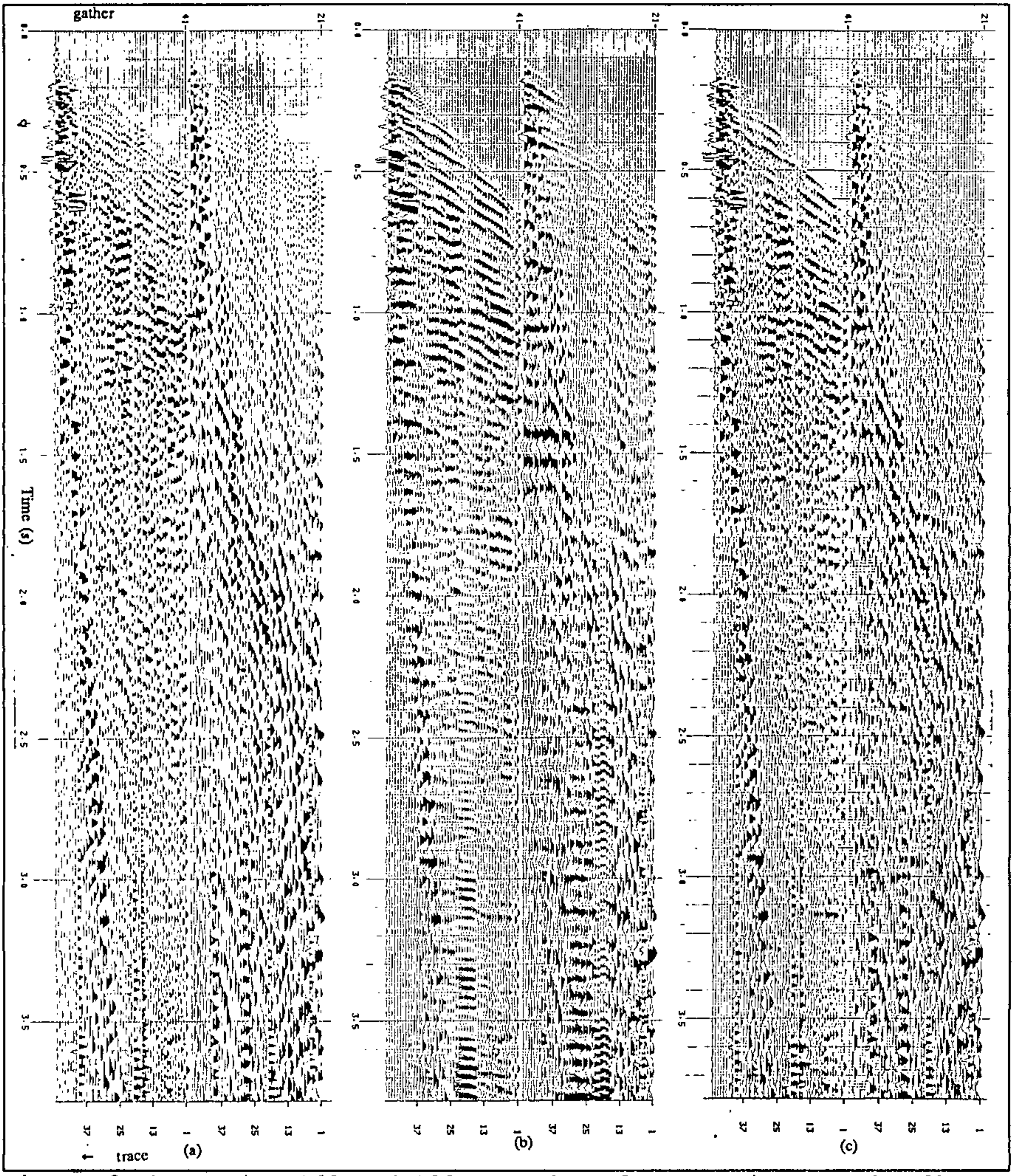

Figure 8 Shot point 168 and 199 P-SV data from Jumping Pound, Alberta a) Input b) after f-k filtering c) after f-k median filtering.

A real data example is applied to P-SV data acquired by the University of Calgary field school in August 1990 from Jumping Pound, Alberta (Twp 25, Rge 3, W5). Two P-SV shot gathers are displayed in Figure 8 after spherical divergence, low pass filtering (60 Hz, -72 dB at -3 dB down) and RMS trace scaling. The trace interval is 30 meters. Three types of noise can be isolated on the input shot gather. Noise glitches caused by dead traces (traces 17 & 20 and 20 & 40 on shot gathers 21 and 41 respectively). Ground roll with a velocity of 200 m/s and near surface refracted reverberations with a velocity of 1470 m/s is observed particularly on the west end of the line (shot gather 21). The spatial filters were applied after initial statics and velocities. Dips greater than +/- 4 ms/trace dips rejected in Figure 8 b) and c). The frequency edges in the f-k domain were tapered with a 2-D moving average filter with a size of 30% of the nyquist frequency (5 by 150 point window smoother). A percentage of the input data set (30%) was added back to the filtered shots to dampen the filter effects. The time domain operation requires a taper to be applied to the edges of each shot gather as the 13 trace 2-D matrix of filter coefficients rolls on and off the data. This was achieved by adding back 100% of the original trace when the 2-D box was centered at the first trace to 0% when the data box was completely filled. The weighted median process for all 62 shot gathers was completed in core on an IBM 3081 with a virtual time of 6 hours and the conventional f-k was processed with the use of an FPS-190L array processor in only 30 minutes. Figure 8 b) demonstrates the smear of an f-k filter. Dead traces on shot gather 41 (ie. trace 17) are spread over approximately 12 traces. The f-k weighted median shows similar dip reject capabilities and does not smear the noise.

2-D f-k transforms of shot gather 41 are shown in Figures 9 to 11. The f-k weighted median filter does not dip reject as severely as the conventional f-k. The aliased ground roll is attenuated comparably at all frequency/wavenumbers (4-14 Hz) in the case of the weighted median (Figure 11) but only the negative dips at 8-12 Hz dominate the conventional f-k filter transform (Figure 10).

The data were further processed, after each pre-stack filterapplication, using the parameters and sequence outline in Table 1. Initial statics and velocities used in the applications of the pre-stack filters were removed before deconvolution. Each data set was processed with the final statics and velocities obtained from the unfiltered data set to maintain consistency. Only the residual trim statics between stacks might be different due to the correlation models dissimilarities after differing pre-stack filters. Figure 12 shows a sample common offset stack between shot point id's 145 and 152. The f-k weighted median filter (Figure 12 c) demonstrates similar dip rejection characteristics to the f-k filter (Figure 12 b) yet there is less noise smearing. Primary events after the median process are better imaged then after the mean f-k application (ie. at the time of 1.5 s.). The final P-SV CDP stacks are displayed in Figures 13 to 15. The median processed data improves the signal-to-noise ratio by removing noise glitches and attenuating aliased noise consistently. This process does not leave the impression of a mixed or smeared section as in the case of the mean f-k but honours the overall character of the original un-filtered input section.

Table 1. Processing sequence and parameters for the radial (P-SV) data.

```
DEMULTIPLEX
GEOMETRIC SPREADING COMPENSATION
     1.0 * e^0.0007
[OPTIONAL PRE-STACK FILTER]
SPIKING DECONVOLUTION
     2 windows, 120 ms operator 1.0% prewhitening
REVERSE POLARITY OF TRAILING SPREADS
APPLY FINAL P-WAVE SOURCE STATICS
INITIAL VELOCITIES
APPLY HAND STATICS FROM COMMON RECEIVER PLOTS
AUTOMATIC SURFACE CONSISTENT STATICS
     Correlation window from 800 to 3200 ms
     Maximum shift of + or - 36 ms
CDP STACK
CONVERTED-WAVE REBINNING
     Vp/Vs ratio of 2.08 independent of depth
VELOCITY ANALYSIS
NORMAL MOVEOUT
FIRST BREAK MUTE
     distance 525  m, time 550  ms
     distance 2880 m, time 1980 ms
TRACE SCALING
     Mean amplitude of 2000
     Windows 0-800, 600-1600, 1400-3400 ms
CDP TRIM STATICS
     Correlation window from 200-3200 ms
     Maximum shifts + or - 20 ms
STACK
     offsets 30-2400 m
BANDPASS FILTER
     Zero-phase, 8-38 Hz
RMS GAIN
     Mean amplitude of 2000
     Window 400-3200 ms
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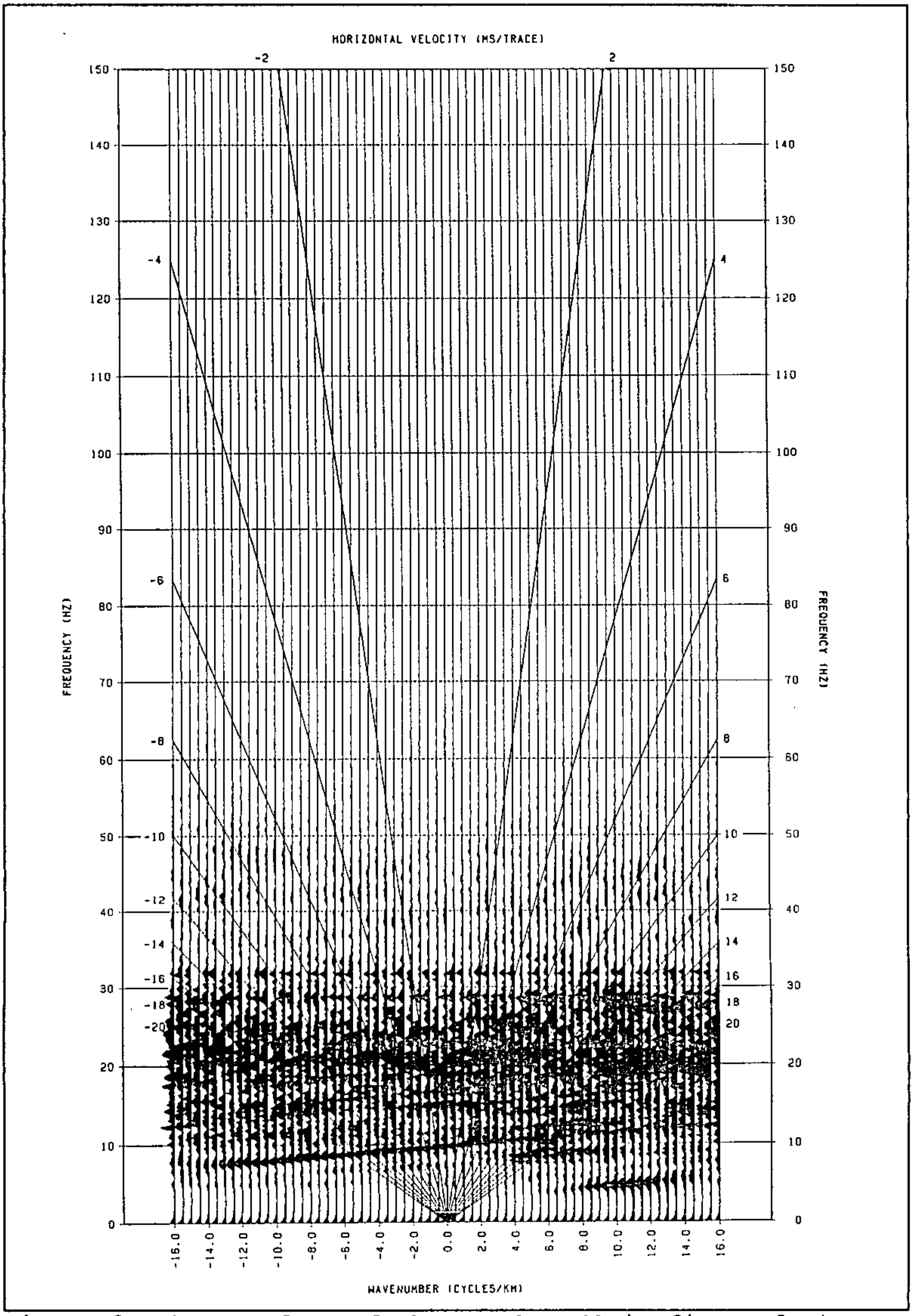

Figure 9 F-k transform of shot gather 41 in figure 8 a)

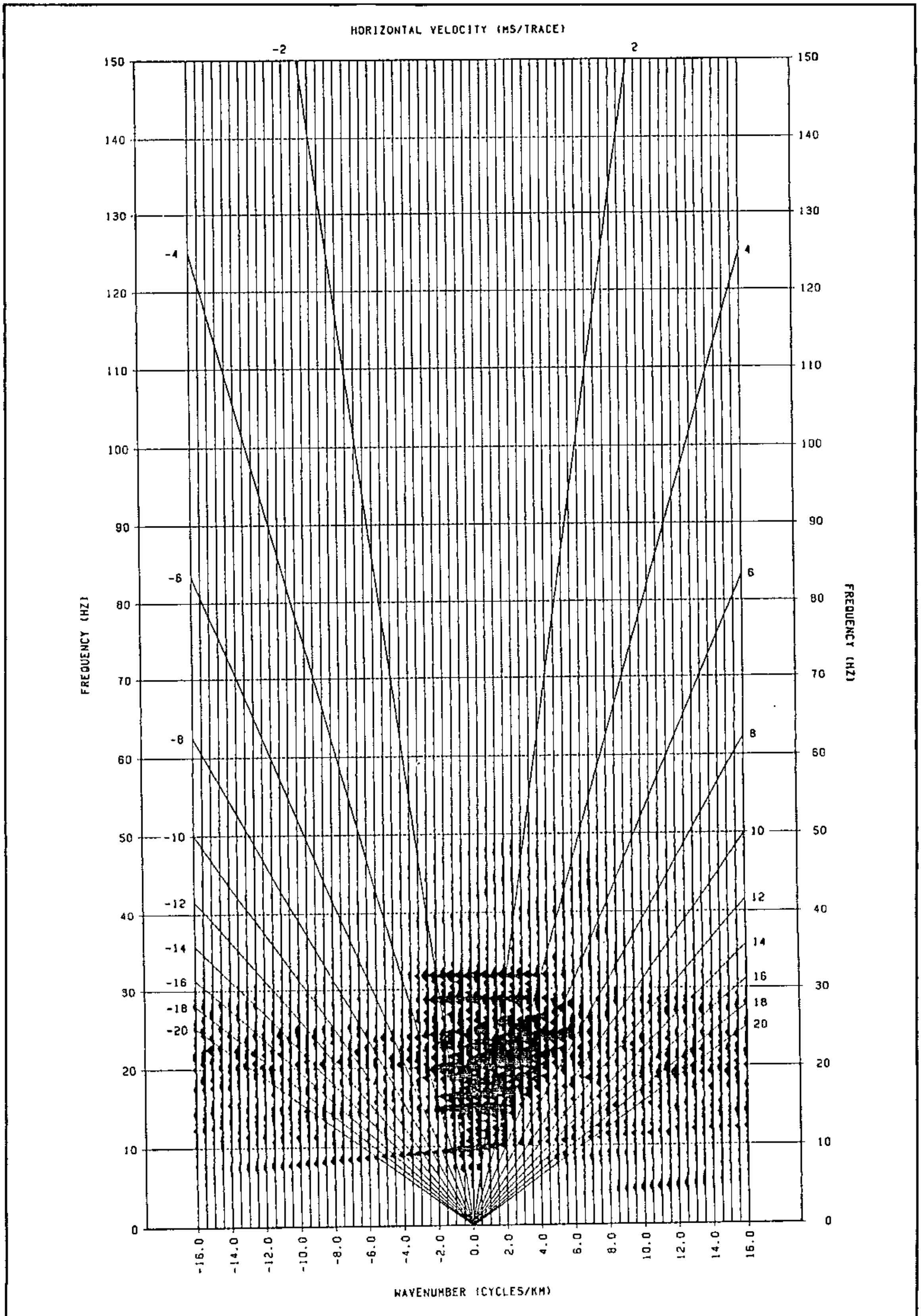

Figure 10 F-k transform of f-k filtered shot from 8 b).

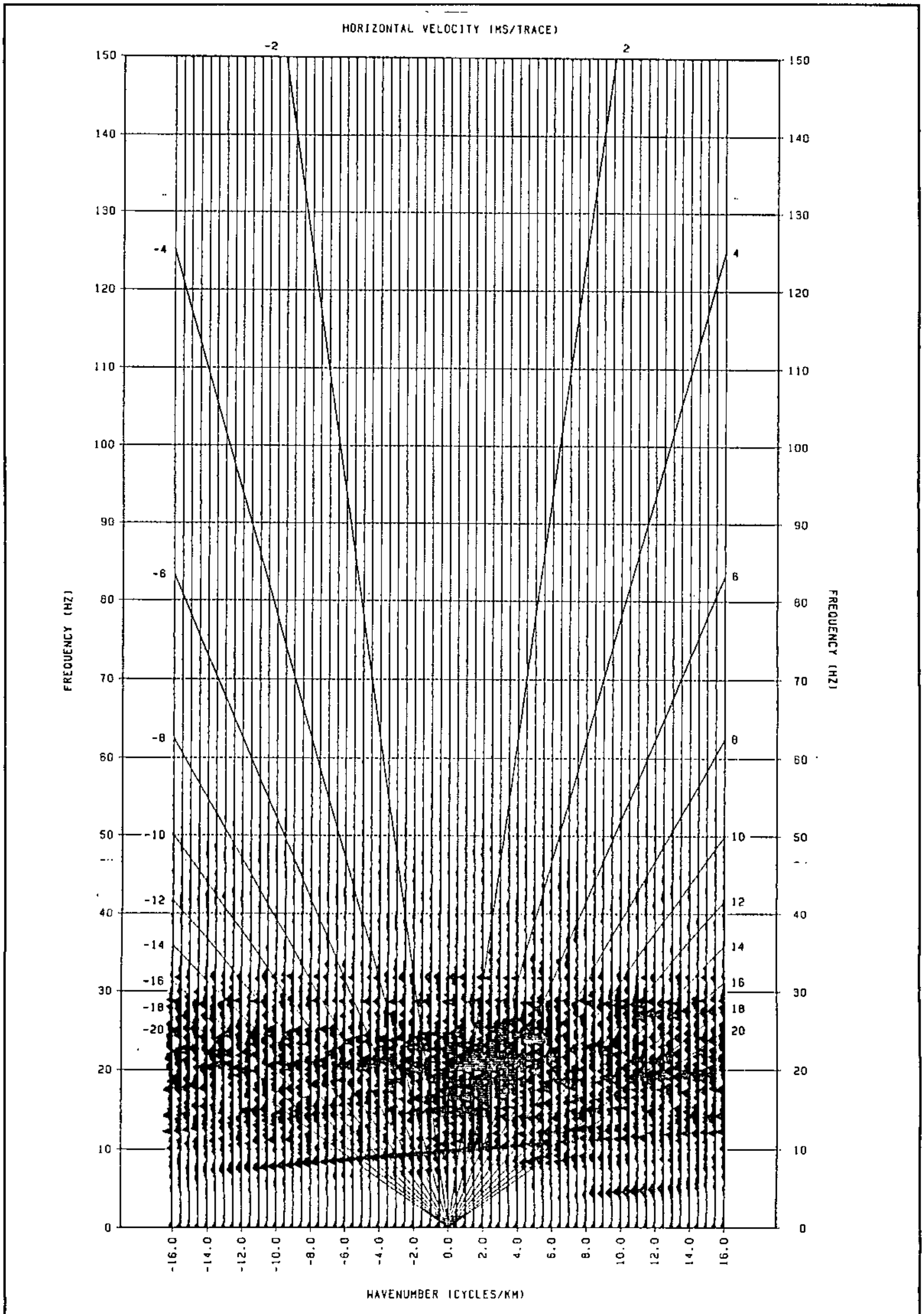

Figure 11 F-k transform of f-k weighted median filtered 8 c)

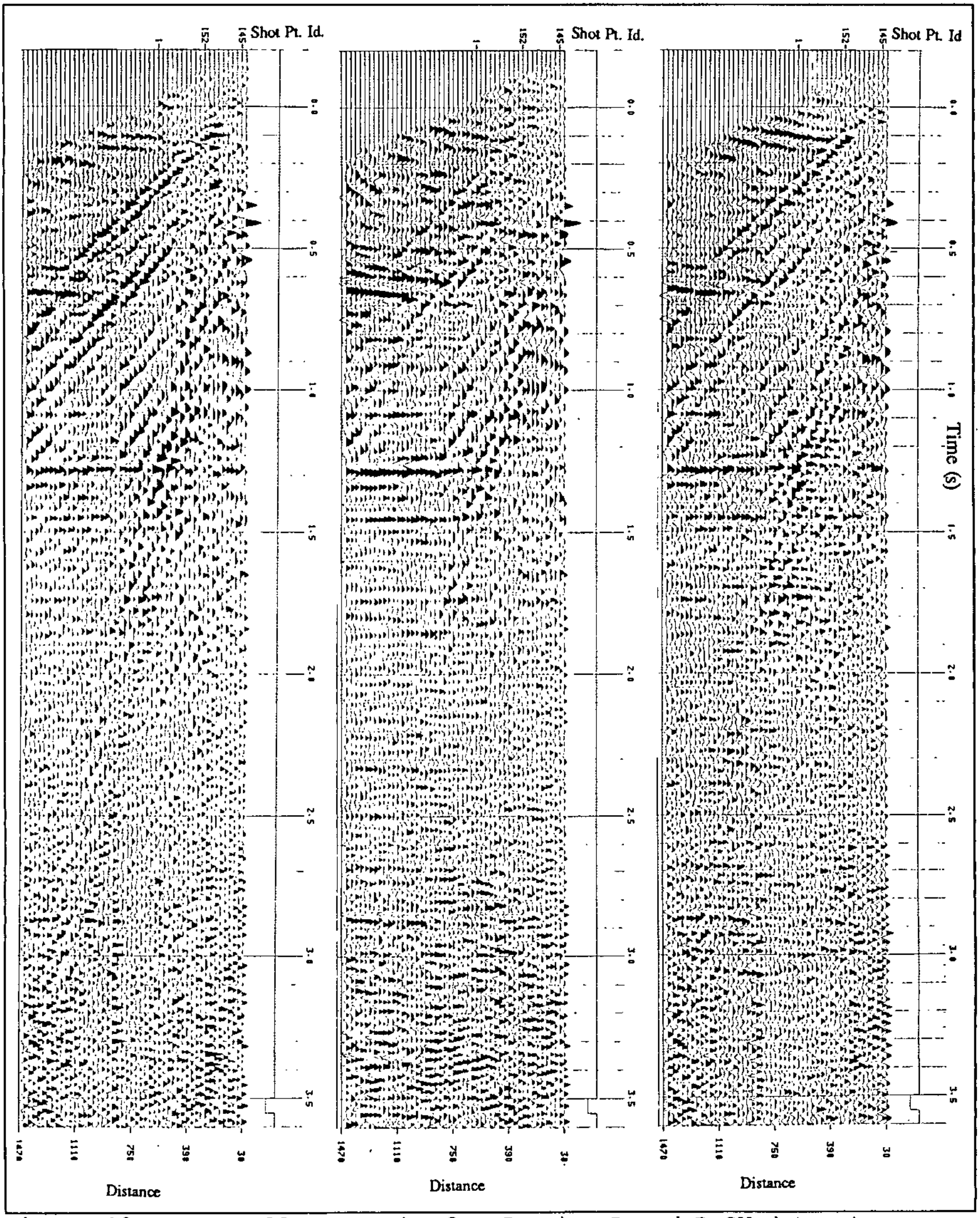

Figure 12 Common offset stacks for Jumping Pound P-SV data a) no pre-stack spatial filtering b) f-k filtered c) weighted median filtered.

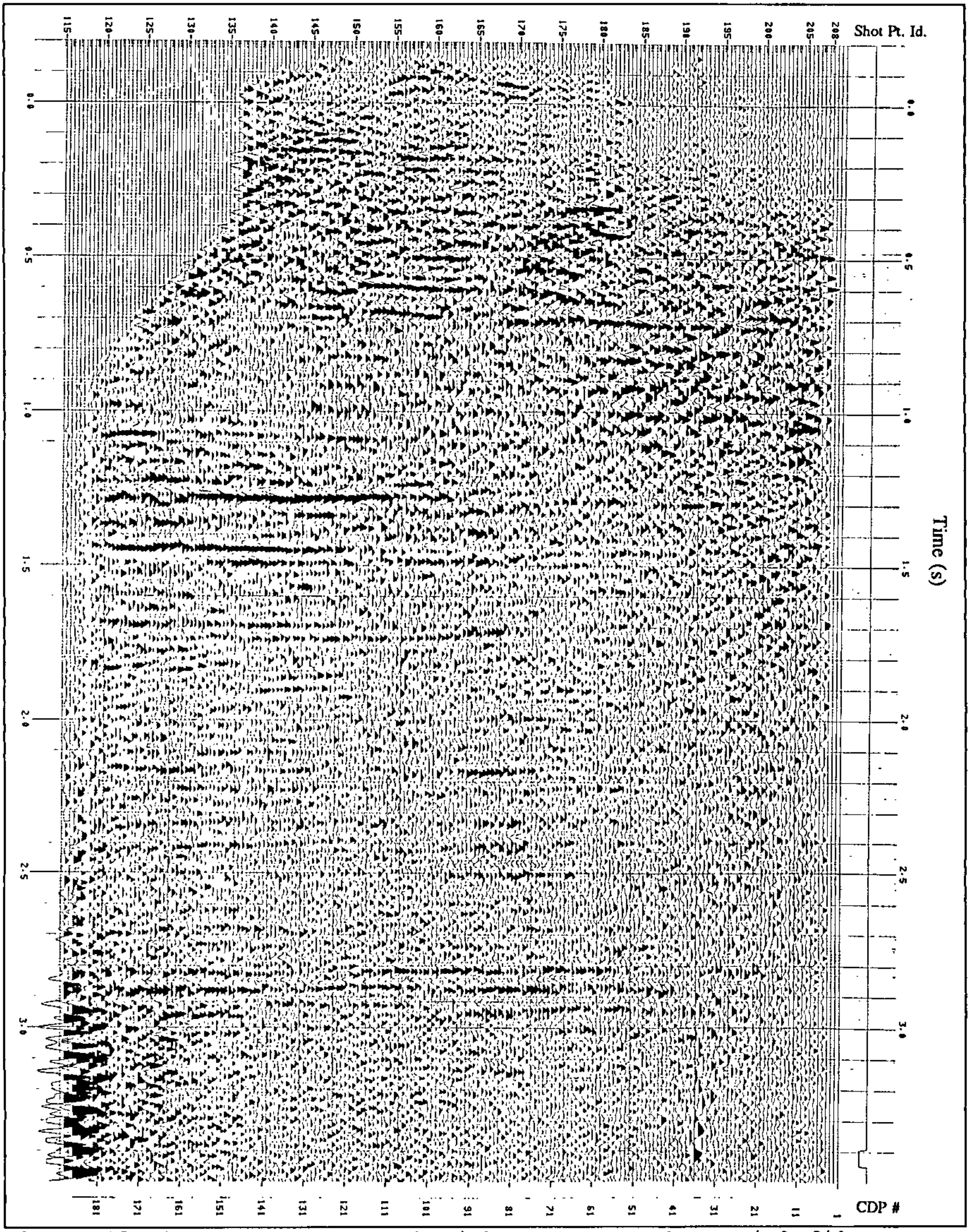

Figure 13 Final P-SV CDP stack without pre-stack spatial filter

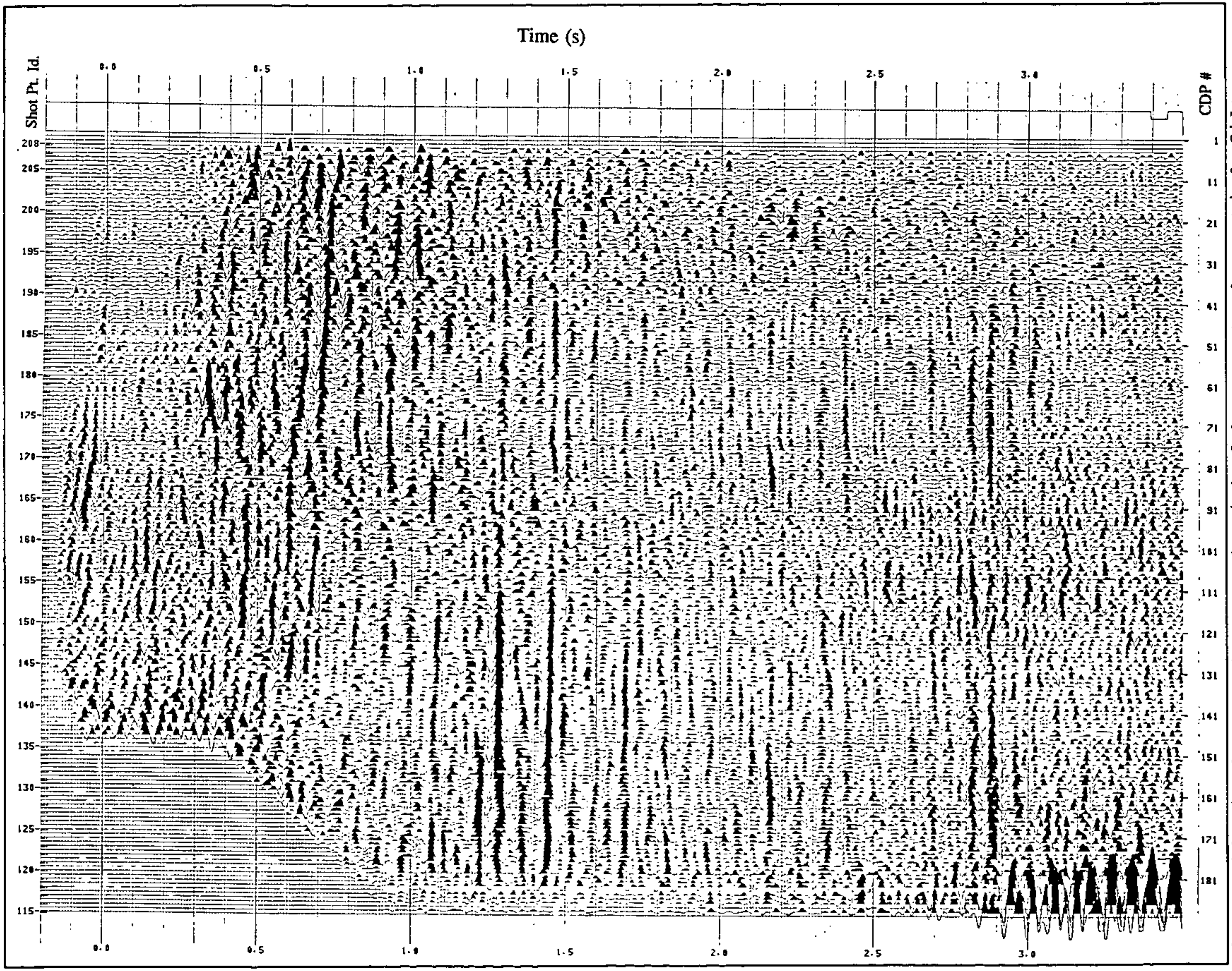

Figure 14 Final P-SV CDP stack with f-k pre-stack spatial filter

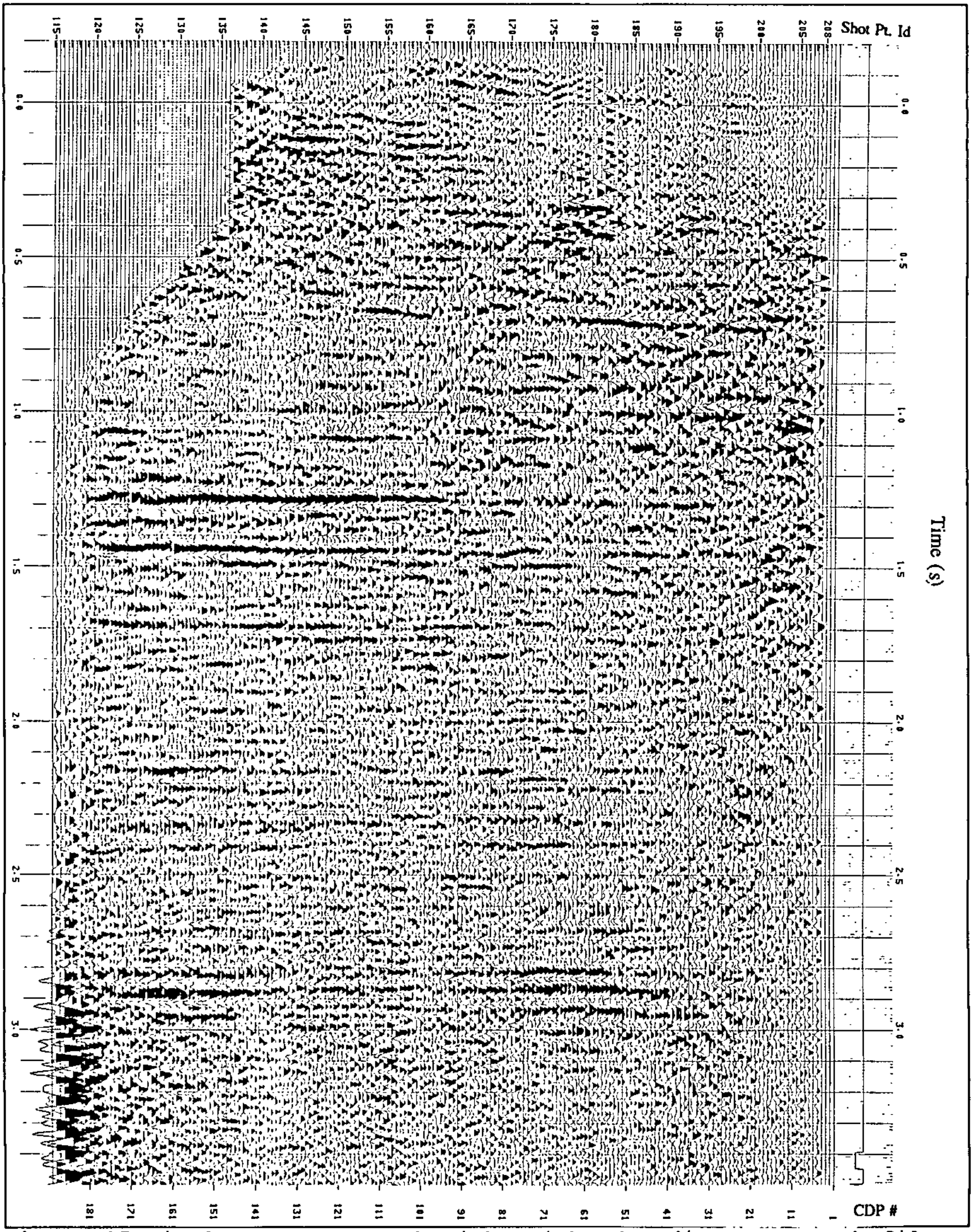

Figure 15 Final P-SV CDP stack with weighted median pre-stack filter

CONCLUSIONS

Pre-stack filters are useful in removing coherent noise on a shot gather before stacking. The edge preserving and de-glitching characteristics of a median process can be applied advantageously to pre-stack seismic records. The aliased dip rejection capabilities of a time domain operation can also be exploited with a f-k weighted median process. The median process is more computationally intensive than mean processes. It appears to reduce random gaussian noise and effectively removes glitches without merely smearing noise spikes in the shape of the time domain operator.

ACKNOWLEDGEMENTS

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