Using P-SV waves to improve conventional AVO estimates: A synthetic study

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ABSTRACT

Linear equations relate small changes in rock properties across an interface to seismic reflectivity (Aki and Richards, 1980). These equations for P-wave and P-SV wave reflectivity can be inverted exactly or in a least-squares sense to provide estimates of relative changes in density, P-wave velocity and SV-wave velocity. By using two observations (P and P-SV reflectivity), this inversion promises better rock property estimates. Based on synthetic seismic data, an error analysis is performed on this joint P and P-SV inversion method. Two steps are taken in this inversion. First, a least squares adjustment is applied to the joint P and P-SV inversion to estimate relative changes in P - wave and SV -wave velocity across an interface. Second, to update the actual P - wave and S - wave velocities from the relative changes, the generalized linear inversion is used. Effects of systematic error and random error in both velocity and reflectivity on the inversion velocity structures are analyzed. Compared with the conventional method (using only P-P reflectivity), the joint P-P and P-SV inversion gives more reliable results for S - wave velocity, especially in the presence of noise in the reflectivities.

INTRODUCTION

In the attempt to understand subsurface lithologies, it is useful to have not just P-wave properties but those of the S wave (e.g. Danbom and Domenico, 1986). Amplitude-versus-offset (AVO) analysis tries to infer S-wave velocities (or Poisson's ratio) from the change of P-wave reflectivity R^{pp} with varying angles of incidence: The change in R^{pp} is partially controlled by the conversion of P-wave into S-wave energy, according to the S-wave velocities. Aki and Richards (1980) give the equations for P-wave reflectivity and P-SV reflectivity, assuming small changes in elastic-wave properties across an interface:

$$R^{pp}(\theta) \sim \frac{1}{2}\left(1 - 4\frac{\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\rho} + \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\alpha} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta\beta}{\beta} \tag{1}$$

$$R^{ps}(\theta) \sim \frac{-\alpha\tan\varphi}{2\beta}\left[\left(1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho} - \left(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\beta}{\beta}\right] \tag{2}$$

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where

θ is the average of the P-wave angle of incidence on and transmission through the interface,
φ is the average of the SV-wave angle of reflection and its associated transmission,
α, β, ρ are the average P-wave and S-wave velocities, and density across the interface,
$\Delta\alpha$, $\Delta\beta$, $\Delta\rho$ are the P-wave and S-wave velocity changes, and density change across the interface.

As seen in the equations above, converted-wave (P-to-SV) reflectivity is only dependent on density and S-velocity changes and not on P-velocity changes. So if our goal is to find S-wave properties, it is reasonable to try to use converted-wave reflectivity, R^{ps}.

Recently, a joint pure-P and P - SV inversion was proposed by Stewart (1991). The use of two independent observations (pure-P and P - SV reflectivity) promises to give better rock property estimates. The implementation of the joint P-P and P - SV inversion is divided into two steps: First, a least squares adjustment is applied to estimate relative changes in P - wave and SV -wave velocity across an interface, assuming a perfectly elastic, horizontally layered medium. Then, to update the P - wave velocity and SV- wave velocity, the generalized linear inversion is introduced.

To test the sensitivity of the velocity estimates to systematic error and random error in both initial velocity guesses and reflectivity, a quantitative error analysis is performed.

Layered Earth Model

Synthetic seismic data are derived from a layered elastic earth model. This model has seven layers, each with a thickness of 500 m, as shown in Figure 1 (after Smith and Gidlow, 1987). The densities and velocities are also given in Figure 1.

Equations (1) and (2) can be simplified by using an empirical relationship between velocity and density (Smith and Gidlow, 1987). The Gardner et al. (1974) relationship

$$\rho \sim k\alpha^{1/4}, \tag{3}$$

can be written in differential form as

$$\frac{\Delta\rho}{\rho} \sim \frac{1}{4}\frac{\Delta\alpha}{\alpha}. \tag{4}$$

Using (4) in equations (1) and (2) gives

$$R^{PP}(\theta) = a(\alpha, \beta, \theta)\frac{\Delta\alpha}{\alpha} + b(\alpha, \beta, \theta)\frac{\Delta\beta}{\beta}, \tag{5}$$

$$R^{PS}(\theta) = c(\alpha, \beta, \theta)\frac{\Delta\alpha}{\alpha} + d(\alpha, \beta, \theta)\frac{\Delta\beta}{\beta}, \tag{6}$$

where

$$a(\alpha, \beta, \theta) = \frac{1}{8}[1 - \frac{4\beta^2}{\alpha^2}(\sin\theta)^2 + \frac{4}{(\cos\theta)^2}],$$

$$b(\alpha, \beta, \theta) = -\frac{4\beta^2}{\alpha^2}\sin^2\theta,$$

$$c(\alpha, \beta, \theta) = -\frac{\alpha\tan\varphi}{8\beta}[1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\varphi],$$

$$d(\alpha, \beta, \theta) = \frac{\alpha\tan\varphi}{2\beta}(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\varphi),$$

and

$$\frac{\sin\theta}{\alpha} = \frac{\sin\varphi}{\beta}.$$

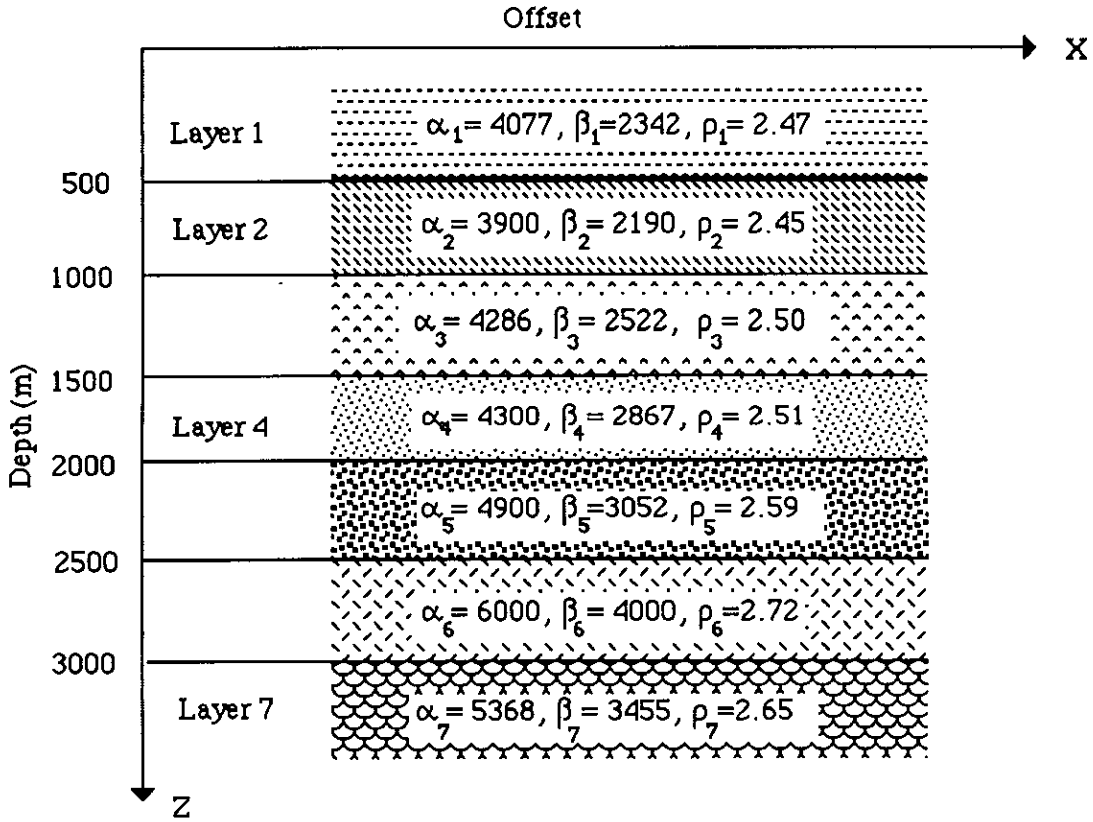


Fig.1: Layered elastic-wave model

The P-wave angle θ of incidence on the interface can be estimated by the following equations (Holzrichter, 1988):

$$\sin^2\theta = \left[\frac{\alpha_{Int}}{\alpha_{rms}}\right]^2 \frac{X^2}{X^2 + (\alpha_{rms}\, T_o)^2} \quad , \tag{7}$$

or

$$\sin^2\theta_i = \left[\frac{(\alpha_{Int})_i}{(\alpha_{rms})_i}\right]^2 \frac{X^2}{X^2 + ((\alpha_{rms})_i\, T_i)^2} ,$$

where

$$(\alpha_{Int})_i = \sqrt{((\alpha_{rms})_i^2\, T_{oi} - (\alpha_{rms})_{i-1}^2\, T_{oi-1})/ (T_{oi}-T_{oi-1})} ,$$

$$(\alpha_{rms})_i = \sqrt{\sum_{n=1}^{i} (\alpha_i\, T_{oi})^2 / \sum_{n=1}^{i} T_{oi}} ,$$

$$T_i = \sum_{n=1}^{i} T_{oi} ,$$

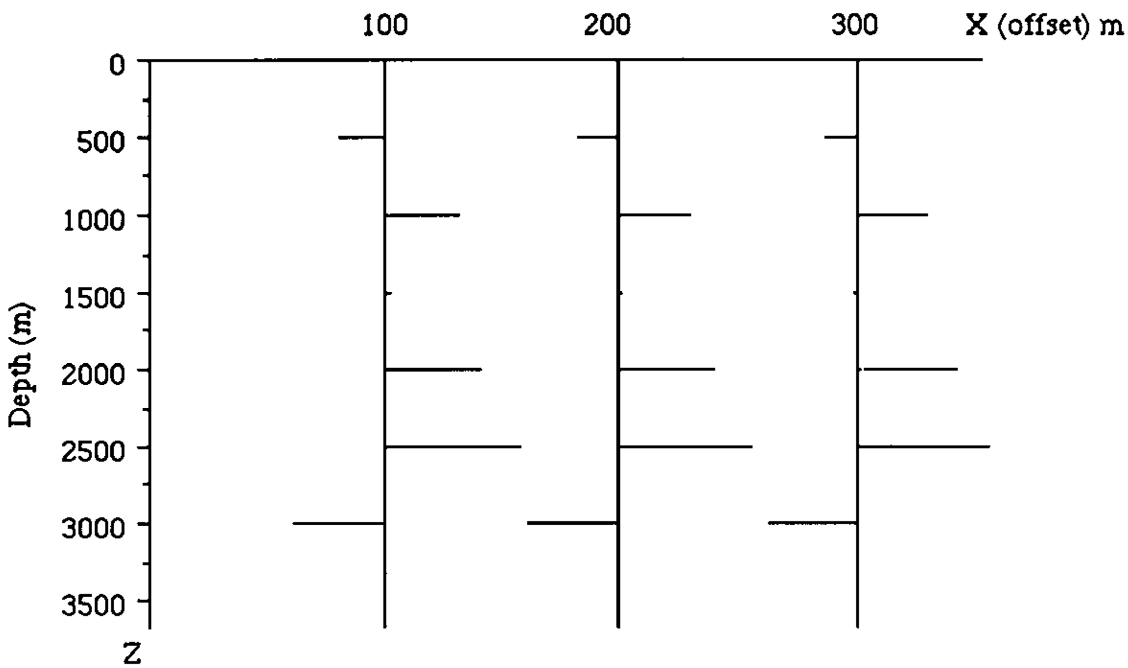
$$T_{oi} = \frac{2\, Z_i}{\alpha_i} .$$

Here T_o and T_{oi} are the two way traveltimes,. Z_i is the thickness of each layer, X is the offset, α_i is the P-wave velocity.

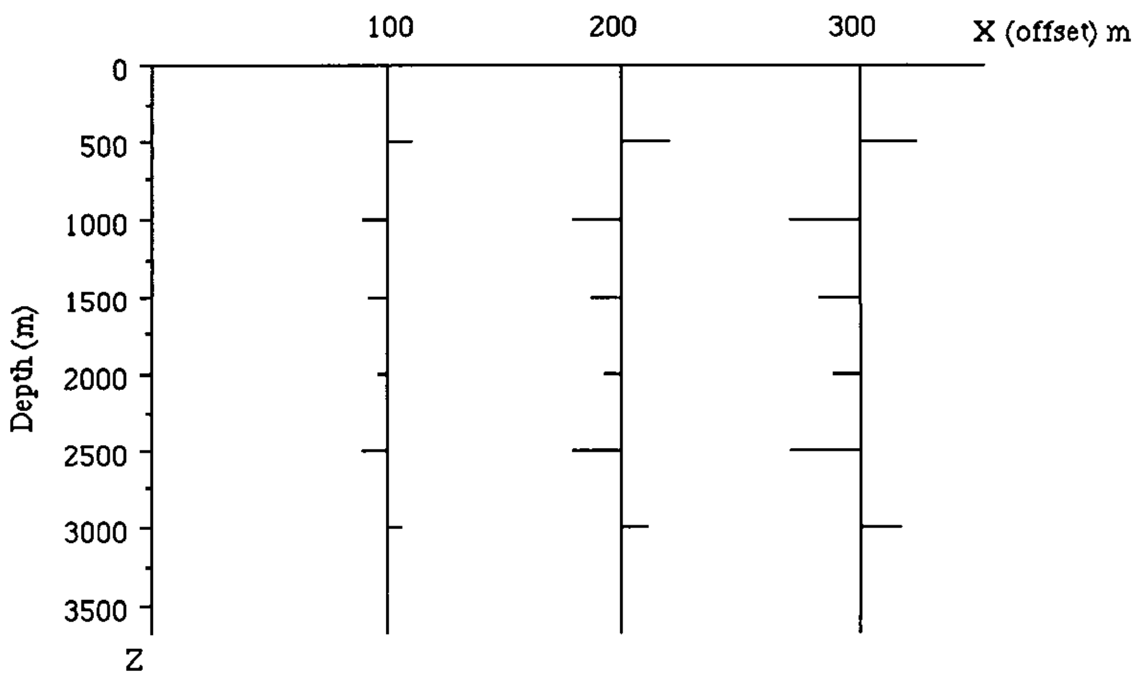
Using the above equations and the model in Figure 1, three traces of synthetic seismic data for P -wave reflectivity and P-SV wave reflectivity across each interface are generated. Figure 2 shows the P-P reflectivity and P-SV reflectivity for the different offsets. In this simple forward model, we have not included the effects of transmission loss, geometric speaking, or attenuation.

Joint P and P-SV inversion

Given the P -wave reflectivity and P-SV reflectivity of each interface for several offsets, we calculate $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ across the interfaces. In processing real data, we conceive of a sparse - spike estimation process that could provide a set of spiked reflectivities similar to the synthetic data under consideration here. An interpretation and registration procedure would also be required to align the P-P and P-SV reflectivities. The inversion procedure is outlined below. Suppose we have a P-P and a P-SV trace, derived from a N-interface model:



(a): P-P reflectivity R^{PP}



(b): P-SV reflectivity R^{PS}

FIG.2 : Reflectivity versus offset

$$R_1^{PP}(\theta_1) = a_1 \left(\frac{\Delta\alpha}{\alpha}\right)_1 + b_1\left(\frac{\Delta\beta}{\beta}\right)_1 ,$$

$$R_1^{PS}(\theta_1) = c_1 \left(\frac{\Delta\alpha}{\alpha}\right)_1 + d_1\left(\frac{\Delta\beta}{\beta}\right)_1 , \tag{8}$$

....

$$R_i^{PP}(\theta_i) = a_i \left(\frac{\Delta\alpha}{\alpha}\right)_i + b_i\left(\frac{\Delta\beta}{\beta}\right)_i ,$$

$$R_i^{PS}(\theta_i) = c_i \left(\frac{\Delta\alpha}{\alpha}\right)_i + d_i\left(\frac{\Delta\beta}{\beta}\right)_i ,$$

....

$$R_N^{PP}(\theta_N) = a_N\left(\frac{\Delta\alpha}{\alpha}\right)_N + b_N\left(\frac{\Delta\beta}{\beta}\right)_N ,$$

$$R_N^{PS}(\theta_N) = c_N\left(\frac{\Delta\alpha}{\alpha}\right)_N + d_N\left(\frac{\Delta\beta}{\beta}\right)_N ,$$

which can be written in matrix form as

$$L_1 = A_1 X, \tag{9}$$

where

$$L_1^T = (R_1^{PP}(\theta_1)R_1^{PS}(\theta_1)R_2^{PP}(\theta_2))R_2^{PS}(\theta_2) \ldots R_i^{PP}(\theta_i)R_i^{PS}(\theta_i) \ldots R_N^{PP}(\theta_N)R_N^{PS}(\theta_N)),$$

$$X^T = \left(\left(\frac{\Delta\alpha}{\alpha}\right)_1 \left(\frac{\Delta\beta}{\beta}\right)_1 \left(\frac{\Delta\alpha}{\alpha}\right)_2 \left(\frac{\Delta\beta}{\beta}\right)_2 \ldots \left(\frac{\Delta\alpha}{\alpha}\right)_i \left(\frac{\Delta\beta}{\beta}\right)_i \ldots \left(\frac{\Delta\alpha}{\alpha}\right)_N \left(\frac{\Delta\beta}{\beta}\right)_N \right),$$

$$A_1 = \left| \begin{array}{cccccccccc} a_1 & b_1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 \\ c_1 & d_1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 & 0 \\ & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & \ldots & a_i & b_i & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & c_i & d_i & \ldots & 0 & 0 \\ & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & a_N & b_N \\ 0 & 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & c_N & d_N \end{array} \right| .$$

From the above, there are 2N reflectivities and 2N unknown parameters in one trace of seismic data. So the solution can be exactly determined from one trace of seismic data by

$$X = A_1^{-1} L_1. \tag{10}$$

If M traces were available then we could appeal to the least-squares criterion to find a solution for $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$. The least squares solution for M traces of seismic data is given as follows:

$$X = (A^T A)^{-1} A^T L, \tag{11}$$

where

$$A^T = (A_1 \;\; A_2 \quad \ldots \;\; A_i \;\; \ldots \quad A_M),$$

$$L^T = (L_1 \;\; L_2 \quad \ldots \;\; L_i \;\; \ldots \quad L_M).$$

Generalized linear inversion

From equations (1) and (2), coefficients a, b, c and d are a function of P - wave velocity, S - wave velocity and reflection angle. During the inversion process, the P - wave velocity and S - wave velocity are not known exactly. Thus, an initial guess at the values of these velocities must be made.

Thus, from the least-squares procedure, the parameters $\Delta\alpha$ and $\Delta\beta$ for each interface are found. We are now interested in finding the actual α and β values. To do this we adopt a generalized linear inversion (GLI) procedure. Taking the estimated parameters $\Delta\alpha$ and $\Delta\beta$ as observations, the following equations can be built:

$$\begin{array}{ll} \Delta\alpha_1 = \alpha_2 - \alpha_1, & \Delta\beta_1 = \beta_2 - \beta_1, \\ \ldots & \\ \Delta\alpha_i = \alpha_i - \alpha_{i-1}, & \Delta\beta_i = \beta_i - \beta_{i-1}, \\ \ldots & \\ \Delta\alpha_N = \alpha_{N+1} - \alpha_N, & \Delta\beta_N = \beta_{N+1} - \beta_N. \end{array} \tag{12}$$

In matrix form, we have

$$\Delta\alpha = B\,\alpha, \tag{13}$$

$$\Delta\beta = B\beta,$$

where

$$(\Delta\alpha)^T = (\Delta\alpha_1 \; \Delta\alpha_2 \; ... \; \Delta\alpha_i \; ... \; \Delta\alpha_N),$$

$$\alpha^T = (\alpha_1 \; \alpha_2 \; ... \; \alpha_i \; ... \; \alpha_{N+1}),$$

$$(\Delta\beta)^T = (\Delta\beta_1 \; \Delta\beta_2 \; ... \; \Delta\beta_i \; ... \; \Delta\beta_N),$$

$$\beta^T = (\beta_1 \; \beta_2 \; ... \; \beta_i \; ... \; \beta_{N+1}),$$

$$\underset{N\mathrm{x}(N+1)}{B} = \begin{vmatrix} -1 & 1 & 0 & 0 ... 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{vmatrix} .$$

Matrix B has no standard inverse matrix as it is not square. To solve equation (13) the generalized linear inversion is used. The generalized linear inversion solution is

$$\alpha = (B^T B + \varepsilon^2 I)^{-1} B^T \Delta\alpha , \qquad (14)$$

$$\beta = (B^T B + \varepsilon^2 I)^{-1} B^T \Delta\beta .$$

We take $\varepsilon^2 = 1$ for this case. Equation (14) is used to update the P - and S - wave velocities.

Coefficient sensitivity analysis

As mentioned before, the distortion of P - wave velocity, S - wave velocity and reflection angle can impact coefficients a, b, c and d in a nonlinear way. Any distortion in the coefficients may lead to an erroneous estimation of parameters $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$. In order to test how sensitive the coefficients are to the errors or changes in P - wave velocity, S - wave velocity and reflection angle, the error in the coefficients in percentage is defined as follows:

$$\Delta a \; \% = \frac{a(\alpha + \Delta, \beta+\Delta, \theta+\Delta) - a(\alpha, \beta, \theta)}{a(\alpha, \beta, \theta)} \times 100,$$

$$\Delta b\ \% = \frac{b(\alpha + \Delta, \beta+\Delta, \theta+\Delta) - b(\alpha, \beta, \theta)}{b(\alpha, \beta, \theta)} \times 100,$$

$$\Delta c\ \% = \frac{c(\alpha + \Delta, \beta+\Delta, \theta+\Delta) - c(\alpha, \beta, \theta)}{c(\alpha, \beta, \theta)} \times 100,$$

$$\Delta d\ \% = \frac{d(\alpha + \Delta, \beta+\Delta, \theta+\Delta) - d(\alpha, \beta, \theta)}{d(\alpha, \beta, \theta)} \times 100.$$

Figure 3 shows the percentage errors of the coefficients caused by distortion of the P - wave velocity for the third layer. Coefficient b is more sensitive to the error in P - wave velocity than the other coefficients. With increasingly deeper layers, the percentage errors of the coefficients are reduced.

Figure 4 shows the distortion of the coefficients caused by the error in S - wave velocity alone. The coefficients in Figure 4 have similar sensitivity magnitude as those in Figure 3. However, the sign of the errors caused by the distortion of P - wave velocity is opposite of that caused by the distortion of S - wave velocity.

The percentage errors of the coefficients from P - wave velocity and S - wave velocity perturbations are shown in Figure 5. From the diagram, it can be seen that the errors are much smaller than that caused by either the error in P -wave velocity or in S - wave velocity. This tells us that errors with the same direction for P - wave and S - wave velocities tend to cancel each other so that the percentage errors of the coefficients are reduced.

Figure 6 shows the results due to errors in the reflection angle. The percentage errors of coefficients are quite sensitive to small reflection angles. With decreasing the reflection angle, the percentage errors of the coefficients increase rapidly. The percentage error of coefficient c are almost identical to that of coefficient d for small reflection angles.

Effects of systematic noise

To extract compressional and shear properties from the joint process of P and P-SV algorithm, it is necessary to have an initial guess values P - wave and S - wave velocities. In practice, the initial guesses could be obtained from seismic velocity analysis, well logs or other geological information. These initial values of P - and S - wave velocities may be contaminated by systematic noise. There are three cases to be examined:

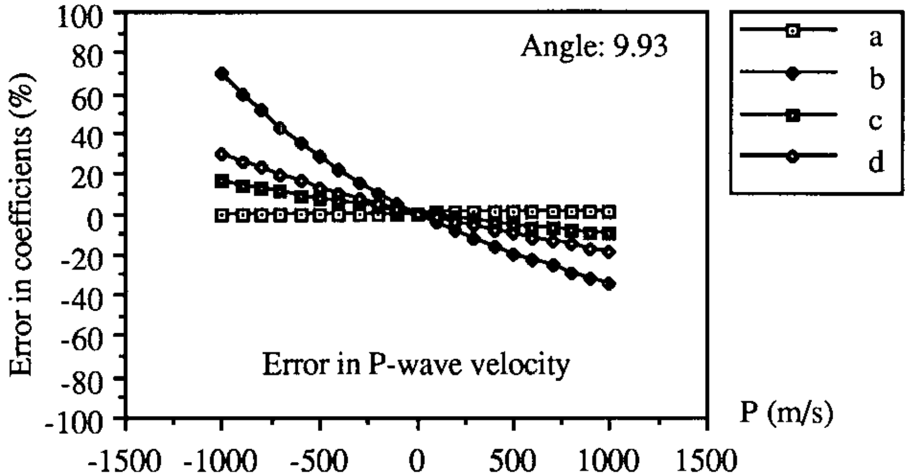


FIG. 3: Coefficient sensitivity to error in P-wave velocity

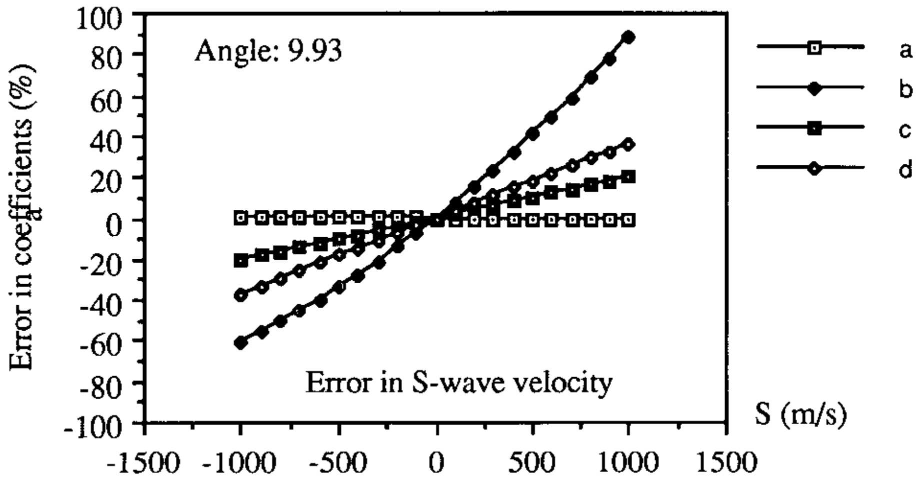


FIG. 4: Coefficient sensitivity to error in S-wave velocity

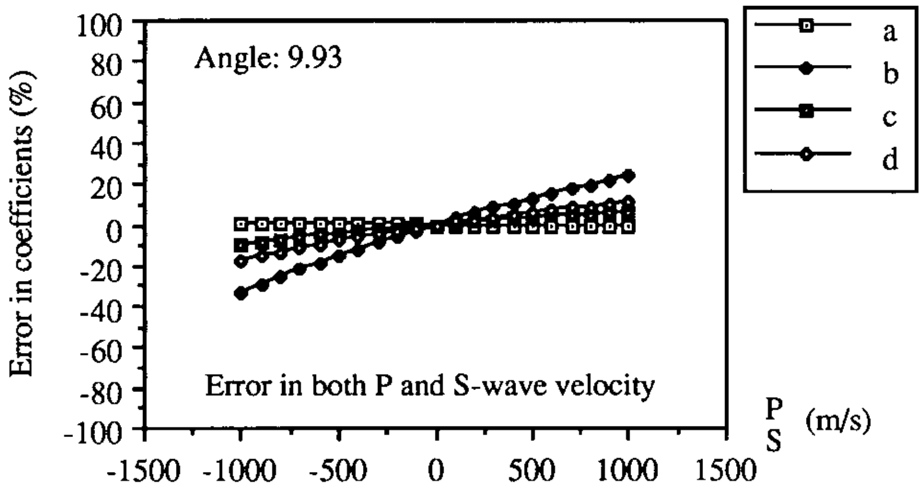


FIG. 5: Coefficient sensitivity to error in both P-wave and S-wave velocity

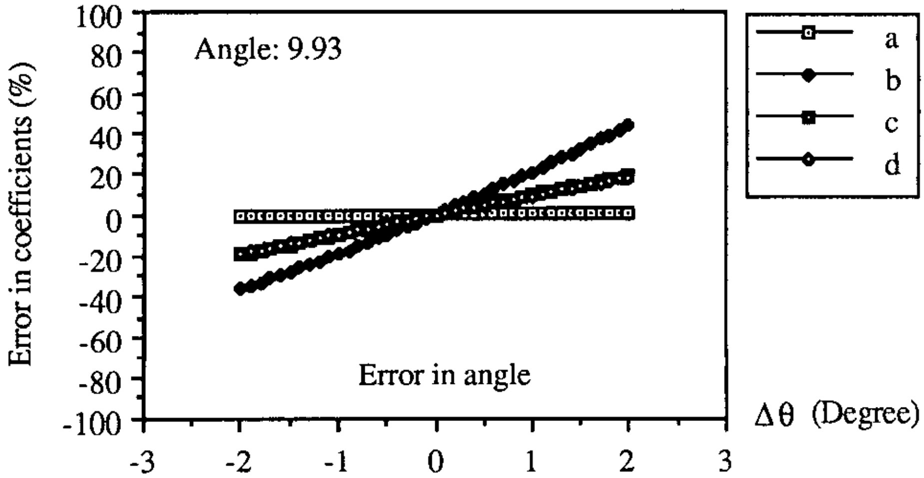


FIG. 6: Coefficient sensitivity to error in deflection angle

Effect of errors in initial guesses

In this case, the P - wave velocity structure is distorted by systematic noise of magnitudes 100, 200, ..., 900 m/s for every layer. Fig.7 (a) shows the resultant P - wave velocities. The joint inversion method could not eliminate systematic noise in P - wave velocity. Systematic error in the S - wave velocity does not effect the P - wave velocity estimate significantly. Figure 8 (a) shows the effects of systematic noise in P - wave on the determination of the S -wave velocity. The S - wave velocity is slightly sensitive to systematic noise in the P -wave velocity. Figures 9 and 10 give us the percentage error of parameters $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$. For example, the mean percentage error of $\Delta\alpha/\alpha$ is about 1% for 100 m/s error in the P - wave velocity, but 5 % for $\Delta\beta/\beta$. For different layers, the percentage error is different from each other. The maximum percentage error of $\Delta\alpha/\alpha$ is at the third layer due to a small value of relative change $\Delta\alpha/\alpha$.

The study of effects of random noise in the joint inversion of P and P - SV data is to add random noise to both P - wave velocity and S - wave velocity. It is also assumed that random errors in P - wave velocity are uncorrelated with that in the S - wave velocity. The following effects are to be considered:

(1): Random noise effects on P - wave velocity and $\Delta\alpha/\alpha$

Figure 11(a) shows the error of the initial guess P - wave velocity in percentage with respect to true P -wave velocity due to random noise. For example, the standard deviation σ of random noise is 100 m/s, which causes about 1 % distortion in P - wave velocity at the first layer.

Figure 11(b) shows the error in P - wave velocity from random noise after the inversion. The standard deviation 100 m/s of random noise causes about 20 m/s error at the first layer.

Figure 11(c) shows the error in the parameter $\Delta\alpha/\alpha$ in percentage. For $\sigma = 100$ m/s, the percentage error of $\Delta\alpha/\alpha$ is less than 1%.

(2): Random noise effects on S - wave velocity and $\Delta\beta/\beta$

Random noise effects on S - wave velocity and the parameter $\Delta\beta/\beta$ are plotted in Figure 12. Compare Figure 12(a) with Figure 11(a), the percentage error of initial guess values of S - wave velocity is a little larger than that of initial guess values of P - wave velocity due to the smaller values of S - wave velocity.

Figure 12(b) shows the error in S - wave velocity from random noise after inversion. For $\sigma = 100$ m/s, the error is about 20 m/s in a negative direction compared with Figure 11(b).

Figure 12(c) shows the error of $\Delta\beta/\beta$ in percentage. For σ = 100 m/s, the percentage error of $\Delta\beta/\beta$ is about 2%, larger than that in Figure 11(c).

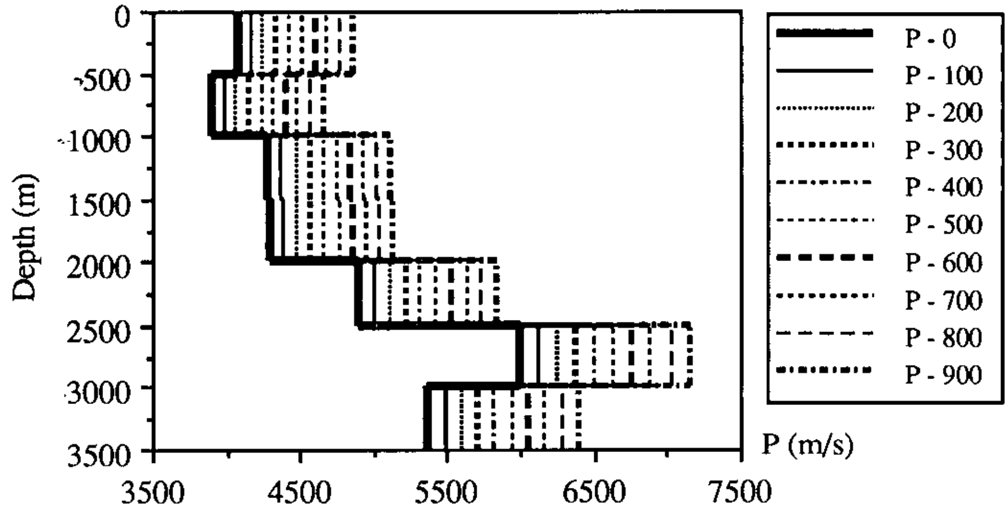


FIG. 7 (a): Inverted P-wave velocity with various systematic errors in the initial P-wave velocity guess

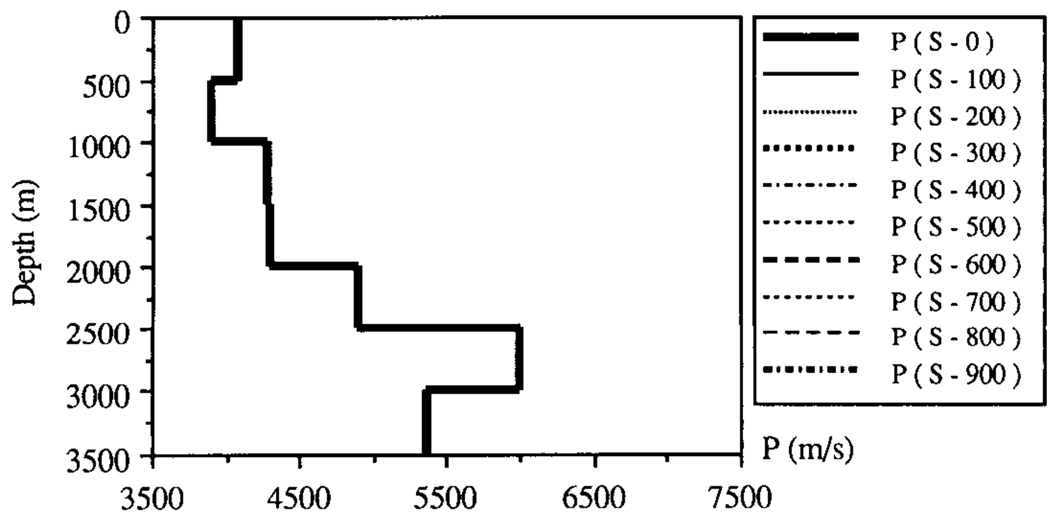
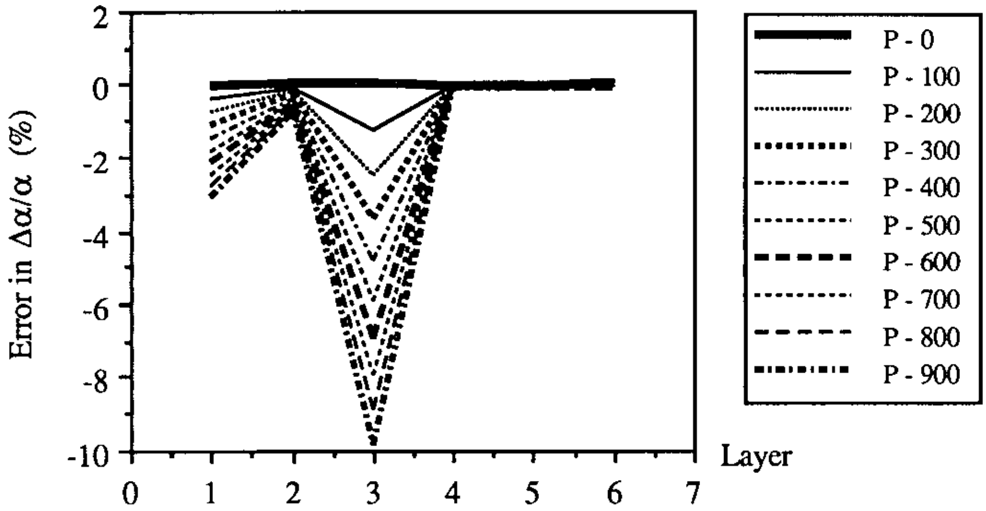
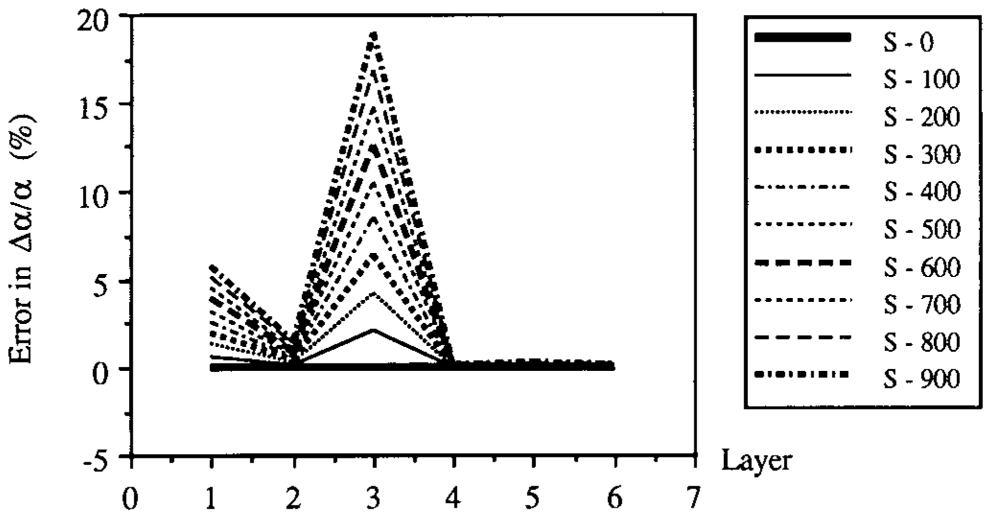


FIG. 7(b): Inverted P-wave velocity with various systematic errors in the S-wave velocity guess.



(a): Error in $\Delta\alpha/\alpha$ from different initial guess velocity in P - wave



(b): Error in $\Delta\alpha/\alpha$ from different initial guess velocity in S - wave

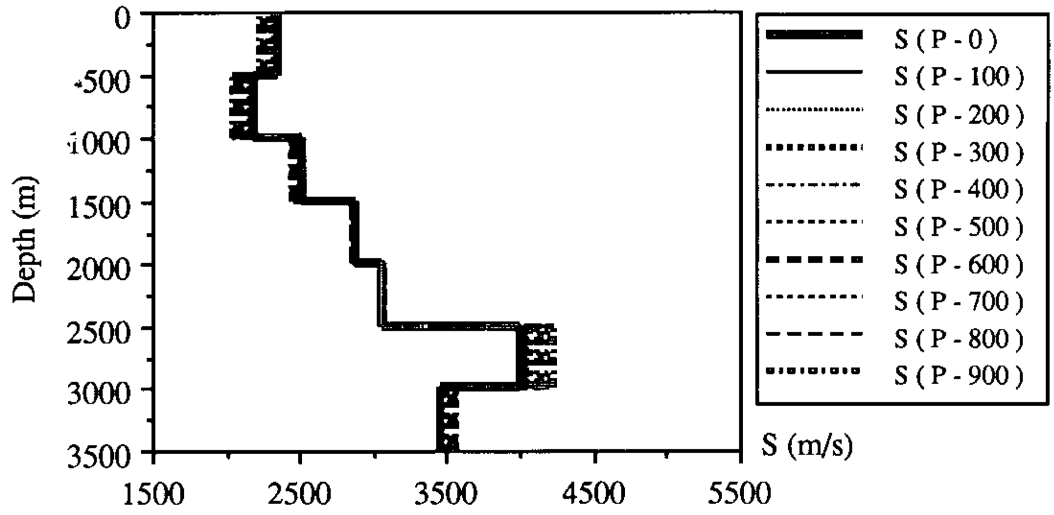
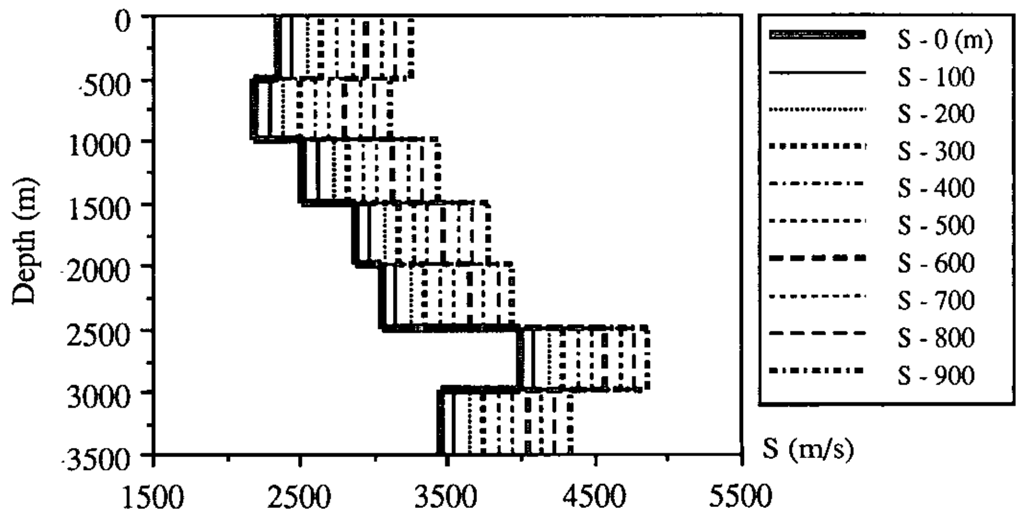
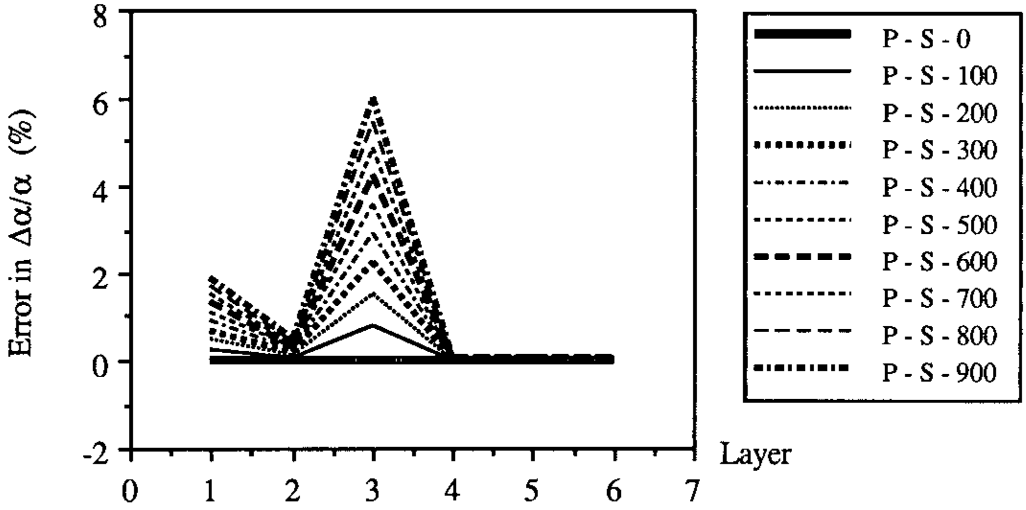


FIG. 8 (a): Inverted S-wave velocity after inversion from different initial guess in P - wave



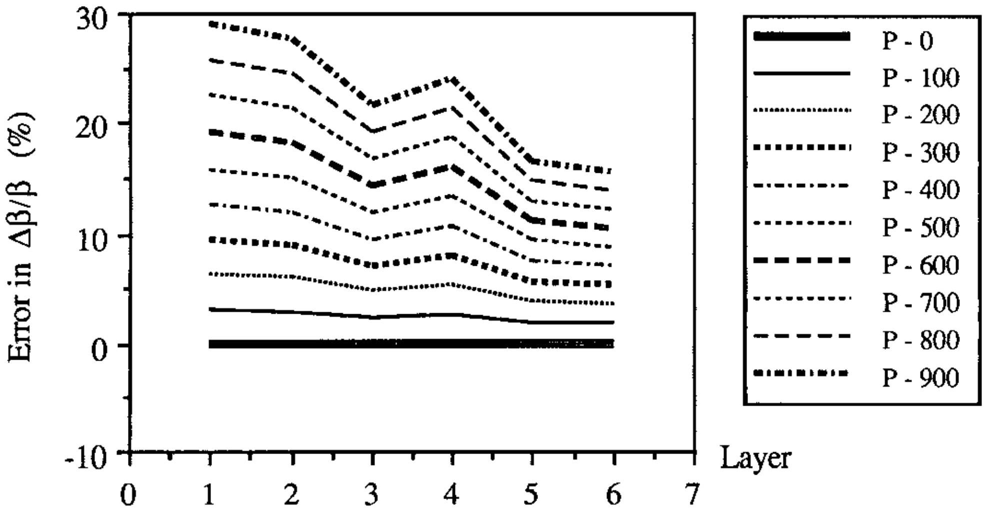
(b): S - wave velocity after inversion from different initial guess in S - wave

FIG. 8 (b): Inverted S-wave velocity after inversion from different initial guess in S - wave

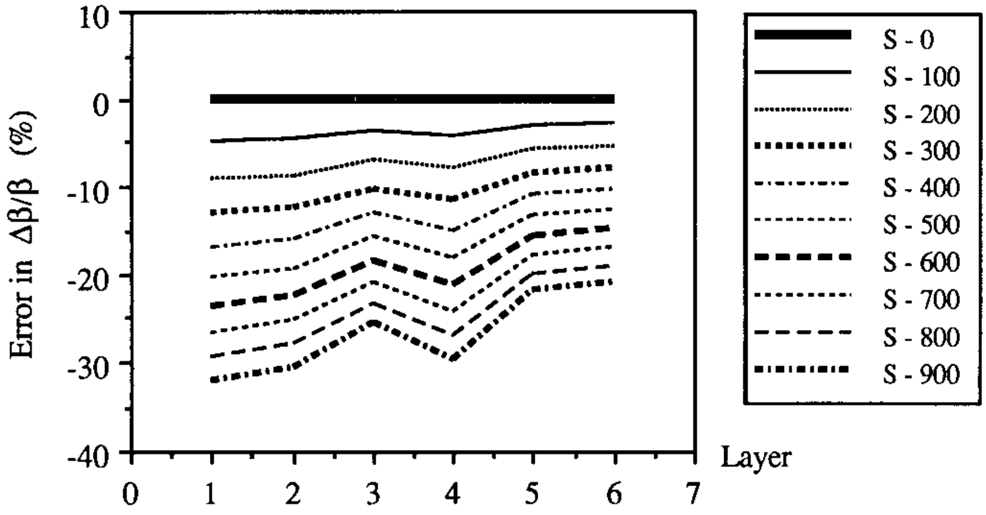


(c): Error in $\Delta\alpha/\alpha$ from different initial guess velocity in P -wave and S - wave

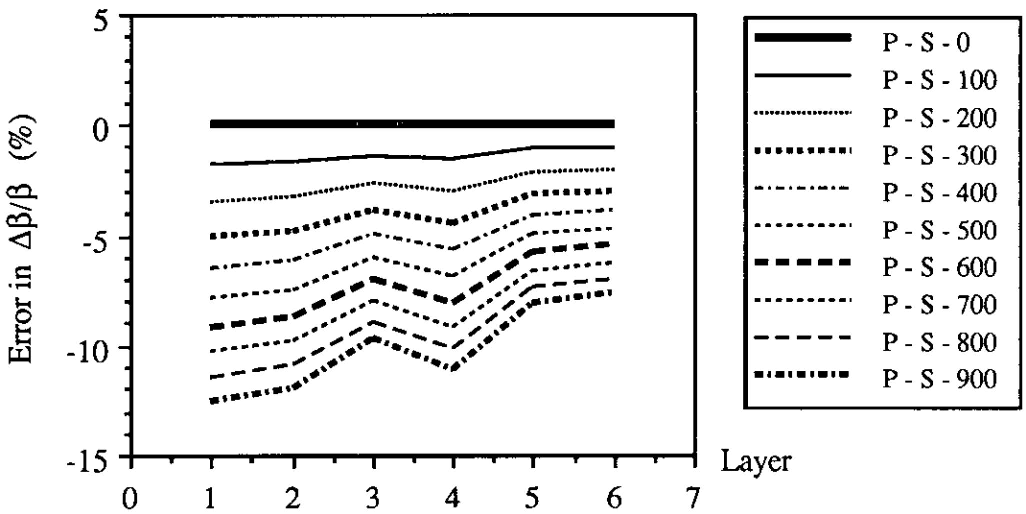
Fig.9: Relative P - wave change $\Delta\alpha/\alpha$ to systematic noise in velocity structures



(a): Error in $\Delta\beta/\beta$ from different initial guess velocity in P - wave

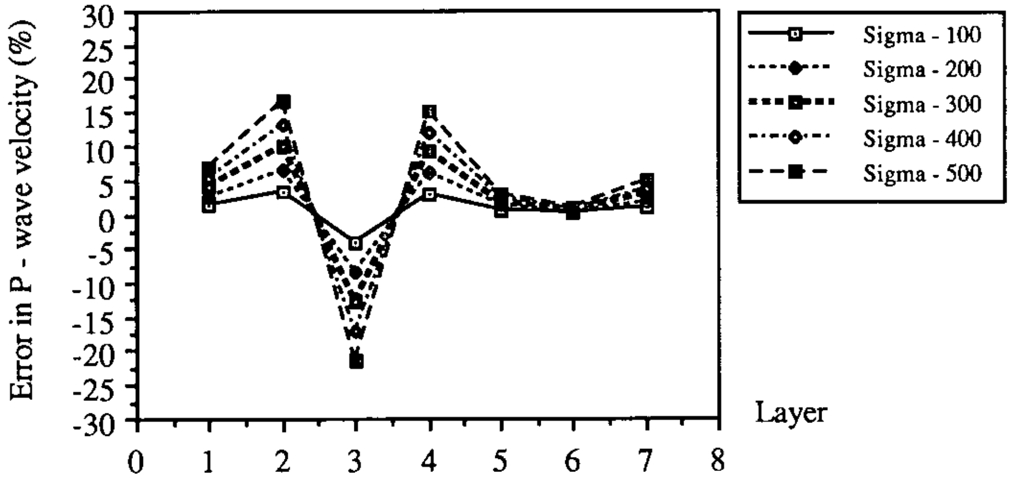


(b): Error in $\Delta\beta/\beta$ from different initial guess velocity in S - wave

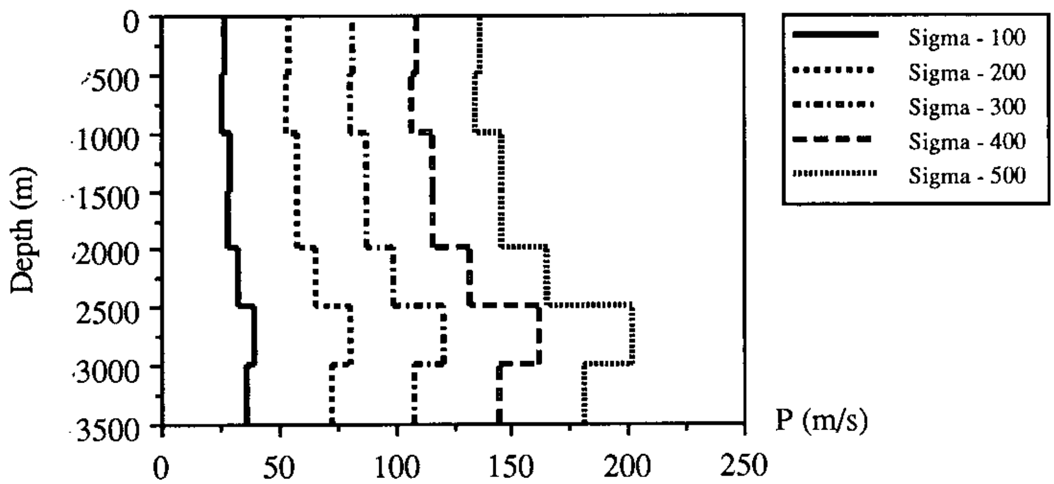


(c): Error in $\Delta\beta/\beta$ from different initial guess velocity in P -wave and S - wave

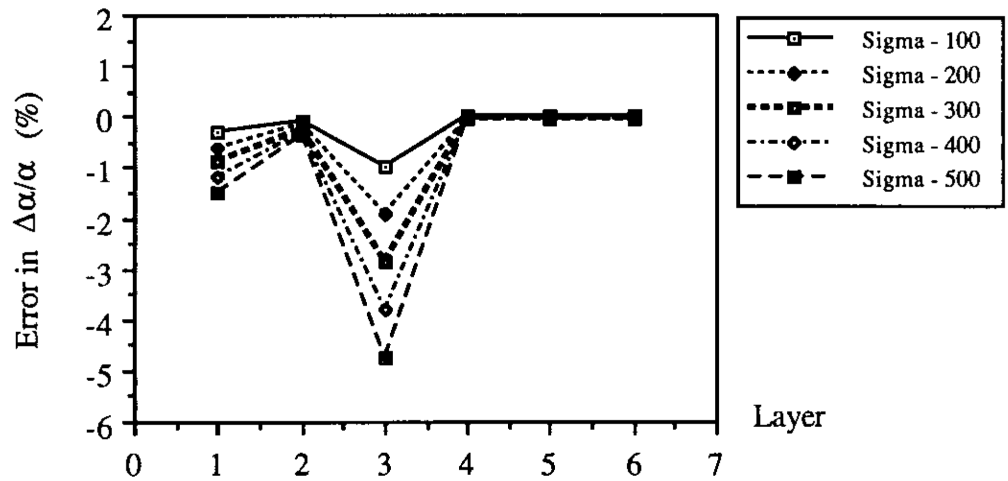
Fig.10: Relative S - wave change $\Delta\beta/\beta$ to systematic noise in velocity structures



(a): Error (%) of initial guess P - wave velocity with random noise in velocity structure

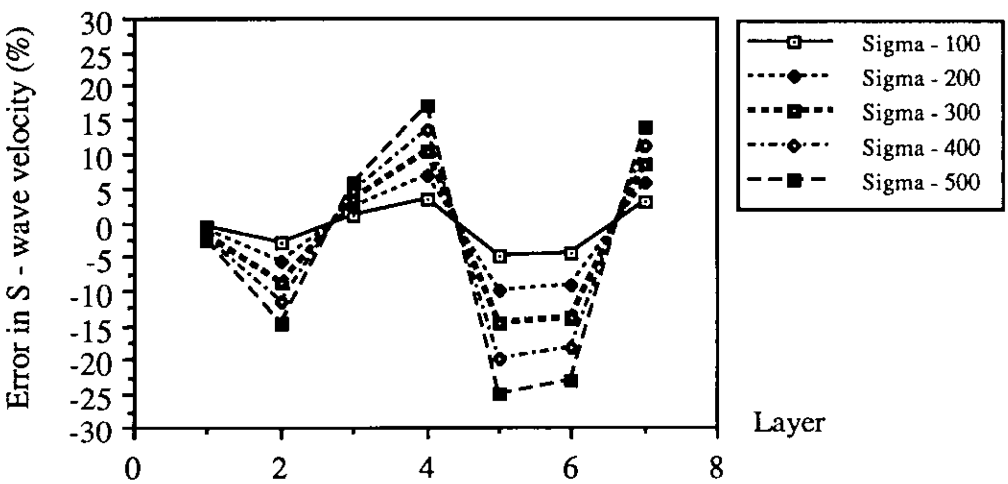


(b): Error in P - wave velocity from random noise in velocity structure after inversion

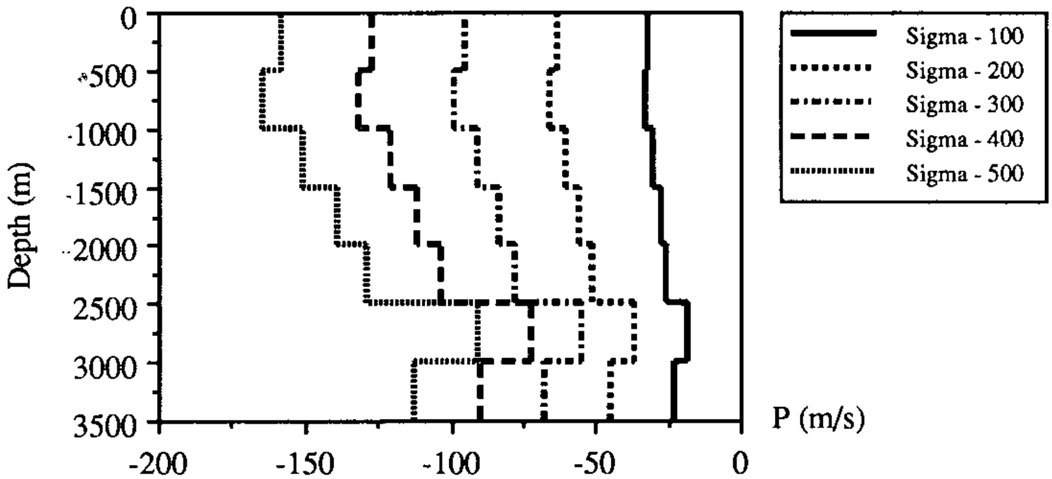


(c): Error in $\Delta\alpha/\alpha$ (%) for each layer with random noise in velocity structure

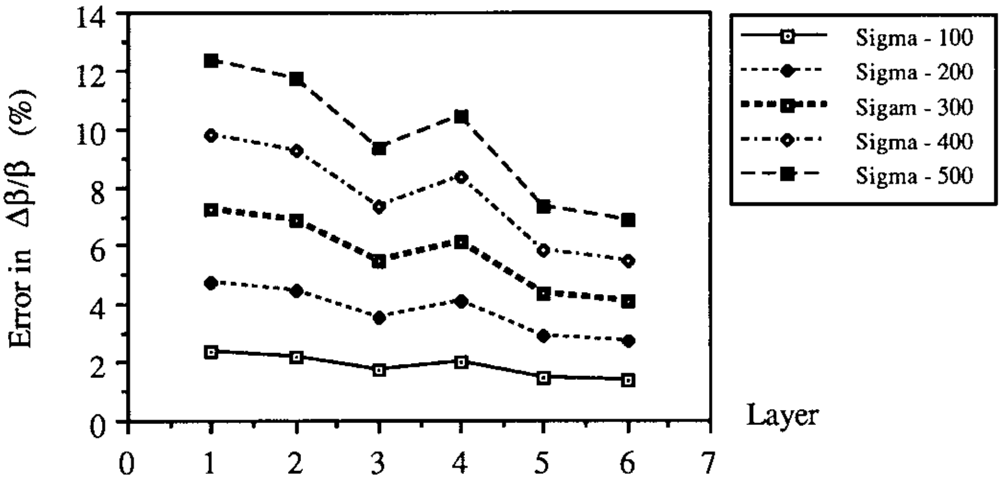
Fig.11: P - wave velocity and relative change $\Delta\alpha/\alpha$ sensitivity to random noise in velocity structures



(a): Error (%) of initial guess S - wave velocity with random noise in velocity structure



(b): Error in S - wave velocity
from random noise in velocity structure after inversion



(c): Error in $\Delta\beta/\beta$ (%) for each layer with random noise in velocity structure

Fig.12: S - wave velocity and relative change $\Delta\beta/\beta$ sensitivity to random noise in velocity structures

Effects of random noise in reflectivity

Figure 13 shows effects of random noise in the reflectivities on P -wave velocity by assuming an accurate initial guess values of P - wave velocity and S - wave velocity. The standard deviation 0.0001 of random noise in the amplitude causes about 2 m/s distortion in P -wave velocity. With increasing the standard deviation of random noise, the error in P - wave velocity becomes larger.

Similarly, Figure 14 displays effects of random noise in the reflectivities on S - wave velocity by assuming the perfect initial guess values of P - wave velocity and S -

wave velocity. With the same standard deviation of random noise in the reflectivities, S - wave velocity are effected more by random errors than P -wave velocity from a comparison of Figure 13 and Figure 14.

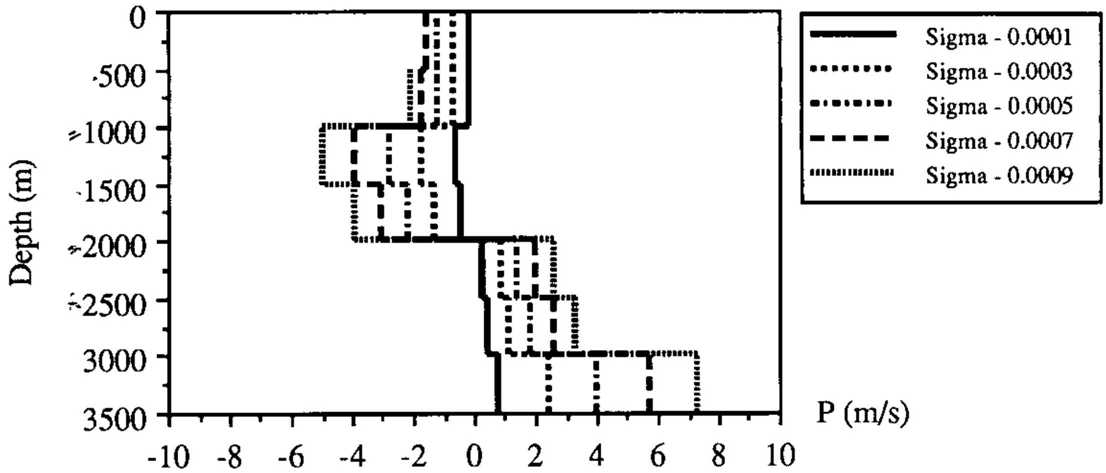


Fig.13: Error in P - wave velocity from random noise in reflectivities after inversion

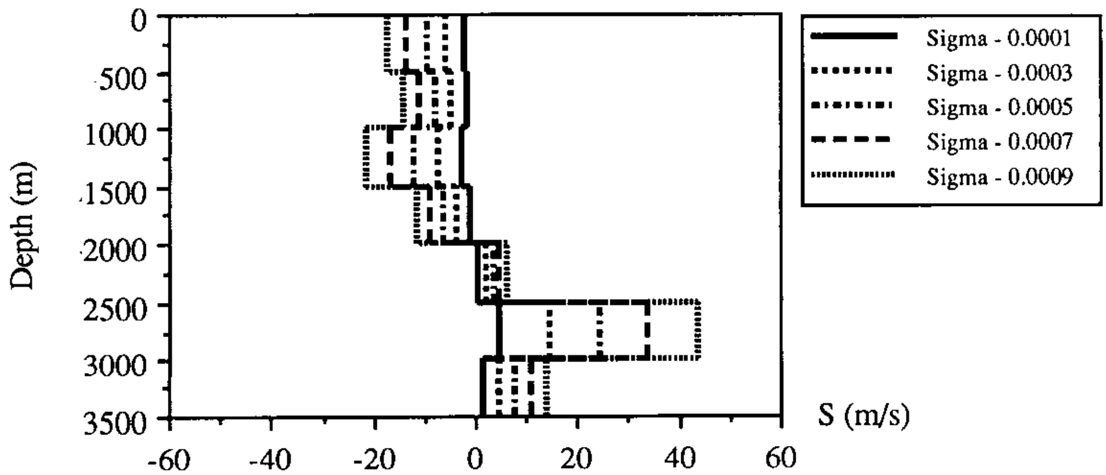


Fig.14: Error in S - wave velocity from random noise in reflectivities after inversion

Effects of random noise in the reflectivities on parameters of $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ are shown in Figure 15 and Figure 16. Random noise effects on $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ in the same way. For example, the errors of different layers in $\Delta\alpha/\alpha$ in percentage are about 1% for $\sigma = 0.0001$. Similarly, the errors of different layers in $\Delta\beta/\beta$ in percentage are 1% for $\sigma = 0.0001$.

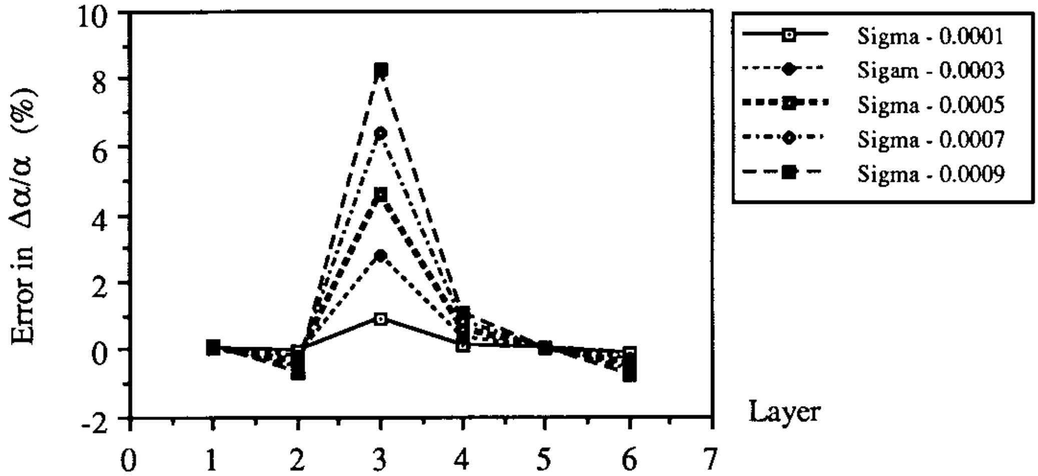


Fig15: Error in $\Delta\alpha/\alpha$ (%) for each layer with random noise in reflectivity

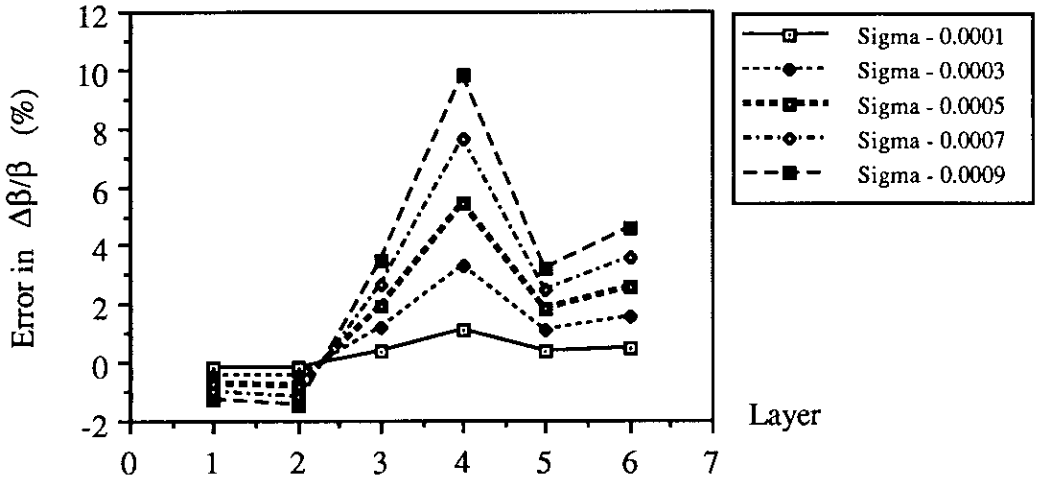


Fig16: Error in $\Delta\beta/\beta$ (%) for each layer with random noise in reflectivity

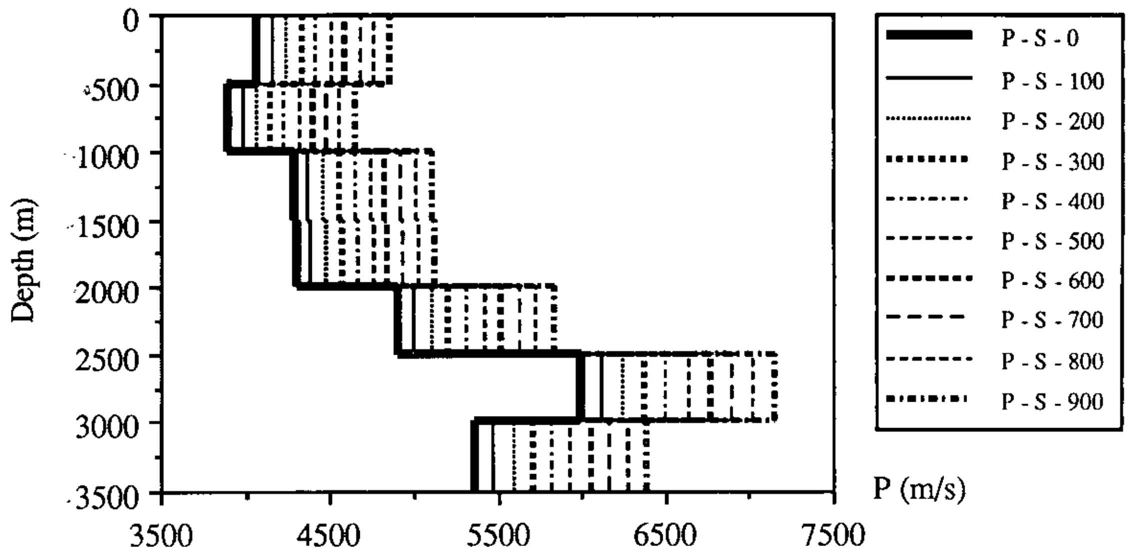
P-SV versus P-P Inversion

In this section, we compare the estimates resulting from the joint inversion with those from P-P reflectivities alone. This P-P inversion means that only equation (1) is used to estimate P - wave velocity, S- wave velocity, and relative changes $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$.

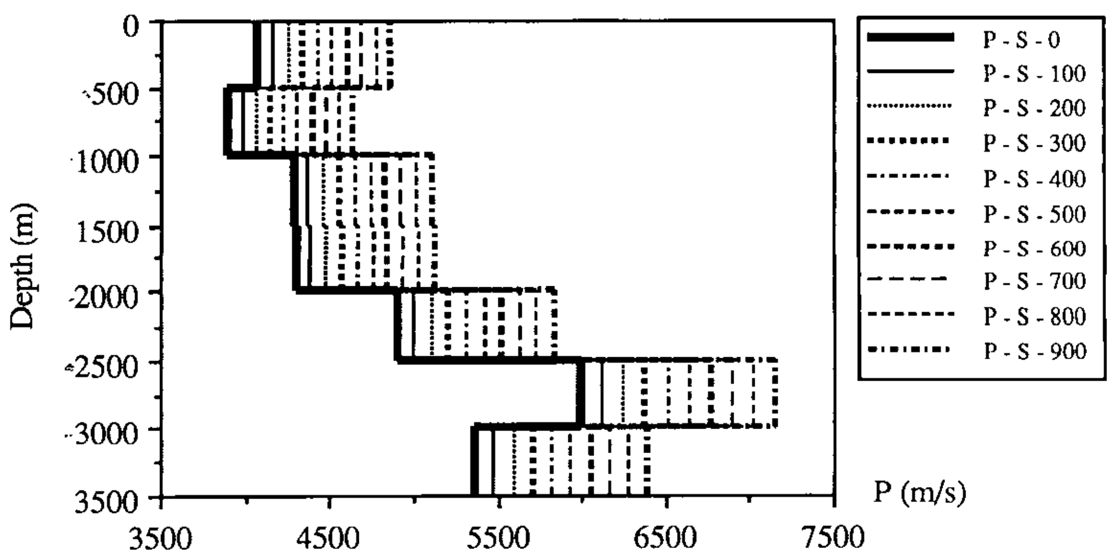
In order to compare these two inversion methods, we generate ten traces of synthetic seismic data to test how sensitive the estimated parameters of P - wave and S - wave to random error and systematic error.

(1): Systematic error in velocity structures

Figure 17 shows the estimated P - wave velocity with systematic errors in velocity structures by these two methods. The errors in P - wave velocity are shown in Figure 18. Compared (a) with (b) in Figure 17 and Figure 18, it can be shown that both methods give us the similar results for the P - wave velocity.

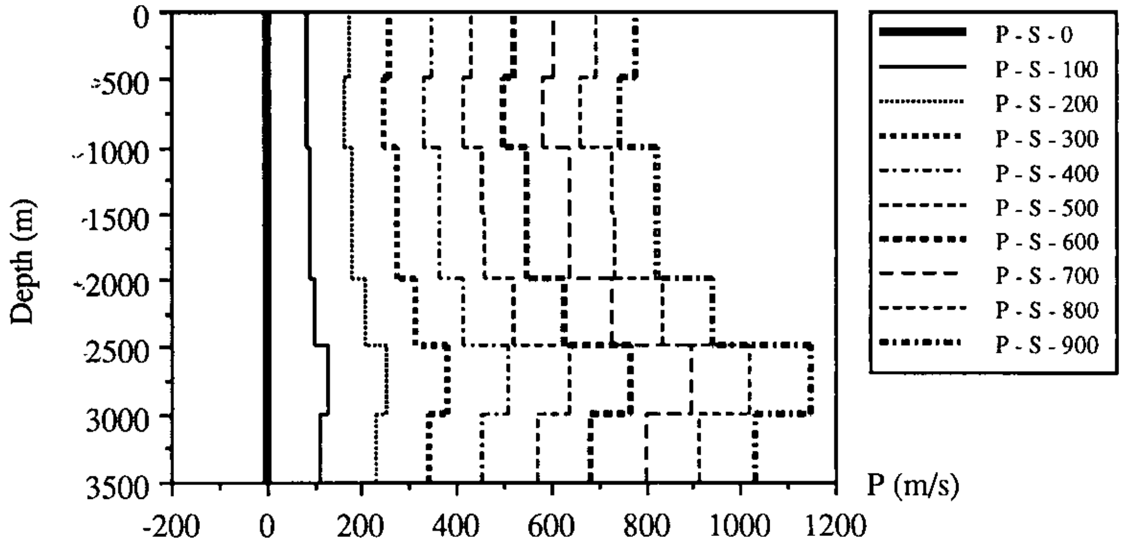


(a): P - wave velocity with systematic error in velocity structures by P-P inversion

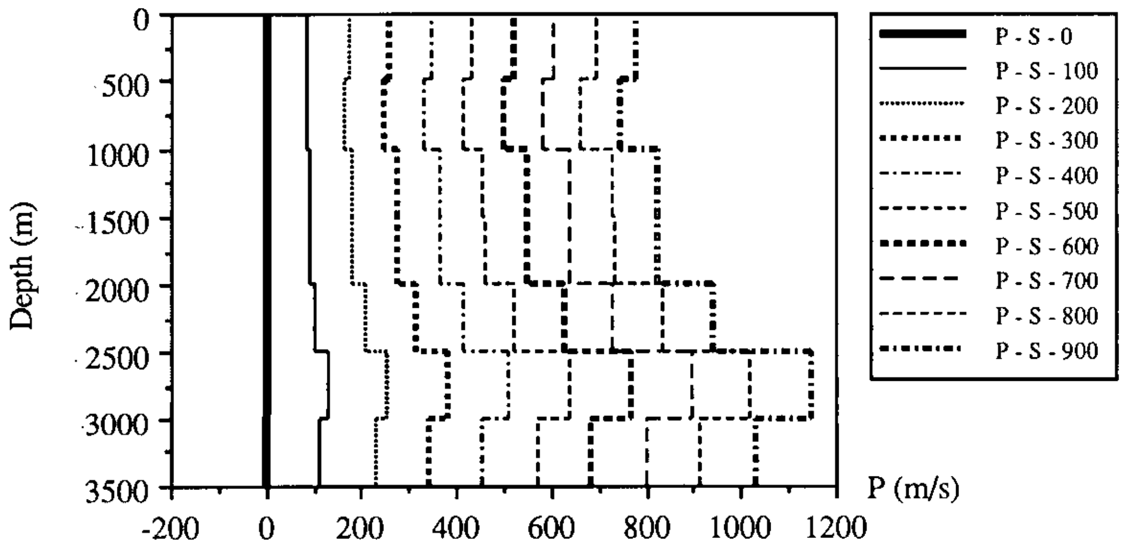


(b): P - wave velocity with systematic error in velocity structures by P-P and P-SV inversion

Fig.17: P-wave velocity with systematic error from joint inversion and P-P inversion

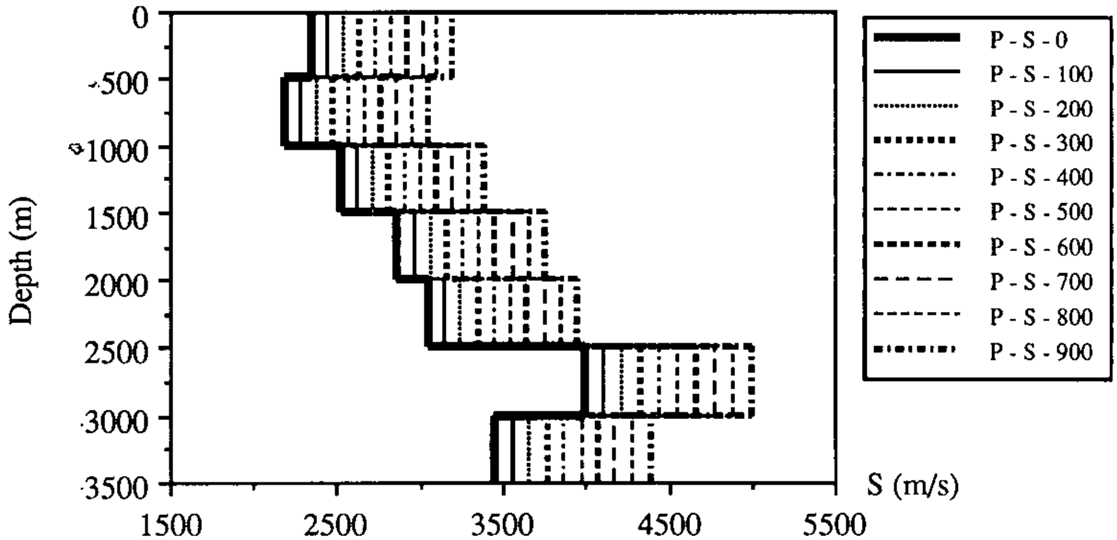


(a): Error in P - wave velocity from systematic error in velocity structures by P-P inversion

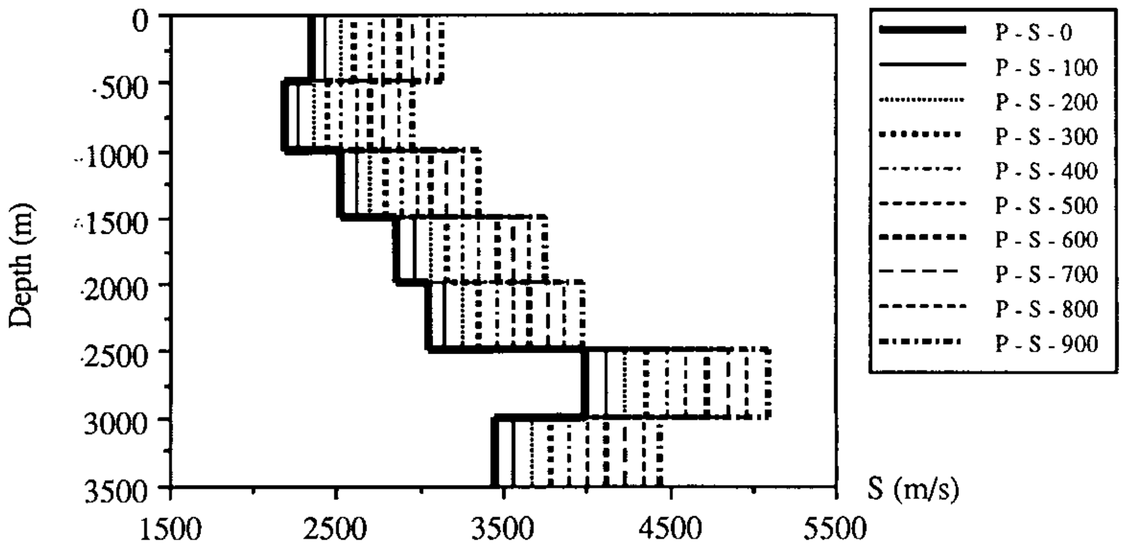


(a): Error in P - wave velocity from systematic error in velocity structures by P-P inversion

Fig.18: Error in P-wave velocity from systematic error in velocity structures by joint inversion and P-P inversion



(a): S - wave velocity with systematic error in velocity structures by P-P inversion



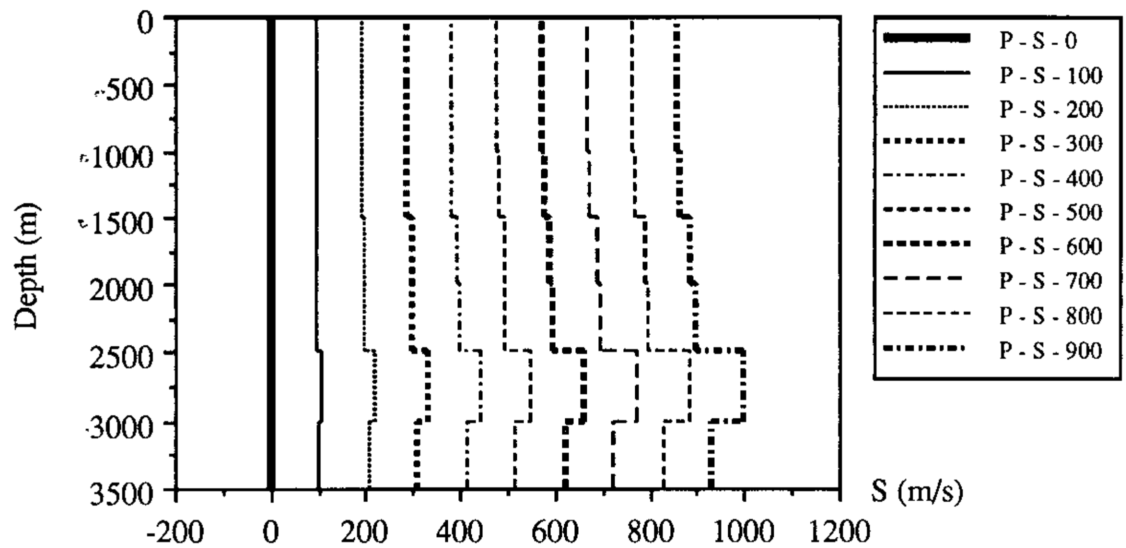
(b): S - wave velocity with systematic error in velocity sturctures by P-P and P-SV inversion

Fig.19: S-wave velocity with systematic error in velocity structures from joint inversion and P-P inversion

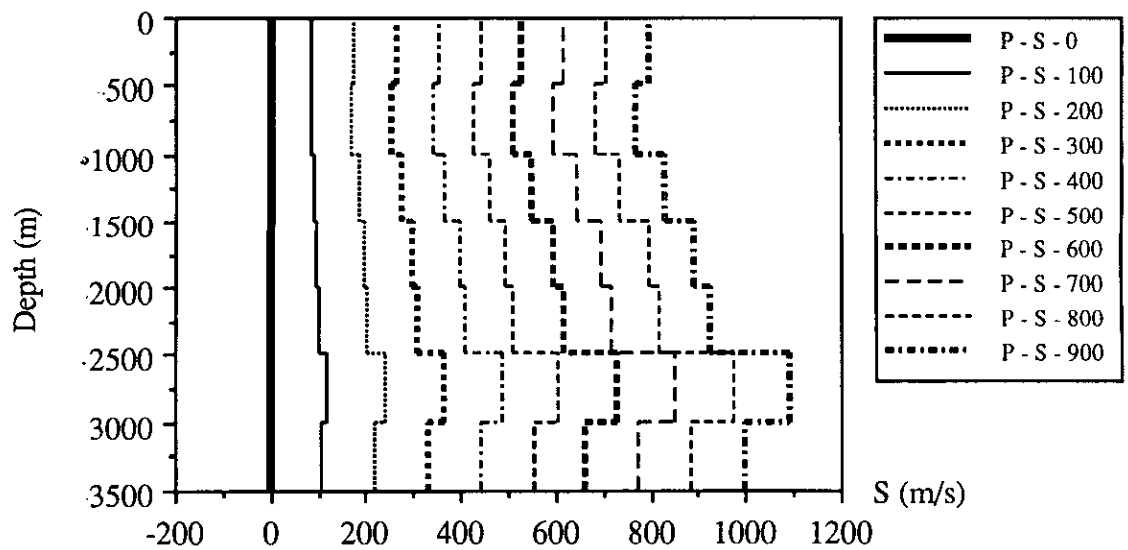
Figure 19 and Figure 20 show the similar situations as above for S - wave velocity. Both methods give almost identical errors to each other.

(2): Random error in velocity structures

Figure 21(a) and (b) show that both inversion methods give the same error in P - wave velocity when there exist random errors in velocity structures. It is easy to see that the error of S - wave velocity from the joint inverse method is somewhat smaller than that from the P-P inversion by comparing Figure 22(a) with (b).

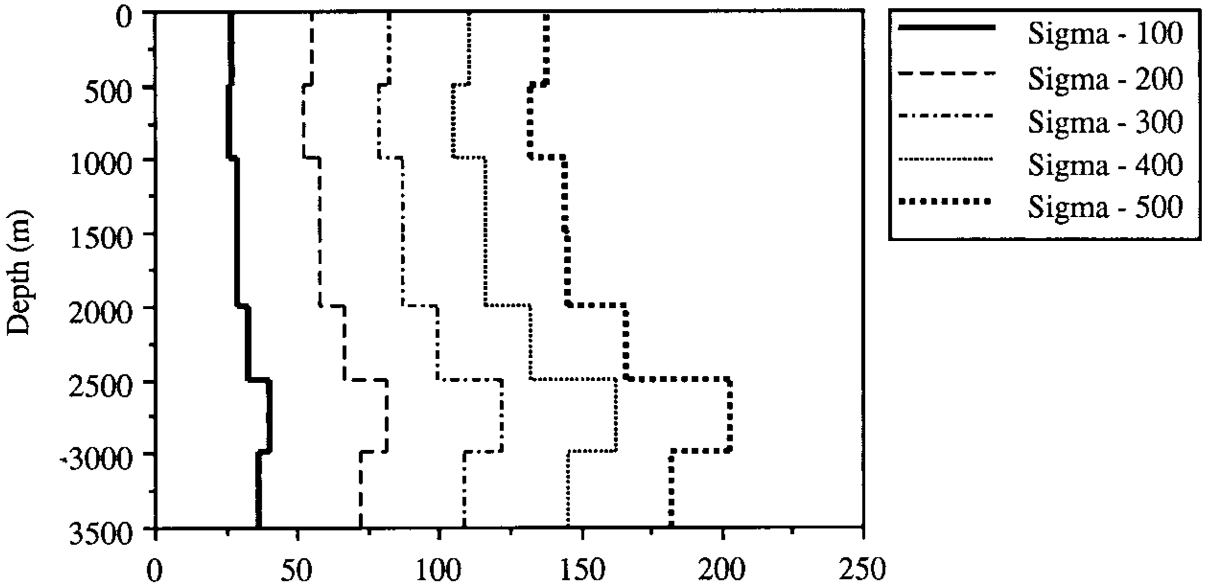


(a): Error in S - wave velocity with systematic error in velocity structures by P-P inversion

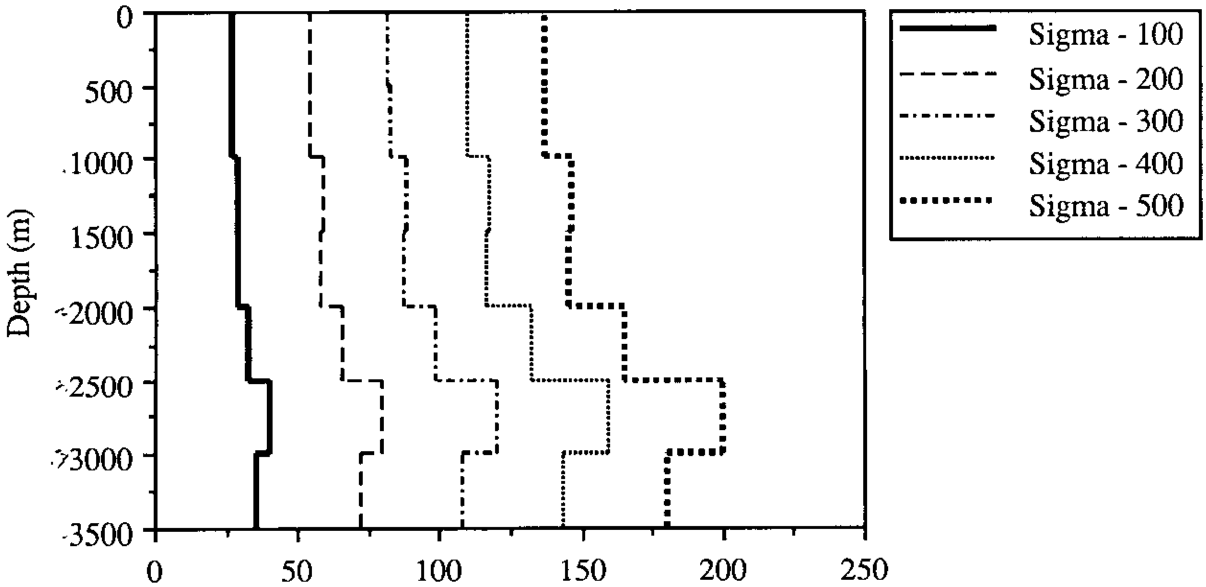


(b): Error in S - wave velocity with systematic error in velocity structures by P-P and P-SV inversi

Fig.20: Error in S-wave velocity from systematic error in velocity structures by joint inversion and P-P inversion

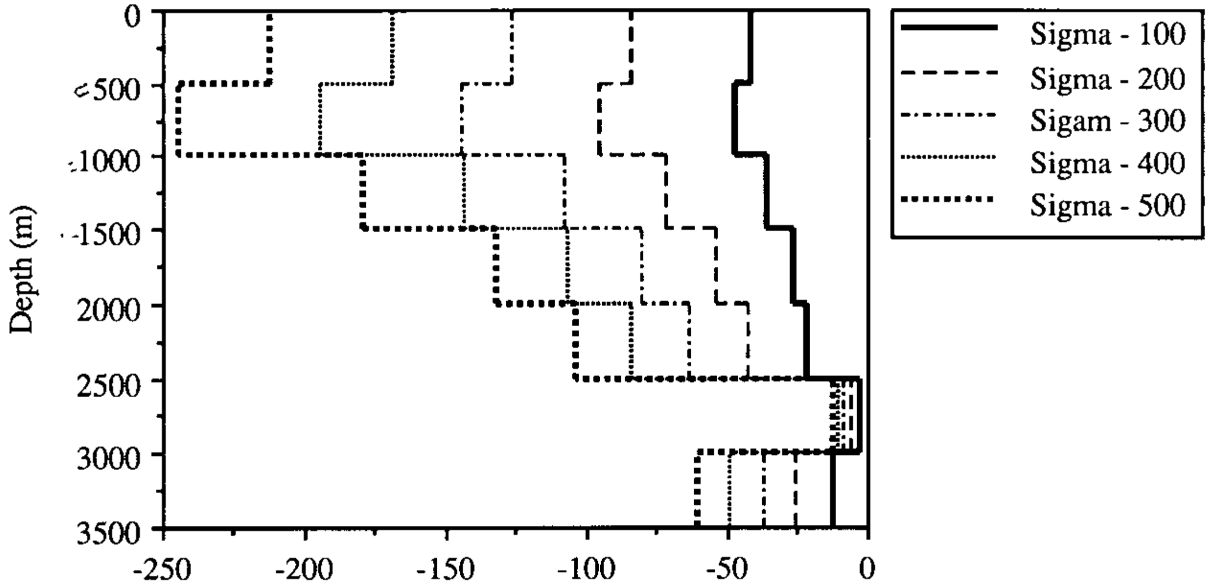


(a): P - wave velocity from random noise in velocity structures by P-P inversion

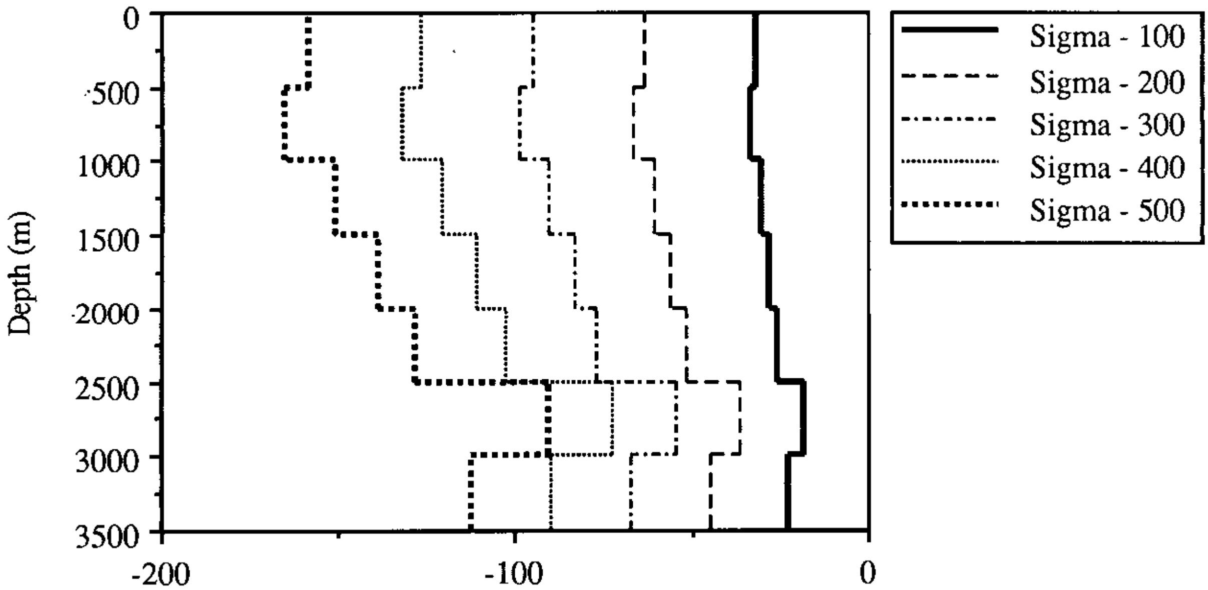


(b): P - wave velocity from random noise in velocity structures by P-P and P-SV inversion

Fig.21: Error in P-wave velocity from random error in velocity structures by joint inversion and P-P inversion



(a): S - wave velocity from random noise in velocity structures by P-P inversion



(b): S - wave velocity from random noise in velocity structures by P-P and P-SV inversion

Fig.22: Error in S-wave velocity from random noise in velocity structures by joint inversion and P-P inversion

(3): Random error in reflectivity

Figure 23 shows the error of P - wave velocity from random noise in reflectivity from these two inversion methods. From (a) and (b) in Figure 23, it can be seen that the error of P - wave velocity from the joint inversion method is about half that from P-P inversion alone. The most dramatic difference is seen in the estimation of S -wave velocity in the presence of reflectivity noise. The error of S -wave velocity from the joint inversion method is much smaller than from P-P inversion as shown in Figure 24(a) and (b).

By examining (a) and (b) in Figure 25 and Figure 26, we can see the errors in relative changes $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ from random error by the joint inversion method are much smaller than that by P-P inversion.

CONCLUSIONS

An analysis of the relationship between noise in the initial velocities and reflectivities and the resultant errors in the P - and S - wave velocities provides a means to assess the effectiveness of the joint P and P - SV inversion. Generally, systematic and random noise in velocity structures and in the reflectivity effects S -wave velocity more than P - wave velocity. The errors in estimates are proportional to systematic and random noise in the velocity structures and reflectivities. As shown, the systematic noise in velocity structures and reflectivities can not be eliminated by the joint P and P - SV inversion. Random noise can be minimized by the joint inversion.

The most dramatic result of this study is the improvement of S - wave velocity estimates from noise reflectivities of the joint inversion over the conventional (P only) inversion.

ACKNOWLEDGEMENTS

This work was partially supported by the CREWES Project (the Consortium for Research in Elastic Wave Exploration Seismology) at the University of Calgary.

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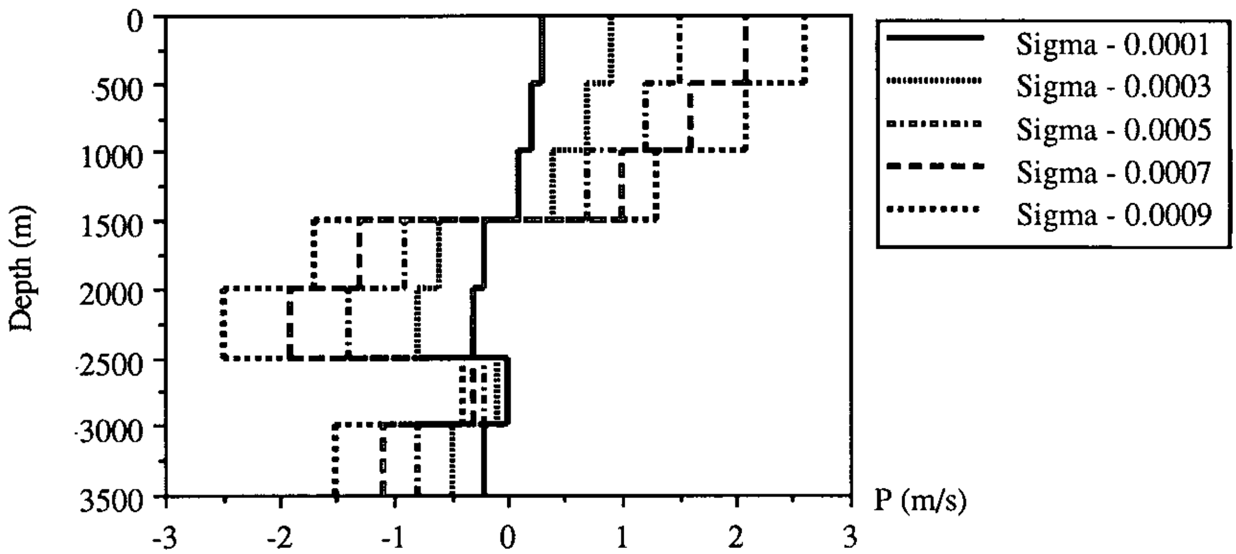
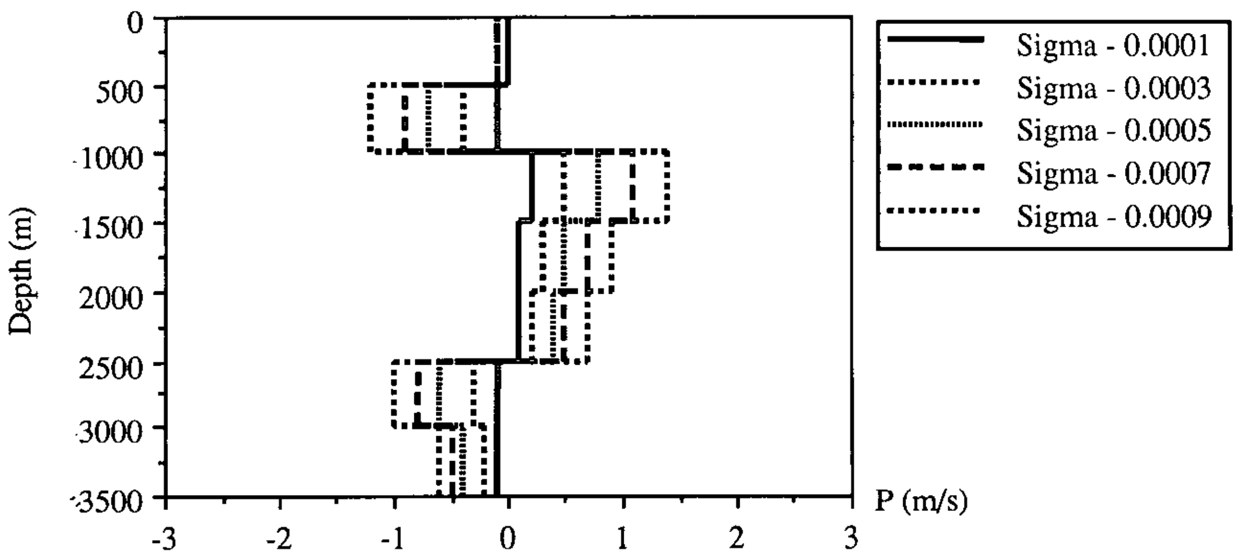
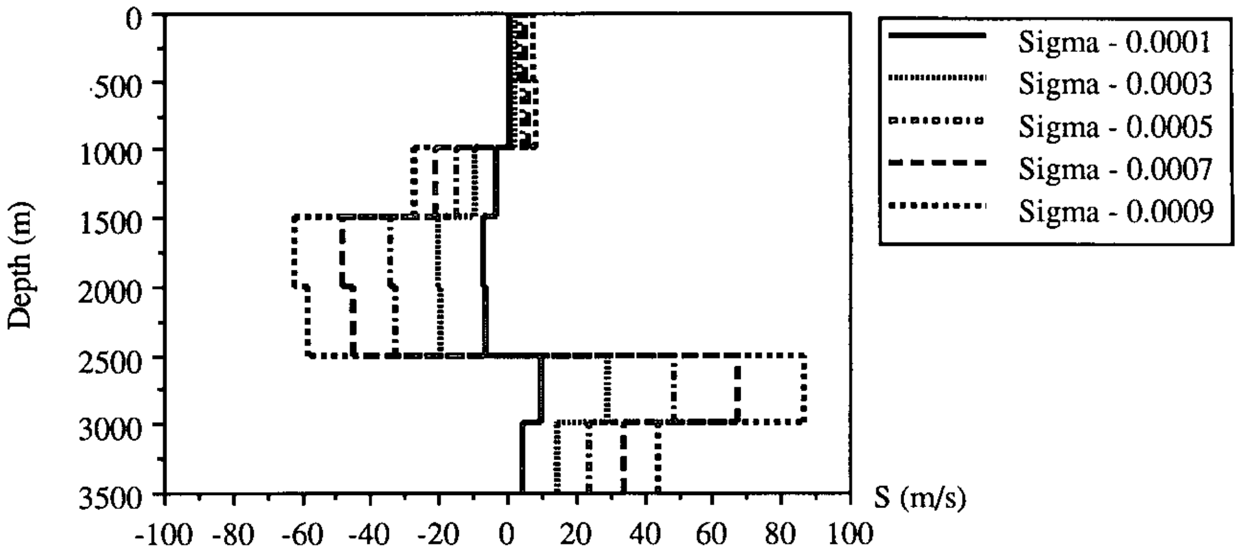


(a): Error of P - wave velocity from random noise in reflectivity by P-P inversion

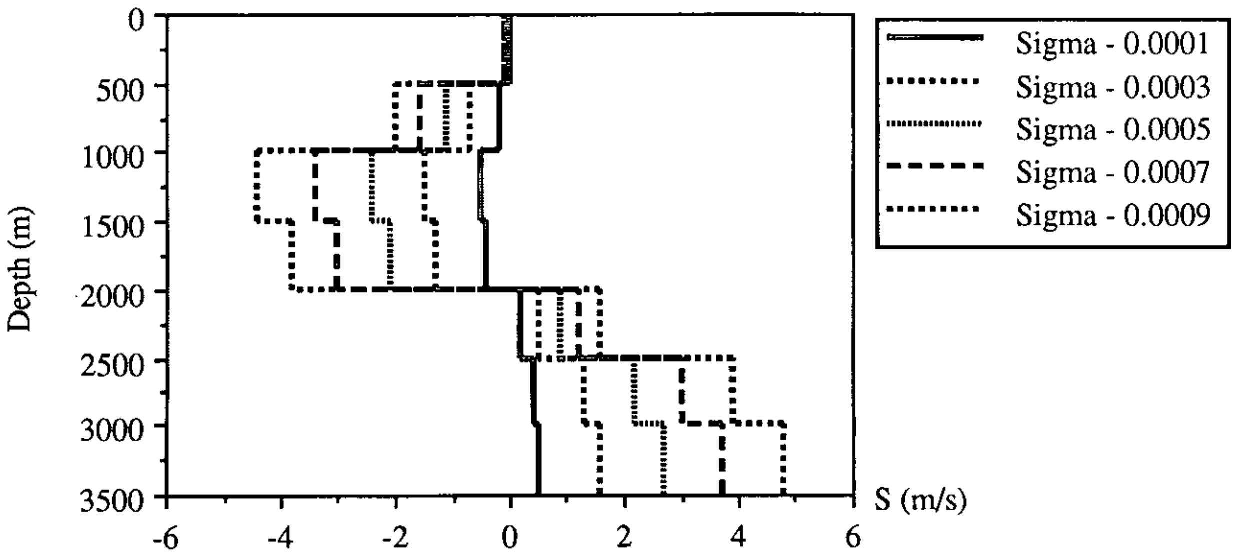


(b): Error of P - wave velocity from random noise in reflectivity by P-P and P-SV inversion

Fig.23: Error in P- wave velocity from random error in reflectivity by joint and P-P in version

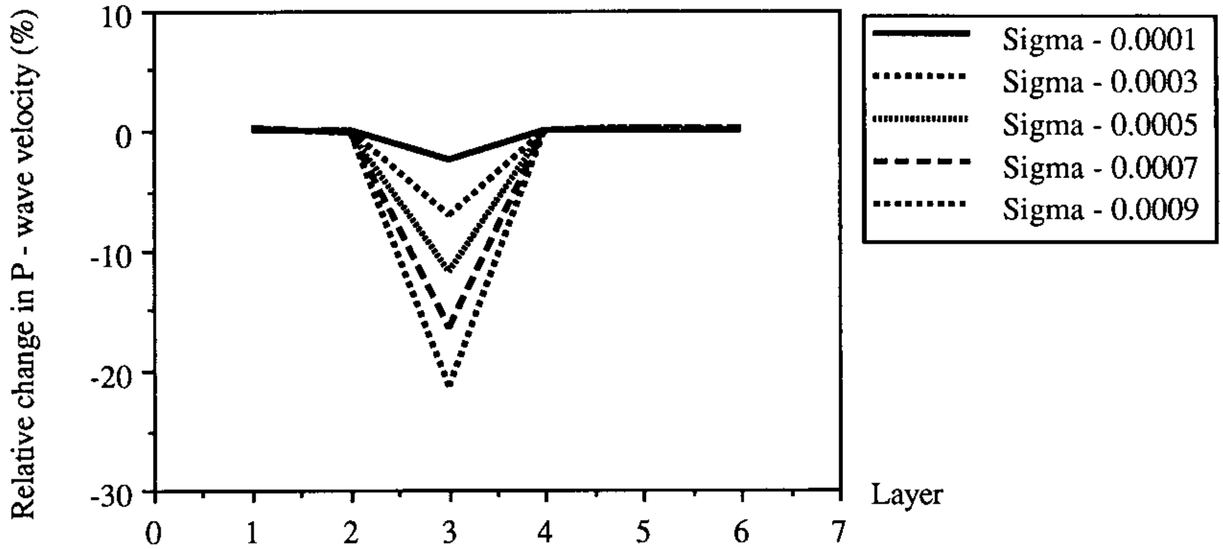


(a): Error of S - wave velocity from random noise in reflectivity by P-P inversion

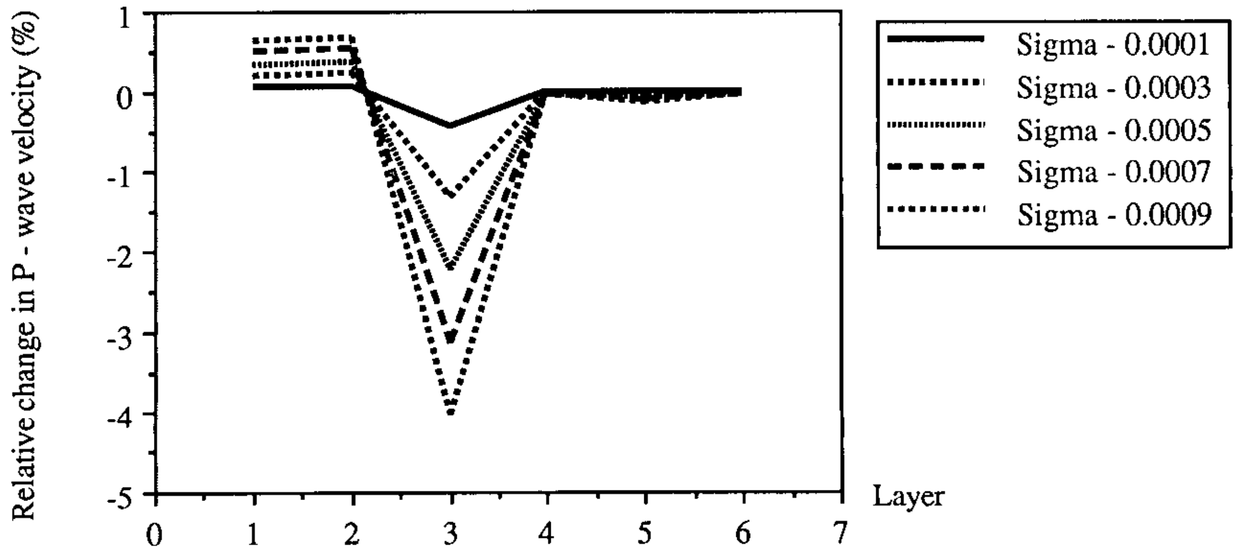


(b): Error of S - wave velocity from random noise in reflectivity by P-P and P-SV inversion

Fig.24: Error in S- wave velocity from random in reflectivity by joint inversion and P-P inversion

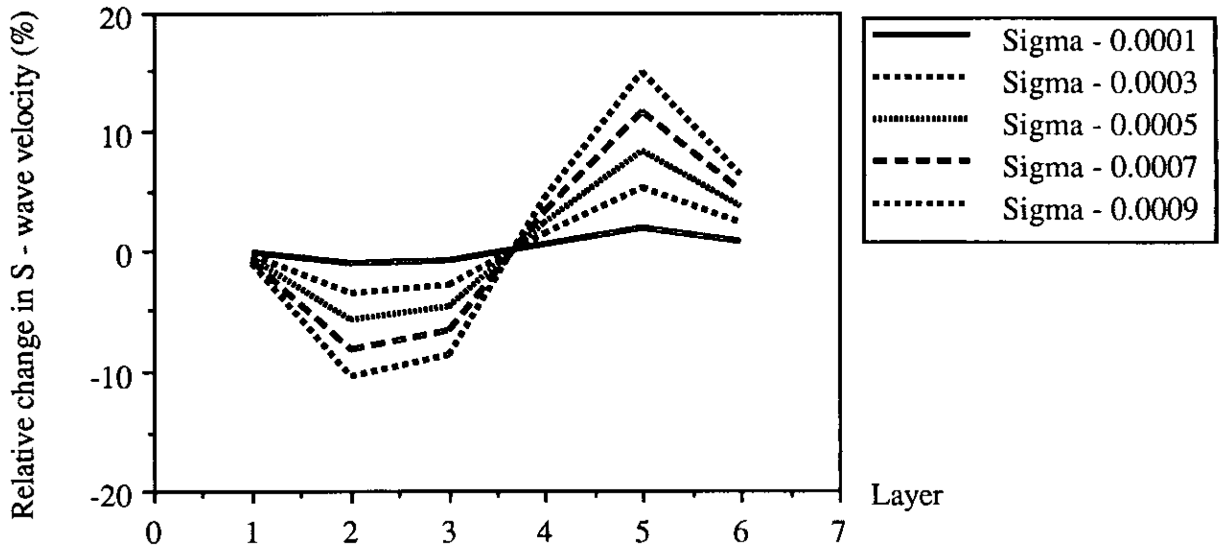


(a): Error (%) in $\Delta\alpha/\alpha$ from random noise in reflectivity by P - P inversion

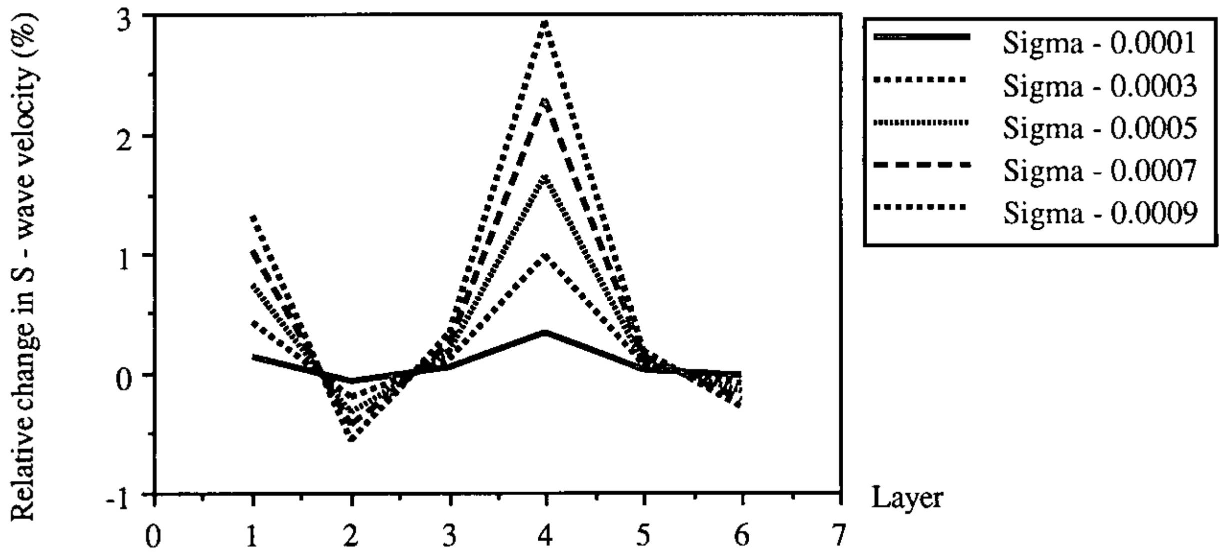


(b): Error (%) in $\Delta\alpha/\alpha$ from random noise in reflectivity by P-P and P-SV inversion

Fig.25: Error in $\Delta\alpha/\alpha$ from random error in reflectivity by joint inversion and P-P inversion



(a): Error (%) in $\Delta\beta/\beta$ from random noise in reflectivity by P - P inversion



(b): Error (%) in $\Delta\beta/\beta$ from random noise in reflectivity by P-P and P-SV inversion

Fig.26: Error in $\Delta\beta/\beta$ from random error in reflectivity by joint inversion and P-P inversion