Modelling shear-wave singularities in an orthorhombic medium

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ABSTRACT

In this paper we report on and compare the results of numerical and physical modelling experiments that focus on the behaviour of shear waves near a point singularity in an orthorhombic material. In the neighbourhood of such singularities, at least one of which is known to exist in a symmetry plane of an orthorhombic material, shear waves may exhibit rapid polarization variations approaching 180°. Our objectives are: to predict theoretically the location of a point singularity in the industrial laminate Phenolic CE using its stiffnesses as determined from experimental measurements; and to locate such a singularity by direct observation of wave propagation in its vicinity in a controlled physical experiment.

Piezoelectric transducer sources and receivers are placed in antipodal positions on a sphere of orthorhombic phenolic laminate and used to acquire traces shot along different directions through the phenolic. Data required for determination of stiffnesses are acquired, as well as traces along two profiles involving only shear sources and receivers. These profiles clearly show rapid variations of polarization direction near the singularity. We believe that this is the first time that a point singularity has been directly identified in either laboratory or field measurements, although the effects of propagation near point singularities have been recognized in VSP data by Bush and Crampin in the Paris Basin and by Yardley and Crampin above the Austin Chalk.

The location of the point singularity on the group-velocity or wave surfaces agrees extremely well with the location computed for the phase-velocity or slowness surfaces from the experimentally determined stiffnesses of the sphere. The slight observed shift of the singularity from phase-velocity to group-velocity surfaces agrees both in magnitude and sense with theory and with previous physical modelling results. We also find that the locations of point singularities are quite sensitive to variations in stiffness values. Small changes in stiffnesses from a cube to a sphere of phenolic cause a shift of some 15° in the position of the point singularity. This could imply that determinations of the directions of point singularities in field cases will act as a very good constraint on the stiffness values of a particular rockmass, which are directly related to its lithologic and fabric-related properties.

INTRODUCTION

Considerable theoretical development and numerical modelling of seismic wave propagation in anisotropic media have been carried out, particularly within the last decade. Only within the past few years have a handful of research groups begun scaled laboratory experiments, or physical modelling, to determine how well the various numerical schemes predict the actual physical results. A particular numerical modelling

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algorithm might be inadequate for several possible reasons. For instance, the basic theoretical assumptions on which the model rests may be in partial error or incomplete; the algorithm itself might incorporate computational approximations that were meant to make the problem numerically tractable, but which introduce significant error; or the routine might be derived on an idealized theoretical basis that is at variance with the physical reality: for example, an elastic model representing an anelastic reality; or a point-source/point-receiver theory representing the real situation.

That there exist significant differences just among different numerical modelling schemes has been elegantly demonstrated by Thomsen et al. (1990). In their ongoing study, several numerical modellers have taken a single anisotropic model with a single acquisition design (both surface and VSP) and are individually applying their own forward-modelling wave-propagation algorithms, on their own hardware platforms, with the intention of comparing results in a common format. Preliminary results show many similarities among the various synthetic seismograms produced, but also some significant differences in the amplitudes of various phases, as well as in computational efficiency. The possibility thus presents itself of carrying out the corresponding physical modelling to put the various numerical schemes to the test. There is therefore considerable interest on the part of seismic anisotropists to compare the results of numerical and physical modelling for *the same model and acquisition design*, that is, where input model parameters for the numerical experiment are identical (or as close as possible) to those of the physical model constructed for the laboratory experiment.

One such area of interest at present is in the behaviour of shear waves near special directions of propagation known as singular directions or *singularities*, which occur at places where the two shear-wave phase-velocity surfaces meet (i.e. touch or intersect). Near the commonest kind of singularity, the *point* singularity, a shear wave will likely exhibit anomalous behaviour such as rapid variation in polarization or amplitude, similar to what might be observed near cusps on the group-velocity surface, even when the anisotropy is not sufficiently strong to cause cusps (Crampin and Yedlin, 1981; Crampin, 1991).

The propagation effects of point singularities have been recognized in VSP data by Bush (1990) and Bush and Crampin (1991) in the Paris Basin, and by Yardley and Crampin (1993) above the Austin Chalk. Such observations may become increasingly important in exploration seismology, not least because point singularities may well occur along nearly vertical raypaths in sedimentary basins and therefore be potentially visible in conventional surface reflection-seismic data. Even for singularities well removed from the vertical, the possibility exists that these will be observable in VSP or crosswell-seismic data. If it were possible to determine the directions of such singularities, it could place tight constraints upon the nature of the internal anisotropy of the rockmass (Crampin, 1991).

Our immediate objectives in the seismic modelling study described in this paper, while not yet including the twinning of numerical and physical experiments, are: (1) to predict theoretically the location of a point singularity in the industrial laminate Phenolic CE using its stiffness values as determined by experimental measurement; and (2) to locate such a singularity by direct observation of wave propagation in its vicinity in a controlled physical experiment. In the longer term, we intend to examine in more detail the variations in polarization and amplitude near such directions, to compare our physical results with detailed numerical results obtained by collaborators, to elaborate how one might use singularity-related observations in geophysical investigations, including hydrocarbon exploration and development, and to develop appropriate processing algorithms for this purpose.

SHEAR-WAVE SINGULARITIES

The concept of singularities in shear-wave slowness and phase-velocity surfaces for anisotropic propagation media has been known for many years. Duff (1960), for example, showed that the two shear-wave sheets come into contact at least twice, and usually much more frequently, in directions (of slowness or phase velocity) known variously as singular directions, singular points, or simply singularities. There are three types of singularity: *kiss* singularities, *line* singularities and *point* singularities, in all of which the slowness and phase-velocity surfaces are analytically continuous. Kiss singularities are points where the two surfaces touch tangentially but do not intersect (Figure 1a). They may occur in any anisotropic symmetry system and, in the case of hexagonal or transverse-isotropy symmetry, there is always one at the cylindrical symmetry axis. Line singularities (Figure 1b), which only occur for transverse-isotropy symmetry, are no more than simple intersections of the two surfaces in a circle centred on the axis of cylindrical symmetry (Crampin and Kirkwood, 1981; Crampin and Yedlin, 1981; Crampin, 1991). As Crampin (1991) states, kiss and line singularities are not expected to cause major disturbances to shear wavetrains; nor are any associated anomalies in polarizations or amplitudes likely to be observable in seismic data.

Point singularities are points where the surfaces not only touch but also cross each other, and in such a manner that they are continuous through the vertices of cones on the inner and outer velocity sheets (Figure 1c). They are sometimes known in the crystallographic literature as conical points. These singularities do not occur in transverse isotropy but necessarily occur in virtually all other symmetry systems, the exception being the triclinic system in which the (at least two) singularities could be either kiss or point types. For solids with orthorhombic symmetry there must be at least

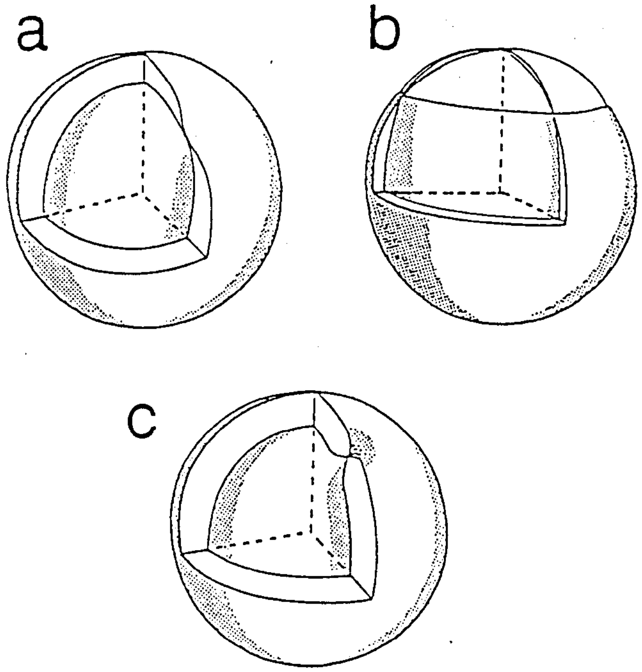


FIG. 1. Sketches of the three kinds of singularity: (a) kiss singularity; (b) line singularity; and (c) point singularity (after Crampin, 1991).

one point singularity somewhere in a symmetry plane (Crampin, 1981). More rigorously, geometrical relationships demonstrate that the three mutually perpendicular planes of mirror symmetry in solids with orthorhombic symmetry must contain an odd number of point singularities. The most common combinations are [0:0:1] (one point singularity in one symmetry plane) and [0:1:2]. The combinations [1:1:1], [0:0:3] and higher odd-number combinations are not thought possible, though we are not aware of any specific investigation having been made.

Point singularities may significantly disturb the behaviour of shear waves on neighbouring rays. Related to the fact that the curvature of the phase-velocity surface near such a point is very great, and that continuous paths on the surface passing through the singular point cross from the inner to the outer sheet (and vice versa), is the complicated behaviour of shear-wave polarizations, which can vary by up to 180° over neighbouring rays in the vicinity of the singularity.

In order to simulate the wave propagation in the Paris Basin (Bush, 1990; Bush and Crampin, 1991) and above the Austin Chalk (Yardley and Crampin, 1993), models of uniform rock with combinations of horizontal periodic thin layering (PTL) and parallel vertical fluid-filled cracks (or extensive-dilatancy anisotropy, EDA) were assumed. PTL alone leads to transverse isotropy (with a vertical symmetry axis for horizontal layering), while EDA alone produces azimuthal anisotropy (with a horizontal symmetry axis for vertical cracks), each of these being a special case of hexagonal or transverse-isotropy symmetry. The combination, however, gives rise to orthorhombic symmetry and, contrary to the case for either PTL or EDA alone, the necessary existence of at least one point singularity. Thus, for the purpose of physically modelling wave-propagation phenomena near point singularities – relevant to the simulation of basins with both PTL and EDA – orthorhombic materials are certainly appropriate, whereas transversely isotropic materials are quite inadequate.

EXPERIMENTAL OBSERVATIONS

Physical seismic modelling on a sphere

Our previous physical modelling experiments using orthorhombic Phenolic CE (Cheadle et al., 1991; Brown et al., 1991), have been carried out on samples having planar faces, namely cubes or slabs, sometimes with edges or corners planed off. In these studies, velocities in various symmetry, and off-symmetry, directions were determined and these showed small but significant differences from one sample to another. Some of these differences were probably due to the nonuniformity of different samples but some were also attributed to the fact that, relative to zero-offset or axial raypaths, oblique raypaths were necessarily of increasing length as offset increased, probably leading to slight variations in effective velocities as a result of source-receiver effects (effective-pathlength and array-attenuation effects), anelasticity (pulse stretching), etc. (Brown et al., 1991). For these reasons, we decided to shoot and record over a sphere of the material, for which all raypaths would not only be of equal length but also will impinge normally on all source and receiver transducers.

We machined a phenolic sphere of diameter 10 cm and initiated experiments on it using the set-up shown in Figure 2, together with the source-receiver transducers and the data acquisition described previously (Brown et al., 1991; Cheadle et al., 1991). The procedure is, having established a line or profile direction, to acquire nine traces, in general, at a particular point on the line. The nine traces correspond to the nine combinations of source and receiver polarizations, using as the three component directions: vertical (normal to the surface), radial (tangential, in-line) and transverse (tangential, cross-line), denoted respectively V, R, and T. This yields, in effect, a vector transfer function (or impulse response) for each point or propagation direction, allowing one to simulate seismic traces for any source polarization (Igel and Crampin, 1990). In the present study, traces involving compressional sources and receivers are used only in the determination of stiffnesses.

Although shooting and recording on a sphere removes some of the difficulties encountered when using a cube or rectangular slab, certain other problems arise that wipe out some of these gains. One such problem is size limitation. One can alleviate the problem of relatively large transducers either with smaller transducers or larger models. Phenolic CE can be acquired in slabs up to about 3 m (10') long, 1.2 m (4') wide and 10 cm (4") thick. However, in view of its high cost, our slab samples have not yet exceeded 60x60x10 cm (2'x2'x4"). So, using split spreads on these slab samples, we have achieved seismic pathlengths only slightly greater than 30 cm, though up to 3 m would be possible. In contrast, in machining a sphere of this material, the largest obtainable seismic pathlength, or diameter, is only 10 cm.

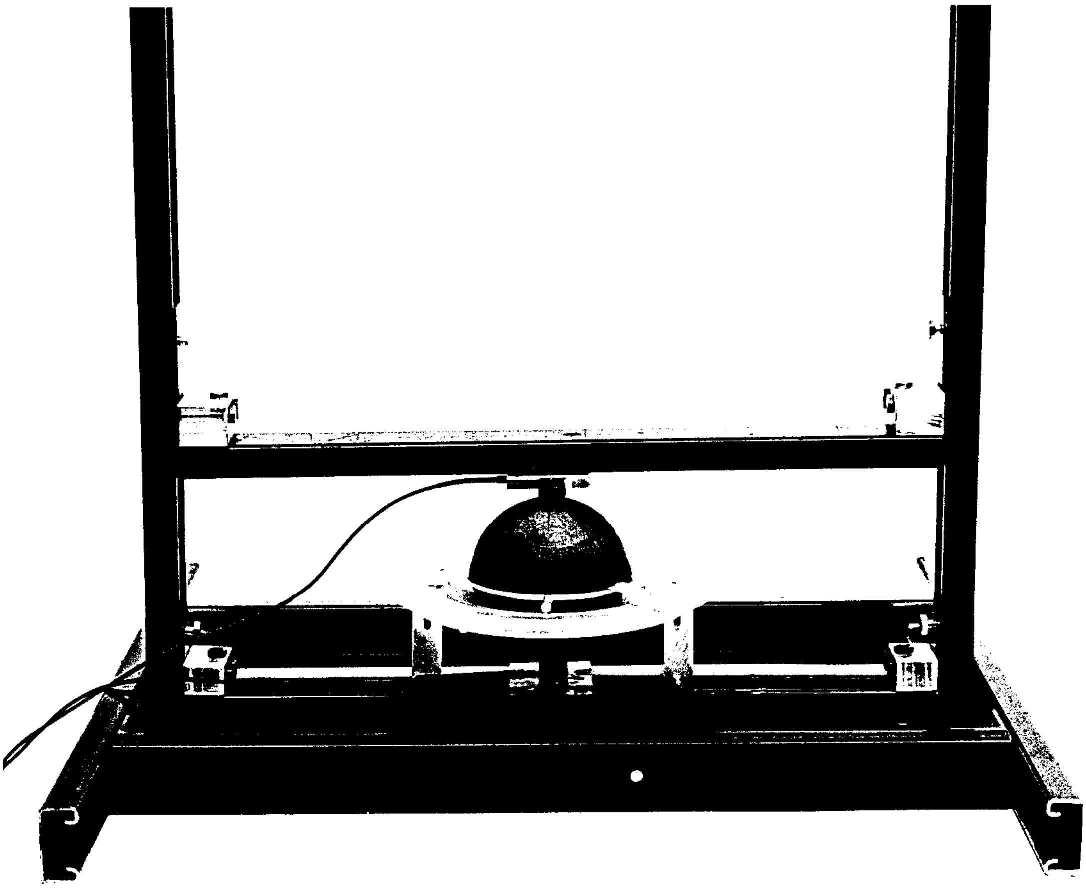


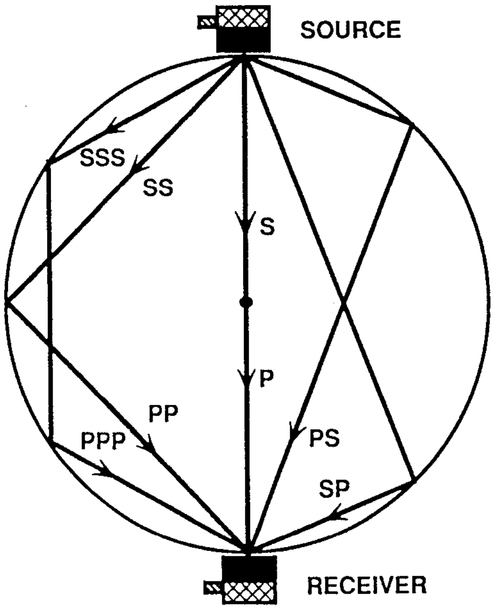
FIG. 2. Photograph of the laboratory set-up used in shooting and recording on the phenolic sphere. Transducers are in contact with the sphere (diameter 10 cm) at its top and bottom.

Another problem is caused by the curvature of the sphere surface. Our transducers have planar faces and naturally do not couple as well to a sphere as to a rectangular slab. Although amplitude reduction resulting from a smaller contact area is not a great problem, one must take pains to ensure consistent coupling and thus consistent source amplitudes. There are undoubtedly also different wave-propagation effects associated with a particular source on a curved surface as opposed to a flat one.

A third trouble source, and potentially the most serious, is the problem of interference of wave phases generated by the spherical surface with the wave phases of our primary interest, particularly *PS* and *SP* with *S*. Fortunately, as illustrated in Figure 3, and as further verified by experiment, the direct *P*- and *S*-wave onsets are, for the most part, uncontaminated.

Numerical model data

The traveltimes measured in these experiments, or eqivalently the velocities, allow one to determine the stiffnesses of the model material. In order to get some idea where to look for a point singularity, we used the only Phenolic CE stiffnesses available at the time, those obtained previously for the phenolic cube (Cheadle et al., 1991). With these as input, the phase and group velocities in the three symmetry planes were computed by the ANISEIS software package (Figure 4). In this we use the knowledge that for solids with orthorhombic symmetry there must be at least one point singularity somewhere in a plane of mirror symmetry (Crampin, 1981). These results indicate a point singularity in the 31-plane about 45° to 50° from the 3-axis, the 1- and 3-axes being, respectively, the directions of highest and lowest *P*-wave velocity (Figure



WAVE PHASE	APPROXIMATE TRAVELTIME (s)
P	0.30–0.35
PP	0.40–0.50
PPP	0.45–0.55
S	0.65–0.75
PS/SP	0.70–0.80
SS	0.90–1.00
SSS	0.95–1.10

FIG. 3. The main body-wave phases that could interfere with direct *P* and *S* arrivals for transmission through a sphere. Traveltimes are scaled up by a factor of 10^4. For simplicity, we here omit the prefix *q* (quasi-).

Table 1. Comparison of velocities (m/s) and stiffnesses (10^{10} N/m^2) for a cube versus a sphere of the industrial laminate Phenolic CE.

Cube				Sphere			
V_{11}	3576	C_{11}	1.7443	V_{11}	3584	C_{11}	1.7521
V_{22}	3365	C_{22}	1.5445	V_{22}	3401	C_{22}	1.5777
V_{33}	2925	C_{33}	1.1670	V_{33}	2935	C_{33}	1.1750
V_{23}	1516	C_{44}	0.3135	V_{23}	1514	C_{44}	0.3127
V_{31}	1606	C_{55}	0.3518	V_{31}	1598	C_{55}	0.3483
V_{12}	1662	C_{66}	0.3768	V_{12}	1670	C_{66}	0.3804
V_{44}	3094			V_{44}	3080		
$V_{4\bar{4}}$	1569	C_{23}	0.6379	$V_{4\bar{4}}$	1579	C_{23}	0.6196
V_{55}	3219			V_{55}	3205		
$V_{5\bar{5}}$	1620	C_{31}	0.6633	$V_{5\bar{5}}$	1605	C_{31}	0.6608
V_{66}	3378			V_{66}	3389		
$V_{6\bar{6}}$	1810	C_{12}	0.7283	$V_{6\bar{6}}$	1838	C_{12}	0.7220

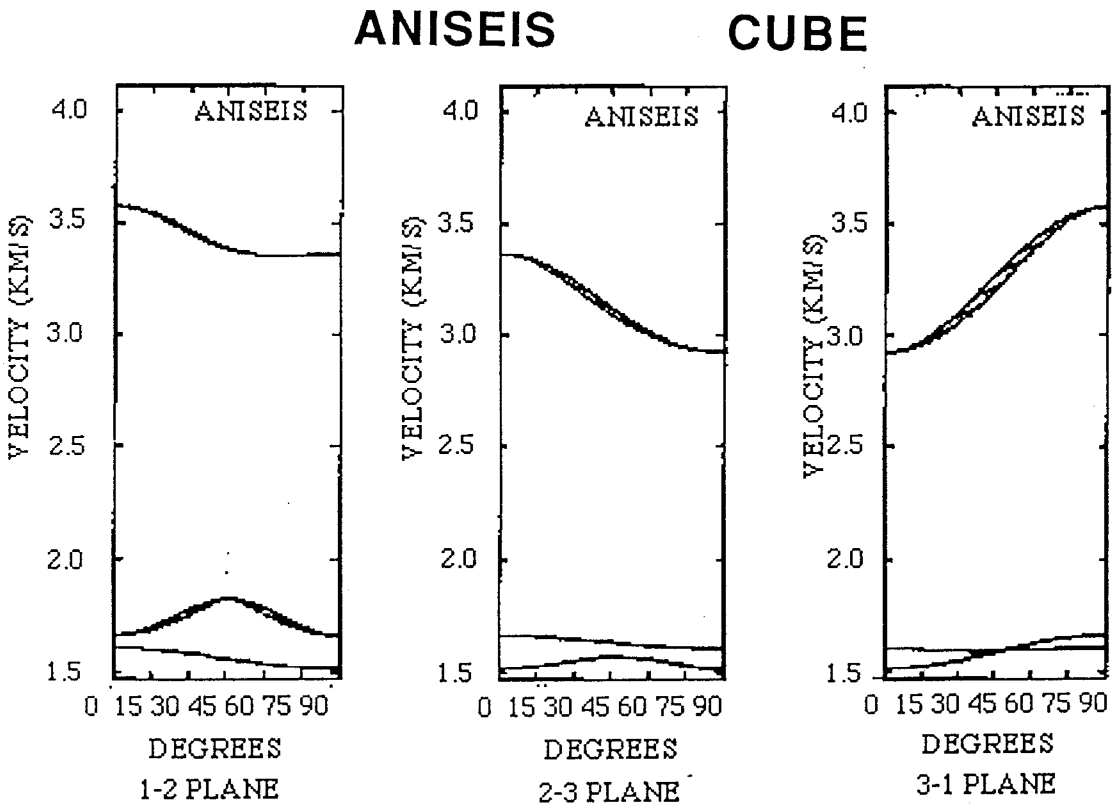


FIG. 4. Phase (solid) and group (dashed) velocities computed by ANISEIS using stiffnesses of the phenolic cube (Table 1) as input. Line segments join points on the two curves corresponding to one and the same point on the wavefront. For these stiffnesses there is one singularity in the 31-plane, where the two shear-wave curves intersect (i.e. a [0:0:1] material).

5). In view of this result, we decided to shoot two perpendicular seismic profiles: line 1 along the meridian spanning the 1- and 3-axes, and line 2 along the great circle bisecting the first at right angles, that is, crossing it 45° from the axes (Figure 5). For convenience, we designate this crossing-point, labelled the 5-direction (Cheadle et al., 1991) in Figure 5, as "near-singularity".

A second ANISEIS computation, using specifically the stiffnesses determined for the sphere itself, could not be run until after the laboratory acquisition of the requisite seismic data, acquired at the same time as the profiles shown below. These results (Figure 6) show that the differences between the velocities and stiffnesses determined for the cube and the sphere are not great (Table 1), the two sets of stiffnesses differing on average by only 1% and at most by 3% (for C_{23}). Nevertheless, the computed location of the singularity has moved significantly, by about 15°, to a point about 60° to 65° from the 3-axis. As shall be seen below, the experimentally observed position of the singularity on the phenolic sphere, while disagreeing with that computed for the phenolic cube by some 15° to 20°, agrees extremely well with that computed for the sphere.

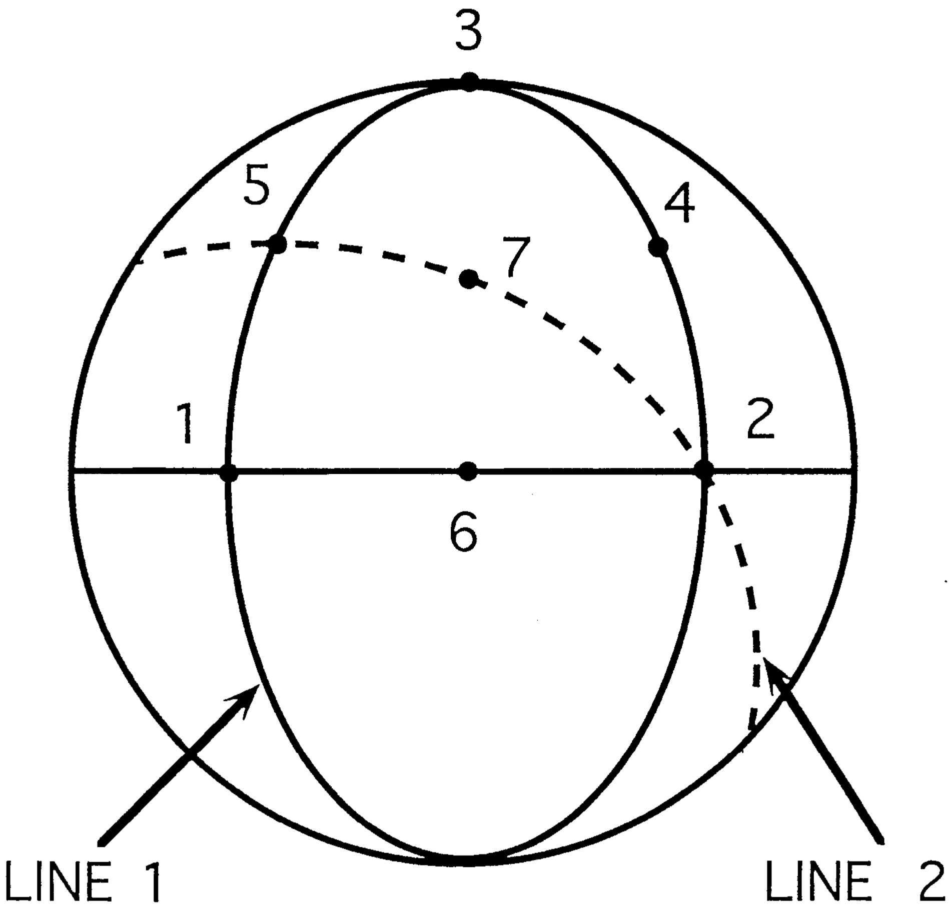


FIG. 5. The two orthogonal profiles, line 1 and line 2, on the sphere. The symmetry axes (1, 2 and 3) are labelled, as well as the directions (4, 5 and 6) halfway between axes. Point 7 marks the direction equidistant from the three axes.

Physical model data

For each of the two lines shown in Figure 5, we acquired profiles of 33 shots, 180° in length, with a nominal shot interval of 5.625°, and with point 5 as the midpoint. At each shotpoint, four tangential (shear) components were acquired, that is, all four combinations of radial and transverse source and receiver polarizations. In every case, the receiver transducer was positioned at the antipode of the source transducer. Figures 7 and 8 show the seismic profiles obtained for lines 1 and 2, respectively.

The most striking feature in these figures is the change in polarity of the S_1 arrival, whose onset is at about 0.65 to 0.75 s, clearly seen on the R-T and T-R (cross-component) records for both line 1 and line 2 (Figures 7b, 7c, 8b and 8c). This polarity change is just as clearly not present on the R-R and T-T records (Figures 7a, 7d, 8a and 8d). One can also again see good examples of negative reciprocity (Knopoff and Gangi, 1959; Brown et al., 1991) in comparing respectively the R-T records (Figures 7b and 8b) with the T-R records (Figures 7c and 8c).

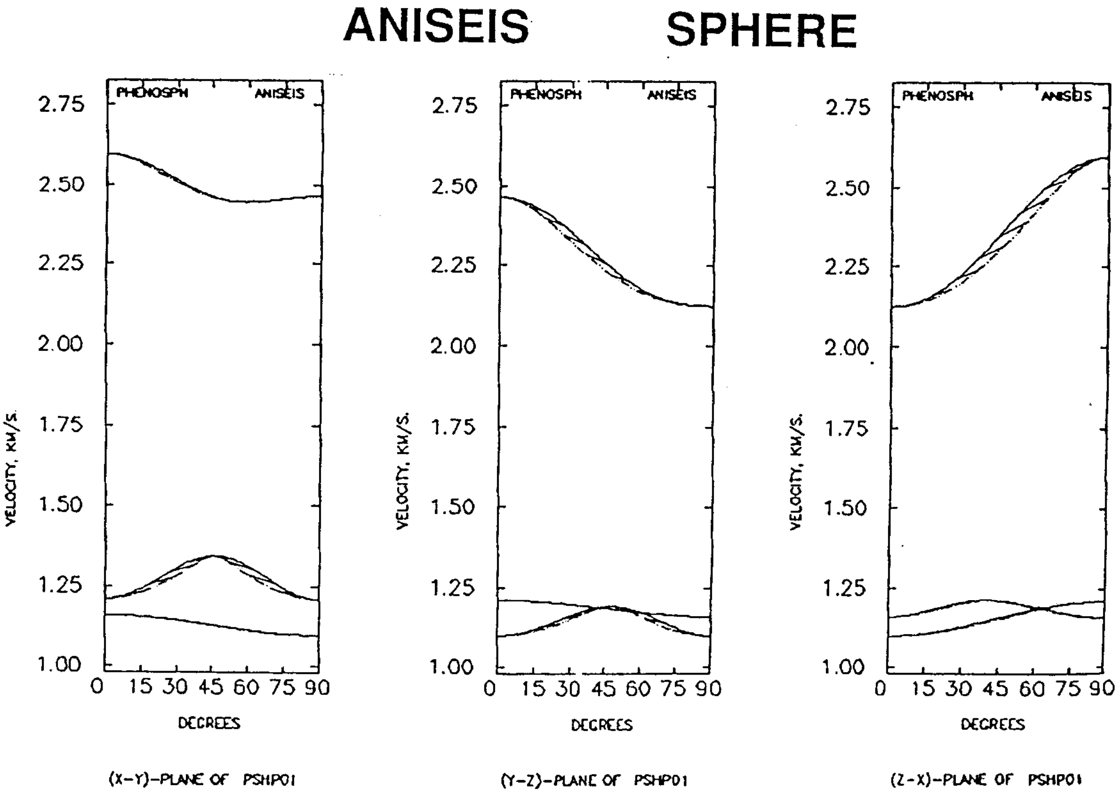


FIG. 6. Phase (solid) and group (dashed) velocities computed by ANISEIS using stiffnesses of the phenolic sphere (Table 1) as input. For these stiffnesses there are two singularities in the 23-plane and one in the 31-plane (i.e. a [0:1:2] material).

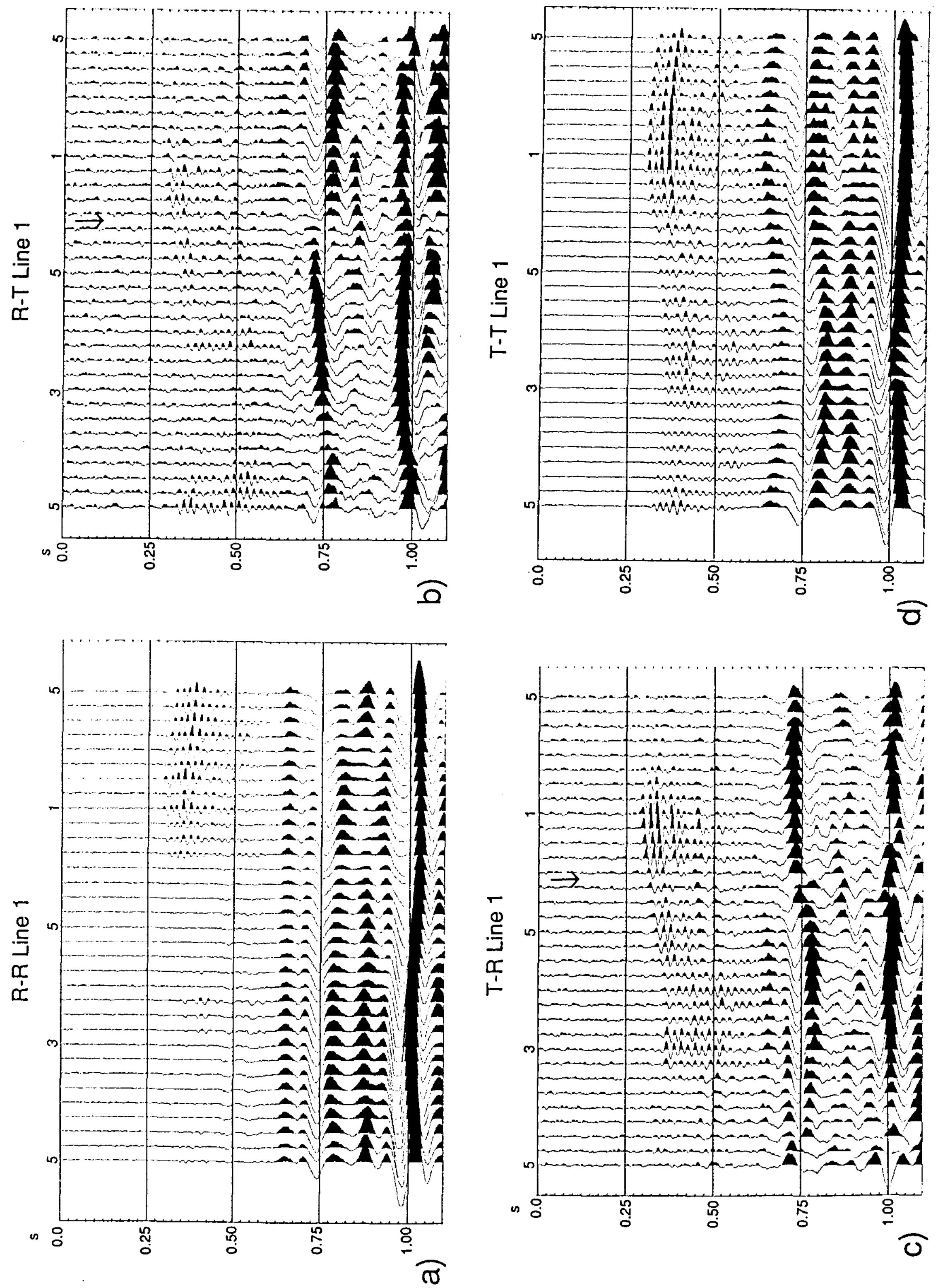


FIG. 7. Line 1, with the 31-plane (Figure 5) as the sagittal plane. There are 33 traces over a 180° profile, giving a nominal trace interval of 5.625°. In reality, we probably have a maximum uncertainty in positioning the transducers for each shot of ~1 mm or ~1°. (a) R-R polarization; (b) R-T; (c) T-R, and (d) T-T. See also Figure 8.

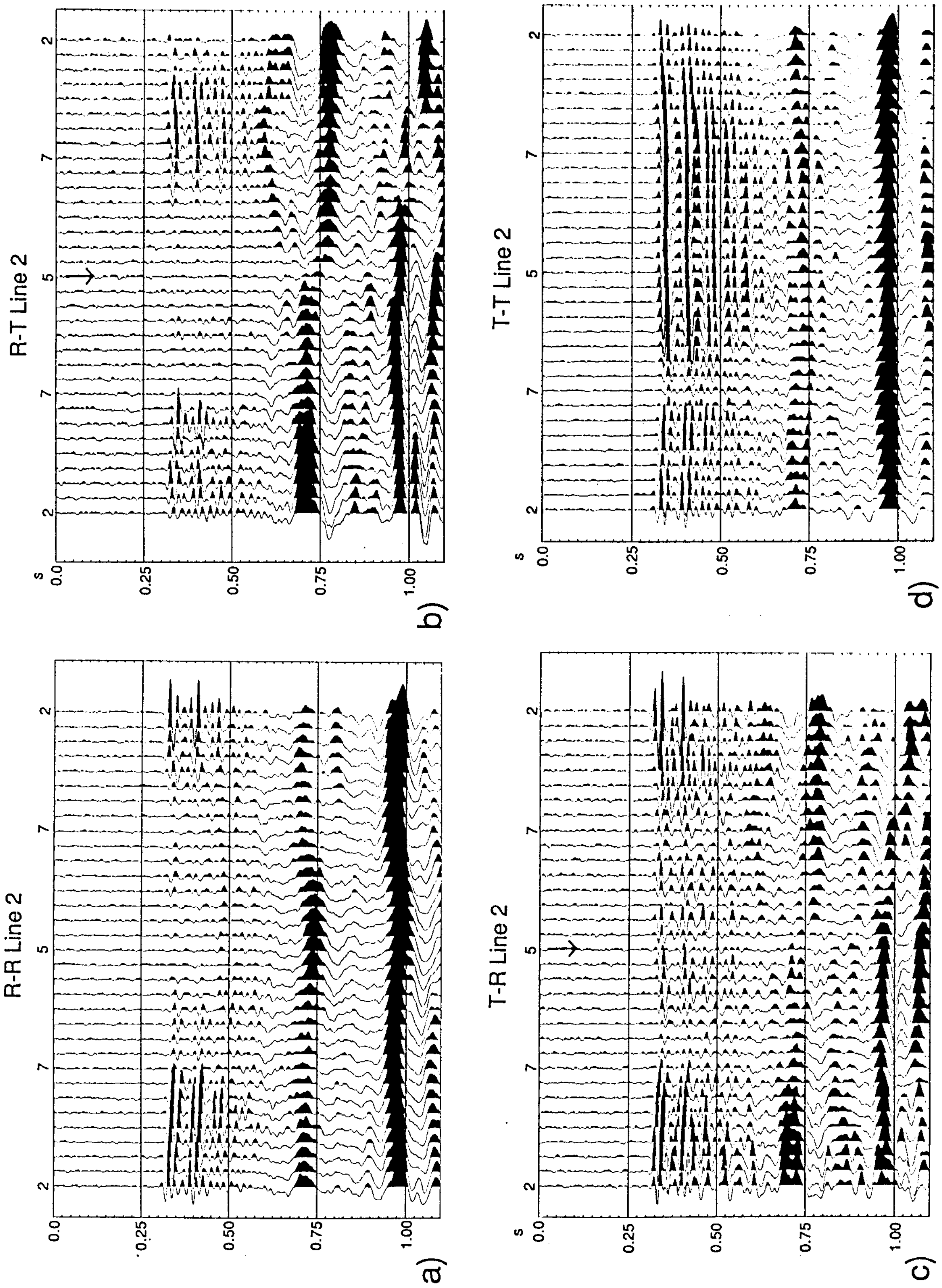


FIG. 8. Line 2, with the 25-plane (Figure 5) as the sagittal plane. Traveltimes and frequencies are scaled up and down, respectively, by a factor of 10^4. (a) R-R; (b) R-T; (c) T-R, and (d) T-T. An arrow near the tops of each cross-component record, (b) and (c), indicates the position of the polarity reversal, or singularity. See also Figure 7.

Supported by the following discussion, we interpret these polarity reversals on the cross-component records as manifestations of rapid polarization changes, and thus as direct evidence of a point singularity in the phenolic material. For example, it is reasonable to expect that a shear wave generated near point 5 with radial polarization (within the sagittal 31-plane) would arrive at the antipodes with a radial component of polarization of this same polarity throughout the neighbourhood of point 5. However, there are no similar momentum-conservation considerations that would determine any fixed polarity for the transverse component (which would vanish in the isotropic case). Indeed we see (Figures 7b and 8b) that it is the transverse component of polarization whose polarity reverses through a close approach to the singularity.

On line 2 the polarity reversal, or the singularity, appears to occur right at point 5, that is at the intersection with line 1. Evidently then, the singular direction is in the symmetry plane subtended by line 1, compatible with the requirement that at least one point singularity lie in a symmetry plane. On line 1 the singularity appears to lie about 20° away from point 5 towards the 1-axis, or about 65° from the 3-axis, in good agreement with the position computed from the sphere stiffnesses (Figure 6).

So far, we have not differentiated between phase- and group-velocity surfaces in locating the singularity. The theoretical singular directions are defined in terms of phase-velocity surfaces and specified in terms of phase angle. In contrast, our seismic traces register energy arrival and so render an image of the group-velocity surface, with directions of wave arrival being group angles, not phase angles. However, our earlier measurements on Phenolic CE have shown that for shear waves near the 5-direction (halfway between 3 and 1) phase and group angles differ by about 5° or less (Cheadle et al., 1991, Table 1). This is also verified by Figure 6, which shows both phase-velocity curves (solid) and group-velocity curves (dashed). Line segments on the two curves join points that belong to the same point on the wavefront. The horizontal displacement of any such pair is the difference between the corresponding phase and group angles. For the shear waves in the 31-plane, this displacement is almost indistinguishable, and in fact is not more than about 5°.

The sense of this displacement, in addition to its magnitude, can also be established. Measuring from 0° at the 3-axis, the phase angles are less than or equal to the corresponding group angles for shear waves in this 31-plane. This has been shown by physical modelling (Cheadle et al., 1991, Table 1) and numerical modelling (Figures 4 and 6). From the ANISEIS output, we have a phase angle for the singularity of 15° to 20° and from the seismic data a group angle of about 20°. The observations are therefore entirely consistent with the theory.

DISCUSSION

Conclusions

We have made a direct experimental observation of a shear-wave point singularity in a sphere of the commercial laminate Phenolic CE. As far as we are aware, this is the first time that a point singularity has been directly identified in either laboratory or field measurements, although the effects of propagation near point singularities have been recognized in VSP data by Bush (1990) and Bush and Crampin (1991) in the Paris Basin, and by Yardley and Crampin (1993) above the Austin Chalk.

The location of the point singularity on the group-velocity or wave surfaces is in the 31-plane of symmetry, about 65° from the 3-axis, the slowest direction for *P* waves. This agrees extremely well with the location on the phase-velocity or slowness surfaces computed by ANISEIS from the experimentally determined stiffnesses of the sphere, that is, about 60° to 65° from the 3-axis. This slight shift of the singularity from phase-velocity to group-velocity surface is wholly consistent with previous findings, namely that this singularity should be a couple of degrees closer to the 3-axis on the phase-velocity surface.

Locations of point singularities are quite sensitive to variations in stiffness values. An average change in stiffnesses of 1% from the cube to the sphere caused a shift of some 15° in the position of the point singularity. This could mean that determinations of the directions of point singularities in field cases will act as a very good constraint on the stiffness values of a particular rockmass, values that are directly related to fabric and lithologic properties such as nature and degree of internal order or alignment of, for example, cracks, grains, or layering.

Future Work

In the longer term, we intend to examine in more detail the variations in polarization and amplitude near such directions, to compare our physical results with numerical results obtained by collaborators, to develop methods that would use singularity-related observations in geophysical studies, for example, in hydrocarbon exploration or development, and to develop the necessary processing code required to this end.

Practical problems will undoubtedly arise in this work. For example, how can we compare recorded amplitudes generated by different source transducers (*P* versus *S*) which may have different piezoelectric responses and probably couple differently to the material surface? We may be able to use the principles of seismic reciprocity, in particular negative reciprocity (Knopoff and Gangi, 1959; Brown et al., 1991), to equalize, for example, a V-T trace (*P* source, *SH* receiver) and a T-V trace, and so on.

We also still have the problem of relatively large transducer size (compared to dominant wavelengths and specimen size), which means that any one trace is a sort of convolution or composite record of a rather thick pencil of rays. This is not as great a problem for traces generated and recorded on a sphere as it is for a cube or slab because the transducer faces are everywhere normal to the raypaths for the sphere. For continued work, however, we are looking into methods to reduce the effective transducer size such as: miniature transducers, deconvolution of recorded traces (incorporating knowledge of the transducer directivity functions or radiation patterns), or more advanced technology based on interferometry or pulsed-laser methods (see e.g. Castagnede et al., 1991).

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