3-D f-k filtering

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ABSTRACT

Stewart and Schieck (1993) show that a conventional 2-pass 2-D f-k filter fails to provide an axially-symmetric response. They suggest using a true 3-D one-pass filter for specific pass ranges and symmetric results. They also approximate a true 3-D f-k symmetric response by axially rotating a 2-D f-k filter. In this paper a true 3-D f-k filter will be compared with this 2-D rotated filter. During the investigation, it is found that an interpolation scheme is required when rotating a discrete filter to avoid an aliasing problem. A special 2-D filter that has a 2-pass symmetric 3-D f-k response can be defined. This Gaussian shaped filter is applied in two passes and produces a symmetric response, however it has a very gradual cut-off region.

INTRODUCTION

The amount of 3-D data collected has been increasing and the need for 3-D filtering can be understood. Stewart and Schieck (1993) show that a simple 2-pass 2-D f-k filter for 3-D fails to have a circular symmetry on application. Instead, they suggest an axially rotated 2-D f-k filter version to maintain the symmetry. However they also claim that this filter is a good approximation for a true one pass 3-D f-k filter but it is not exact. In this paper a true 3-D f-k filter is derived and is compared with a 2-D rotated version. A sampling problem occurred during rotating the 2-D filter. This problem is also examined. A special filter that has a two-pass symmetric 3-D f-k response is developed and is discussed.

Methods

A 3-D f-k filter has the following expression in ω-k_x,k_y domain.

$$F(\omega,k_x,k_y) = \begin{cases} 1 & \frac{\omega}{k} > v_L \\ 0 & \frac{\omega}{k} \le v_L \end{cases}$$

where $k = (k_x^2+k_y^2)^{\frac{1}{2}}$.

From this expression, each frequency slice has an amplitude of zero outside a circular radius k and one inside this radius. This radius k is a function of frequency and cut-off velocity. In this paper, the analysis will be focused on a frequency slice, hence the 3-D f-k can be reduced into 2-D problem. Bracewell (1978) shows that the Fourier transform of a 2-D circular symmetrical function is equivalent to the Hankel transform of the same function in 1-D. Furthermore he also shows that

$$\text{Hankel transform of } \Pi\left(\frac{r}{2a}\right) = \frac{aJ_1(2\pi q)}{q}$$

$$\text{where } \Pi(x) = \begin{cases} 1 & |x| \le \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$

and J_1 is 1st order of Bessel function.

Therefore the response of a true 3-D f-k filter in ω-x,y domain is an axially rotated 1-D Bessel function of first order. Stewart and Schieck (1993) approximate this filter by an axially rotated 1-D sinc function. The differences are shown in Figure 1a. In the x-domain the Bessel function has less oscillation than the corresponding sinc function, therefore the roll-off of amplitude for a Bessel function is very gentle and can be thought as a special taper being applied to the box-car function (Figure 1b). After 2-D Fourier transforming, a rotated Bessel function gives a cylindrical shape of amplitude spectrum with much less over-shooting at the edge than a 2-D Fourier transform of a equivalent sinc function (Figure 1c).

In the test above, analytical expressions exist for both sinc and Bessel function, and they can be easily rotated. However if an analytical expression for a filter is not easily available (for example a filter with a taper), it may be economical to just rotate a discrete filter. In this case, sampling a discrete filter during the rotation can be an issue. In Figure 2, cross-sections of a 2-D amplitude spectrum for Bessel function, sinc function and sinc function with 5 Hz linear taper, are shown. It is generated by rotating a discrete filter without interpolation. In Figure 2a the Nyquist of the filter is 125 spatial frequency, the amplitude spectrum shows a lot of ripple inside the pass-band. In Figure 2b the pass-band of the filter is reduced into 50 spatial frequency, and in Figure 2c the pass-band is kept at 100 spatial frequency, however the Nyquist is increased into 250 spatial frequency. Both spectra have relative better flat top in the pass-band than that in Figure 2a. In addition, significant amplitude of high frequency noise outside the pass-band are noticed. In Figure 3, same filters as in Figure 2 are rotated but with linear interpolation. This time the spectra are much better than that in Figure 2. High frequency noise, however, are not noticed in Figure 3. Therefore it is concluded that nearest sampling (no interpolation) can generate high frequency noise and even worse they can wrap around and alias inside the pass-band.

For large data sets, a two-pass 2-D f-k filter may be more efficient than one-pass 3-D f-k filter. Conventional two-pass 2-D f-k filter, however lacks circular symmetry (Stewart and Schieck, 1993) after application. In the following paragraph, a circular symmetric two-pass 2-D F-K is introduced and discussed.

A 2-D filter slice can be separated into two-pass, if the filter f can be written as:

$$f(x,y) = g(x)h(y) \tag{1}$$

where g and h are some filters.

Also circular symmetry requires the filter f to have the following properties:

$$f(x,y) = f(y,x) = f(r) \tag{2}$$

where $r = \text{sqrt}(x^2 + y^2)$

Combining (1) and (2),

$$f(x,y) = g(x)g(y) = f(r)$$

or

$$f(r) = g(x)g(y) \tag{3}$$

A Gaussian function of the form $f(x) = \exp(\alpha x^2)$ satisfies equation (3) with $g(x) = f(x)$ if

$$\begin{aligned} f(r) &= \exp(\alpha r^2) \\ &= \exp(\alpha(x^2+y^2)) \\ &= \exp(\alpha x^2)\exp(\alpha y^2) \\ &= f(x)f(y) \qquad (4) \end{aligned}$$

This function behaves like a low-pass filter with a smooth build-in taper. The pass-band is determined by α. This filter has another interesting property that the Fourier transform of this filter is another shape of a Gaussian filter (Bracewell, 1978). Figure 4 shows an example of f-k filter with a Gaussian function in ω-x and ω-k. When this filter is applied in two-passes, it produces a circular symmetric response with the same shape as the filter being rotated and rejects low velocity events.

Although this filter has some good properties and is efficient when applying in 3-D, it cannot well-define a cut-off velocity due to the gentle shape of the taper. This filter also cannot reject high velocity without saving the original input or the symmetry of response will be destroyed.

Conclusion

In this paper a true 3-D f-k filter is compared with a 2-D rotated sinc function. This 2-D rotated filter requires a taper to reduce it's Gibb phenomena in 2-D. A interpolation scheme is required to rotate a discrete filter in order to avoid the aliasing. A 2-pass 2-D symmetric filter is also discussed and it is attractive in terms of speed, however it also has some disadvantage.

REFERENCES

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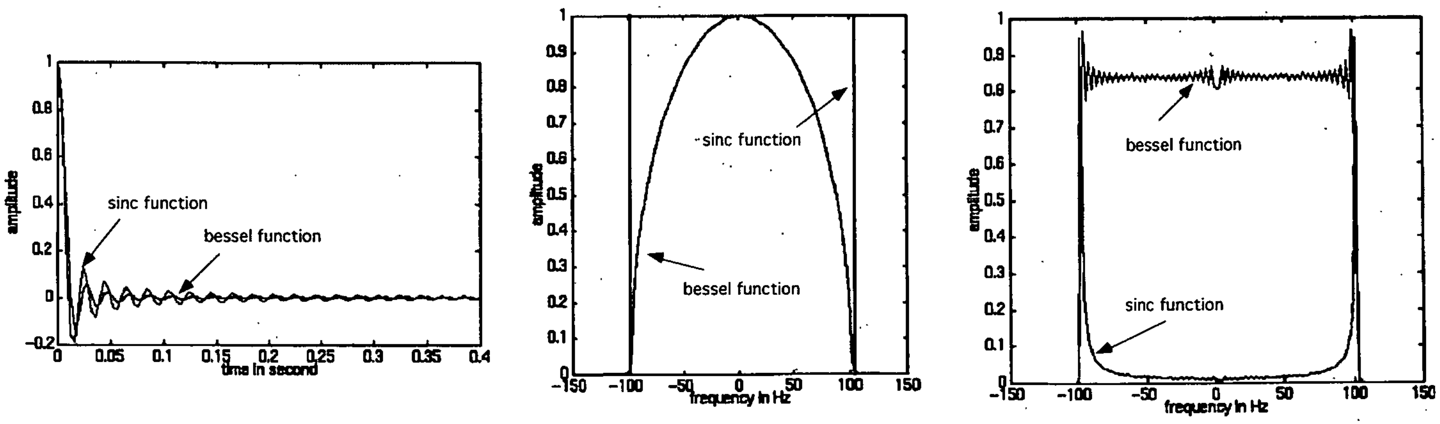

FIG. 1. Differences between a sinc and a bessel functions. (a) time domain difference. (b) frequency domain difference. (c) frequency domain difference after rotation.

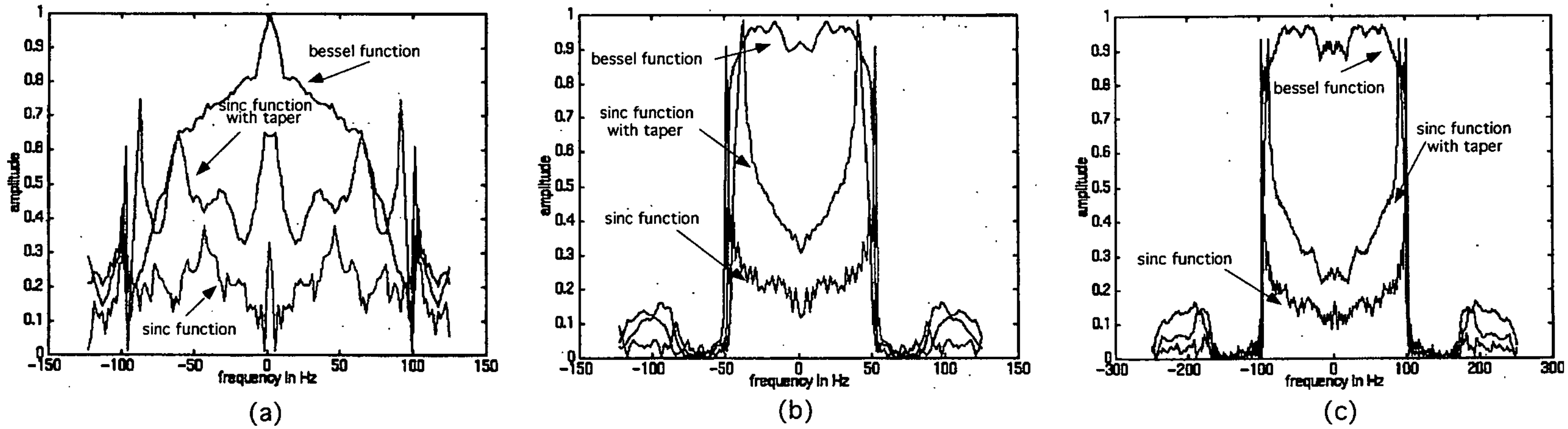

FIG. 2. Cross-sections of 2-D amplitude spectrum of rotated discrete functions without interpolation. (a) pass-band is 100Hz with Nyquist at 125Hz. (b) pass-band is 50Hz with Nyquist at 125Hz. (c) pass-band is 100Hz with Nyquist at 250Hz.

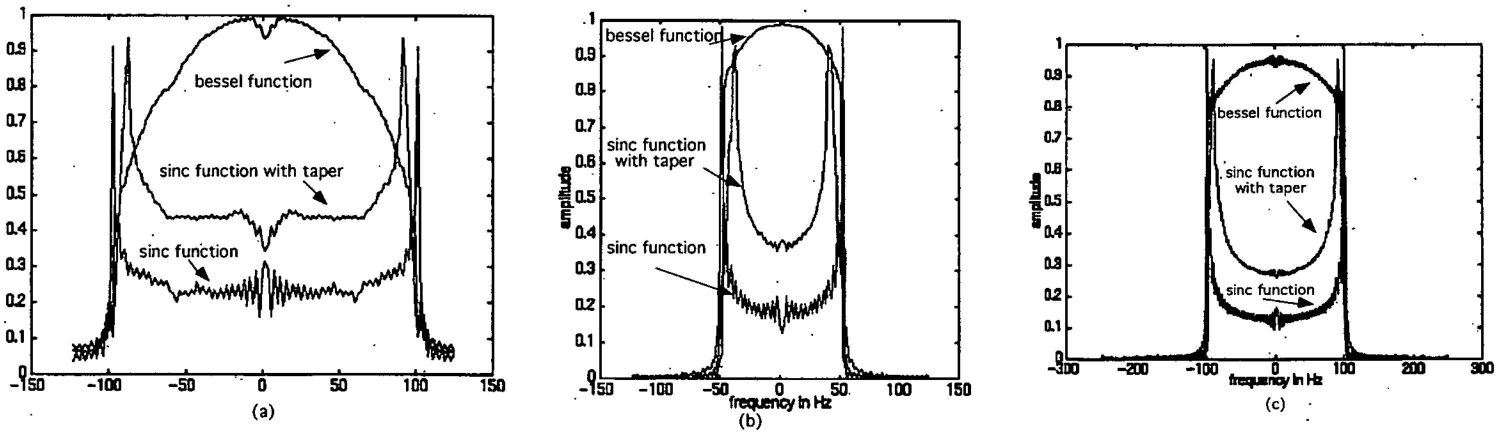

FIG. 3. Cross-sections of 2-D amplitude spectrum of rotated discrete functions with interpolation. (a) pass-band is 100Hz with Nyquist at 125Hz. (b) pass-band is 50Hz with Nyquist at 125Hz. (c) pass-band is 100Hz with Nyquist at 250Hz.

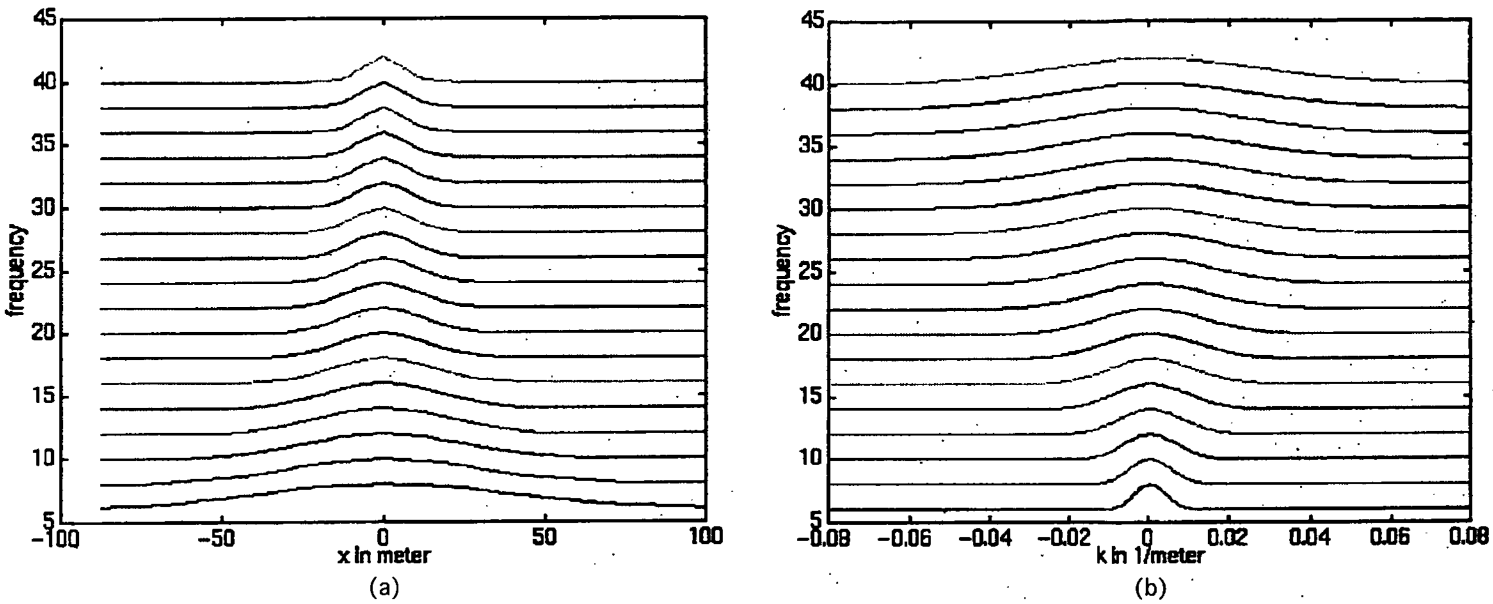

FIG. 4. Schematic diagram of using a gaussian function as a 2-pass 2-D symmetric 3-D f-k filter to reject velocity up to 500m/s. (a) filter in ω-x domain. (b) filter in ω-k domain.