

## **Prestack migration by equivalent offsets and CSP gathers**

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### **ABSTRACT**

A method of prestack time migration is presented that is simpler, faster, and more flexible than the conventional methods. The method is based on the principles of prestack Kirchhoff time migration and can be applied to both 2-D and 3-D data. Common Scatter Point (CSP) gathers are created for each output migrated trace, replacing the common midpoint (CMP) gathers of conventional processing. Samples from each input trace are assigned an equivalent offset for each output scatter point position, then copied into the appropriate offset bin of the CSP gather. The time sample position of the input data remains the same when copied to the CSP gather. Data in the CSP gathers may be scaled, filtered, or have noise attenuation process applied. Normal moveout (NMO) and stacking is all that is required to complete the prestack migration process.

The CSP gather is defined at the same position as a CMP gather relative to the output migrated trace, but contains all traces within the migration aperture. The CSP gather also contains larger offsets than the CMP gather, and each bin in the CSP gather may have large fold. These attributes of the CSP gather permit more accurate velocity analysis than conventional methods using super gathers from CMP locations.

### **INTRODUCTION**

Migration is a process that attempts to reconstruct an image of the original reflecting structure from energy recorded on input seismic traces. Prestack migration is a direct process that moves each input sample into all the possible reflection positions, and invokes the principles of constructive and destructive interference to recreate the actual image. An alternate description of the migration process starts by selecting an output migrated sample. All input traces are searched to find energy that contributes to the output sample. This second description is the basis of Kirchhoff migration.

Prestack migration is an expensive process. Many simplifying processes have been developed to reduce the cost of processing. A typical approximation to the prestack migration process involves normal moveout (NMO), stacking, and post-stack migration. It has long been recognized that the stacking process only works correctly for horizontally layered reflectors. Although dipping events may be stacked by modifying the velocity, reflection point smear will still occur. Commonly, dip moveout is introduced into the processing sequence to eliminate problems associated with the stacking of dipping events. However, these processing sequences require two estimates of the velocity model; one for the NMO process, and a second for the post-

stack migration. Ideally, the migration velocity should be the same as the DMO corrected NMO velocities, but in practice, considerable effort is spent iterating between the two before optimal migration velocities are obtained. In contrast, prestack migration uses one velocity model.

The Kirchhoff approach to prestack migration is based on a model of the subsurface as an organized set of scattering points. The model assumes that energy may come from a source located anywhere on the surface. The energy is scattered by points in the subsurface, then returns to a receiver located anywhere on the surface. The location of energy on a recorded trace is the total travel time along the ray path from the source down to the scatter point and back up to the receiver. The energy on the recorded input trace is weighted by an amplitude function then summed into the sample location on the desired output migrated trace.

The surface position of a vertical array (or trace) of scatter points is referred to as the *common scatter point* (CSP) location. The collection of all input traces that record energy from a given scatter point is referred to as the *migration aperture*.

Ray path travel times may be estimated by a number of different methods such as ray tracing, wave front computations, or travel times based on RMS velocity. The shape of these travel times in the neighboring traces is often referred to as the diffraction shape. When velocities in the data vary smoothly, the diffraction shape can be approximated by an hyperbola for 2-D data and an hyperboloid for 3-D data. The shapes are computed using estimates of the RMS velocities. The RMS velocity is defined at the scatter point location, which permits ray paths to the surface to be approximated by straight ray paths with a constant velocity. When the RMS velocities are defined on a time section, the resulting migration is referred to as a *time* migration.

## 2-D PRESTACK DATA VOLUME

Each data trace recorded along a 2-D line may be considered as part of a volume of sorted data, where the sort parameters define the axes of the volume. For conventional data, the axes are CMP location, time, and source-receiver offset. Typically, time on the vertical axis and distance on the horizontal axes are normalized by RMS velocity and CMP spacing so that curves within the data volume can be described simply as circles, ellipses or hyperbolas. An example of a data volume is illustrated in Figure 1.

The top surface of the data volume may be viewed as a stacking chart in conventional processing. After stacking, the front face of the data volume may be viewed as the output stacked section. Within the data volume, collections of data traces sharing a common parameter value are called records, gathers, or panels, and can be displayed as planes.

### Source (Shot) Records

Prestack data is generally acquired as source gathers or records. Source records in a data volume are illustrated in Figure 1. Each trace is assigned a location in the data volume based on the surface location of the midpoint between the source and receiver (CMP), and the offset (h), equal to half the distance between the source and receiver. The bold vertical lines represent traces from a one-sided source record. The offset of each trace increases as distance from the source to the receiver increases.

The dots on the top surface of the volume represent traces from one-sided source records. The intersection points of the grid show all possible locations of traces within the data volume and indicate that, for the prestack data illustrated in Figure 1, the sources were located every second receiver station, or every fourth CMP station.

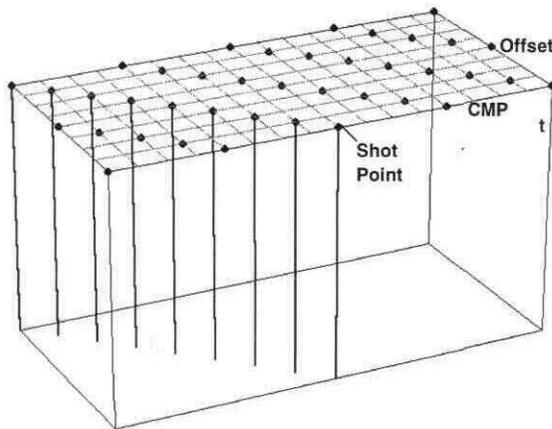


FIG. 1. Schematic representation of a source record displayed in a prestack volume.

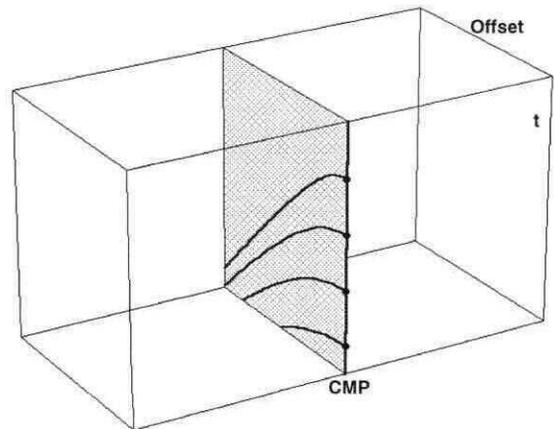


FIG. 2. Identification of a CMP gather in the prestack data volume.

### CMP gathers

Prestack data can be sorted into CMP gathers. Figure 2 contains a shaded area in a prestack volume that represents traces with the same CMP location sorted by increasing offset. Reflected energy from flat events located directly beneath the CMP location will arrive at progressively later travel times as offset increases. The reflection arrivals will appear as a series of curved lines in the CMP gather. Normal moveout (NMO) corrects these to be horizontal, and allows the traces in the CMP gather to be added or stacked. Note that each CMP gather in the prestack data volume illustrated in Figure 1 contains only three traces.

### Constant offset section

Prestack data can be sorted into constant offset sections as illustrated in Figure 3. The maximum number of offsets in a 2-D line may be large and is typically equal to one half the number of receivers for a split spread. However, the spacing of live traces in a constant offset section is equal to the spacing of the shots, as can be seen by examining Figure 1.

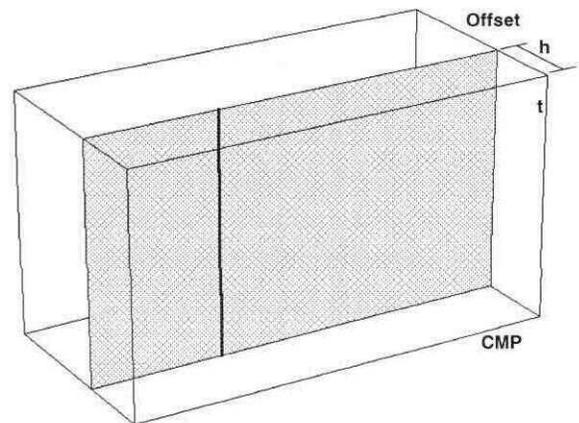


FIG. 3. Schematic of a constant offset section displayed in a prestack volume.

The number of offset sections is often reduced by gathering traces over a range of offsets to form limited offset sections. An additional advantage of creating limited offset sections is to increase the number of live traces, or equivalently, decrease the spacing between live traces. Referring again to Figure 1, collecting over a range of

four offsets creates a limited offset panel with live trace spacing equal to the CMP spacing. Limited offset sections are usually assumed to have a constant offset, and are often referred to as constant offset sections.

## CONVENTIONAL MIGRATION

There are a number of conventional methods of prestack and poststack migration. One common method is Kirchhoff migration, which was described in the previous section. To summarize, hyperbolic shapes are computed, the input traces are scanned for energy along the hyperbolas, and a weighted sum of the energy is placed on the output migrated trace. Kirchhoff migration can be accomplished a number of ways depending on the arrangement of the input data.

### Post-stack Migration

Post-stack migration is performed on stacked sections. The post stack operators create semi-circles from points on a zero-offset CMP trace, as illustrated in Figure 4. Arbitrary input and output traces are indicated by bold vertical lines in the shaded CMP gather and in the output zero-offset section, respectively. Note that energy from the input trace follows the path of NMO, stacking, then migration. All other input traces within the migration aperture may be mapped in a similar manner to contribute energy to the output migrated trace.

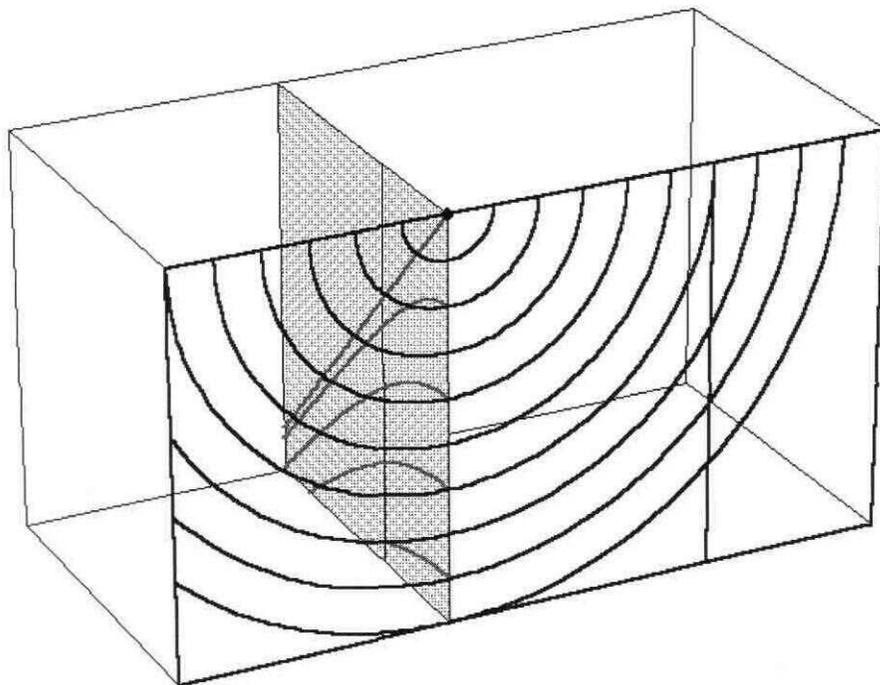


FIG. 4. The path of post-stack migration in the prestack volume.

### Prestack Source (Shot) Record Migration

Prestack data may be gathered into source records and each record migrated separately. The migration of one trace in a source record is illustrated in Figure 5 by the series of prestack migration ellipse, where the source and receiver positions are the foci of the ellipse. Although an output trace after migration appears to be located at a non-zero offset in a given CMP gather, the migrated trace should be considered as zero offset. The collection of all migrated traces in the CMP gather can be stacked directly to complete the migration process. Energy moves directly from the input trace to the output CMP position, as illustrated in Figure 5.

There are many algorithms to migrate source records. Typical methods use Kirchhoff directly, or combinations of downward continuation and Kirchhoff.

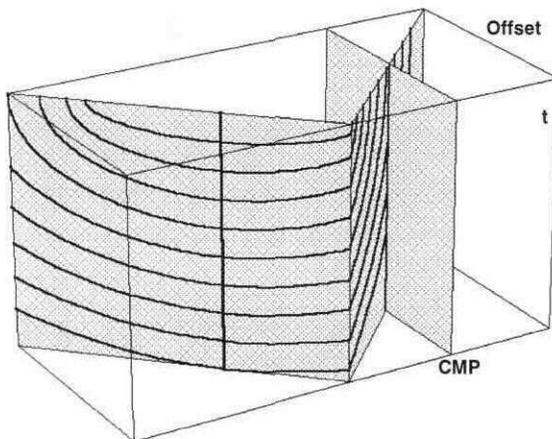


FIG. 5. Prestack source record migration illustrated in the prestack volume.

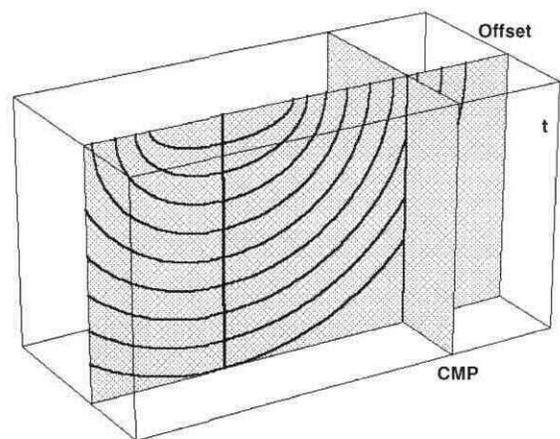


FIG. 6. Prestack constant offset section migration illustrated in the prestack volume.

### Prestack Constant Offset Migration

Migration of a constant offset section is illustrated in the 2-D prestack volume of Figure 6. The prestack migration of an input trace is shown by the series of prestack migration ellipse with the source and receiver at the foci. As before, energy in the input trace moves directly to the CMP position. The migration of data in the source record (Figure 5) may be compared with migrated data in the constant offset section. Both methods produce identical results when the migrated traces are projected (stacked) to the zero offset section.

There are a number of ways to migrate constant offset sections, but the most common algorithms seem to be based on the Kirchhoff method.

## Full Prestack Kirchhoff Migration

Full prestack Kirchhoff migration creates one output migrated trace by summing energy from all input traces within the migration aperture. Equivalently, each sample from a given input trace could be moved in time and position to all possible output traces in the migration aperture. As mentioned previously, this is an expensive procedure and as a result has limited application.

## CSP GATHER

The surface position of a vertical array (or trace) of scatter points is referred to as the *common scatter point* (CSP) location. Input traces with energy from a common scatter point location can be collected into CSP gathers. The CSP gathers are similar in function to the CMP gathers of conventional processing. Each CSP gather contains all the traces in the migration aperture. In the new migration method *CSP gathers replace the CMP gathers of conventional processing*.

A prestack data volume is shown in Figure 7. A CSP gather is in the same plane as the CMP gathers described previously, but contains information from all traces within the migration aperture. The movement of zero-offset CMP traces into an offset position in the CSP gather is illustrated by the curved lines on the upper face of the data volume. For a zero-offset CMP trace, the CSP offset position will be the distance from the CMP location to the CSP location. The offset in the CSP gather is referred to as the *equivalent offset*.

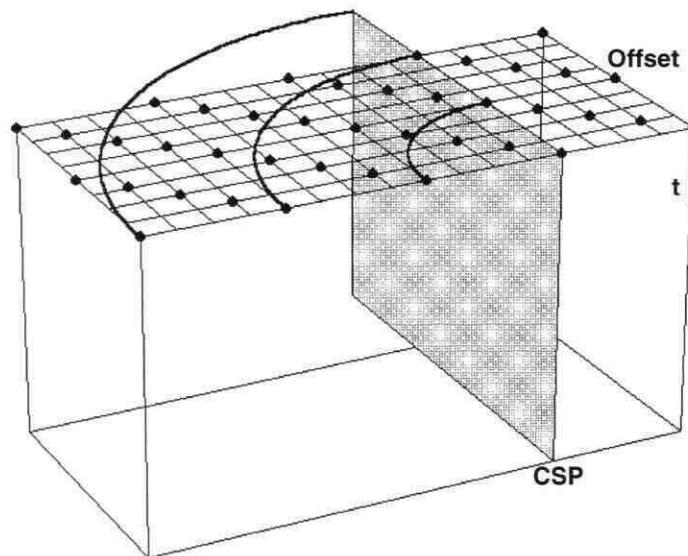


FIG. 7. Formation of the CSP gather is illustrated in the prestack volume.

Observe that each offset in a CSP gather can contain energy from more than one trace. Conversely, energy from a particular trace will appear in every CSP gather. A CMP gather at the CSP location in Figure 7 contains only three traces with offsets that are limited to the maximum source-receiver offset. In contrast, the CSP gather will include many traces at all equivalent offsets, and include equivalent offsets that are much larger than the maximum source-receiver offset.

The range of equivalent offsets may be continuous, and requires binning of the offsets that form the CSP gather. Many input traces fall in the same bin. Since all these traces have the same equivalent offset, all may be migrated as one trace. The time saving for the arithmetic computation will be proportional to the fold in the bin. For 2-D data the fold may be in the ten's of traces, and much higher for 3-D data.

An important feature of the CSP gather is the time position of all the samples in the trace remains the same.

The CSP gather has many advantages over the CMP gather, including more accurate prestack-migration velocity analysis, fully-coupled surface-consistent statics, and a simplified migration process. Once the CSP gather has been formed, only NMO and stacking are required to complete the migration process.

## THE EQUIVALENT OFFSET

The definition of equivalent offset is straightforward for zero-offset CMP traces: it is just the distance from the CMP location to the CSP location. Deriving an equivalent offset for non-zero offset CMP traces is more challenging.

A CMP collection of input traces is illustrated in Figure 8. The *one-way* normal moveout time  $T$  is computed from the half offset  $h$ , the zero offset *one way* time  $T_0$ , and the RMS velocity  $V_{rms}$  defined at  $T_0$ , i.e.

$$T^2 = T_0^2 + \frac{h^2}{V_{rms}^2(T_0)}.$$

It is informative to imagine the source  $s_1$  in Figure 8 to be located at the receiver  $r_1$ , and the dip of the plane at  $T_0$  modified to be perpendicular to the ray path. The computation of the ray path travel time would still be the same, but the *offset* used would be the distance from the source-receiver position to the reflector position. This *offset* defines the equivalent offset when the reflector point on the plane becomes the scatter point.

Even though the velocity layering may be complex, the RMS velocity  $V_{rms}$ , allows us to assume linear ray paths from the reflector to the surface. A pseudo depth  $Z_0$  that is equivalent to the one-way time  $T_0$  may also be defined from

$$Z_0 = T_0 \times V_{rms}(T_0).$$

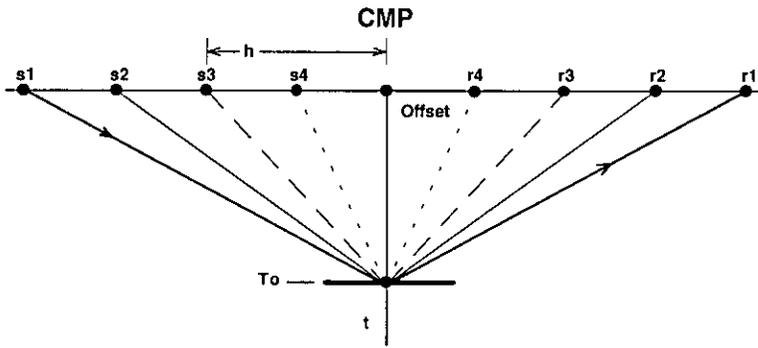


FIG. 8. Ray paths and reflector that make up the CMP gather.

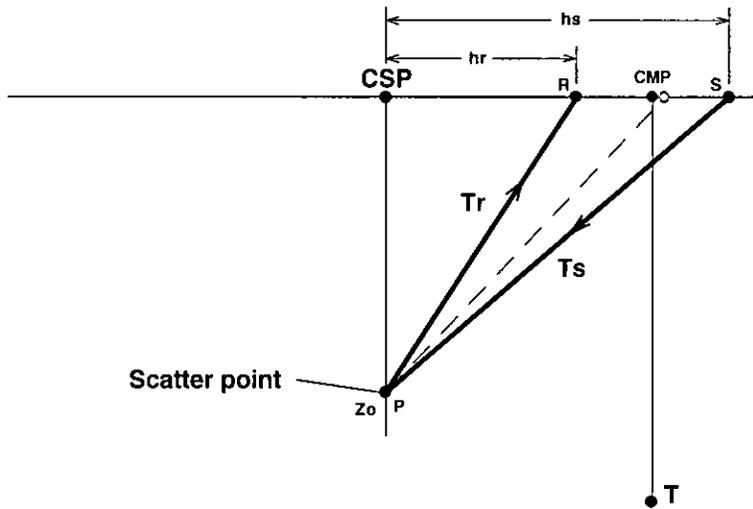


FIG. 9. Ray paths and travel times for a scatter point.

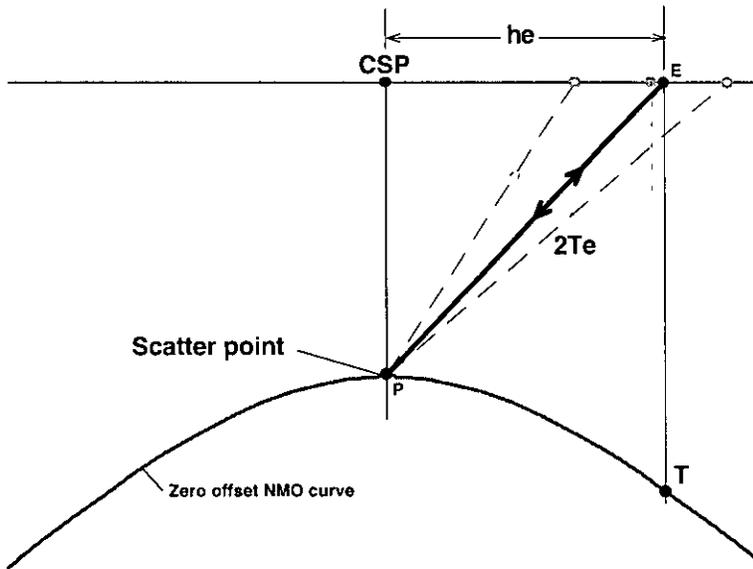


FIG. 10. Ray paths that define the equivalent offset.

The total (two-way) travel time from source to receiver will be defined by  $T_t$  as

$$T_t = T_s + T_r = 2T$$

Ray paths in Figure 9 illustrate the CMP trace that contains energy from a scatter point. The distance from the source to the CSP is  $h_s$ , the distance from the receiver to the CSP  $h_r$ , and the pseudo depth of the scatter point  $Z_0$ . The travel times from the source to the scatter point is  $T_s$  and the travel time from the scatter point to the receiver is  $T_r$ . The total travel time  $T_t$  is  $T_s + T_r$ . Conventional prestack migration moves energy on the input trace at  $T_t$  and adds it to the CSP migrated trace at  $Z_0$ . The value of  $T_t$  is found from,

$$T_t = \left( T_0^2 + \frac{h_s^2}{V_{rms}^2(T_0)} \right)^{\frac{1}{2}} + \left( T_0^2 + \frac{h_r^2}{V_{rms}^2(T_0)} \right)^{\frac{1}{2}}$$

The problem is to define an equivalent offset for the trace and scatter point defined in Figure 9.

The equivalent offset is defined by co-locating the source and receiver at the equivalent offset, while maintaining the same travel time  $T_t$ . Figure 10 illustrates the equivalent offset position with one-way travel times for the source and receiver to the scatter point being  $T_e$ . The time constraints, based on prestack time migration, are

$$2T_e = T_s + T_r,$$

or, when expanded, becomes

$$2 \left( T_0^2 + \frac{h_e^2}{V_{rms}^2} \right)^{\frac{1}{2}} = \left( T_0^2 + \frac{h_s^2}{V_{rms}^2} \right)^{\frac{1}{2}} + \left( T_0^2 + \frac{h_r^2}{V_{rms}^2} \right)^{\frac{1}{2}}.$$

Solving for  $h_e$  we obtain

$$h_e = V_{rms} \left\{ 0.25 \left[ \left( T_0^2 + \frac{h_s^2}{V_{rms}^2} \right)^{\frac{1}{2}} + \left( T_0^2 + \frac{h_r^2}{V_{rms}^2} \right)^{\frac{1}{2}} \right]^2 - T_0^2 \right\}^{\frac{1}{2}},$$

or

$$h_e = \left\{ 0.25 \left[ (Z_0^2 + h_s^2)^{\frac{1}{2}} + (Z_0^2 + h_r^2)^{\frac{1}{2}} \right]^2 - Z_0^2 \right\}^{\frac{1}{2}}.$$

When the distance from the CMP to the CSP  $h_{cmp}$  is used, we get,

$$h_e = \left[ h_{cmp}^2 + h^2 - \left( \frac{2h_{cmp}h}{T_t V_{rms}} \right)^2 \right]^{\frac{1}{2}}.$$

The equivalent offset  $h_e$  is time or depth varying, and is also a function of velocity. This means the an input trace may have its sample spread over a number of offset bins. Note however, that the time samples still remain at the same time.

An example of the time varying equivalent offset is shown in Figure 11. Note the slight increase in offset with increasing time. Only the lower portion of the CMP trace is used for the given CSP trace. The first useful sample on the CMP trace corresponds to the travelttime of a ray path from the source and receiver to a scatter point located at the surface. i.e.

$$T_{\alpha} = \frac{2h_{cmp}}{V_{rms}}$$

The offset of this first point  $h_{e\alpha}$  is defined by

$$h_{e\alpha} = h_{cmp}$$

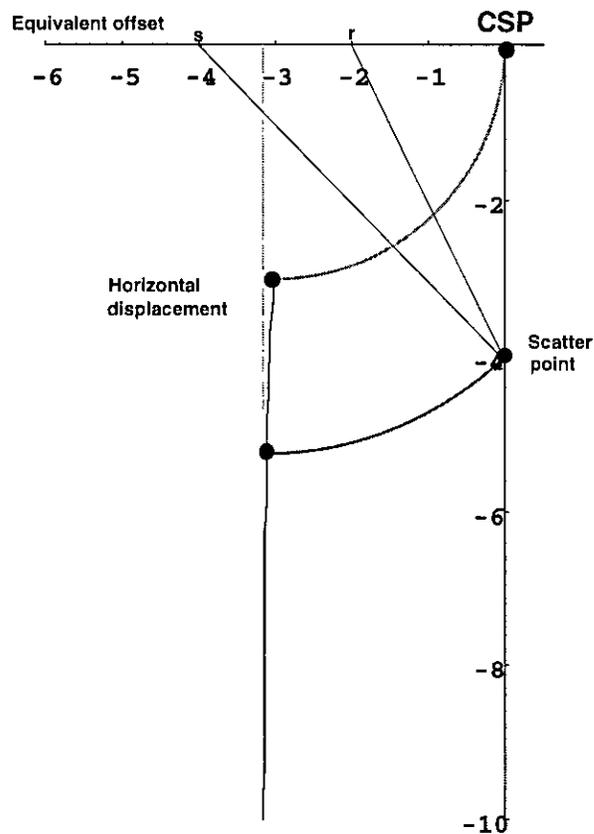


FIG. 11. Example of the time varying equivalent offset position.

As the time on the input trace increases, the equivalent offset tends to an asymptote  $h_{e\omega}$  given by,

$$h_{e\omega} = (h_{cmp}^2 + h^2)^{\frac{1}{2}}$$

When half offset  $h$  is small relative to  $h_{cmp}$ , the time-varying portion of the equivalent offset  $h_e$  becomes insignificant. These traces may be moved to a fixed offset bin in the CSP gather. When the half offset  $h$  is large relative to  $h_{cmp}$ , samples in the input trace may be spread over a number of offset bins in the CSP gather. The spread of samples is illustrated in Figure 11.

Note that the first useful time sample in Figure 11 migrates to a ninety degree dip. If the dip limit of the migration is sufficiently restricted, the time-varying portion of the equivalent offset  $h_e$  will not be included, and a constant value of  $h_{e\omega}$  can be used for the remaining samples in the trace.

The offsets used in the CSP gather are based on the equivalent offset of the source and receiver from the CSP. The ray paths from the equivalent offset lie on one side of the scatter point. Note the similarity to the CMP source and receiver rays which are identical, but lie on either side of the reflection point. The CMP processing requires NMO and stacking to produce the section. In a similar manner, NMO and stacking may be applied to the CSP gather to produce a section that is virtually identical to one produced by conventional prestack migration.

Since the new method is based on Kirchhoff prestack time migration, an appropriate amplitude scaling is required for each offset, along with the root differential filter for 2-D data, and the differential filter for 3-D data.

## PROPERTIES OF THE CSP GATHER

An important property of the CSP gather is that the equivalent offset approximation  $h_{e\omega}$  is independent of time and velocity. This independence from time and velocity provides stability to the CSP gather. The CSP gathers may be formed with an arbitrary velocity, and the gather used to define a more accurate velocity.

Conventional velocity analysis requires a super gather of CMP traces to fill all the offsets in the gather. This super gather will attenuate the energy of dipping events. A single CSP gather contains high fold information at all possible offsets, eliminating the need for super gathers, and preserves the energy of dipping events for velocity analysis.

The fold of offset bins in the CSP gather is usually quite high. Fold may range from twenty to thirty for 2-D data to hundreds for 3-D data. NMO will be applied once to all the summed traces in a CSP offset bin. Conventional prestack migration requires a complex NMO-type operation to be applied many times to each input trace, once for each output migrated trace. In comparison, the stacking of traces in CSP offset bins before NMO results in a considerable saving in computation time over the conventional method. Once the CSP gathers are formed, a number of analysis may be performed quite economically which would otherwise be too expensive with the conventional method.

The maximum offset of the CSP gather is defined as the maximum offset in the migration aperture. These offsets are much larger than the half source-receiver offset  $h$ . Consequently, the velocities on semblance plots focus to smaller points than on conventional semblance plots formed from a CMP gather. It is emphasized that velocities derived from the CSP gather are RMS velocities, and that these velocities take data from an input trace directly to the prestack migrated position.

A number of features of the CSP gather suggest new possibilities for evaluating coherent and incoherent noise such as multiples. Standard algorithms designed for use with CMP gathers may be used directly on CSP gathers. It is apparent that, in migrated data, the attenuation of multiples is more a function of the migration aperture than the source-receiver offset. This feature of the CSP gather may have substantial implications on future field designs.

Statics analysis is an area in which the CSP gather may be of significant benefit. When conventional 2-D lines are recorded with source points at four-station intervals, four independent (de coupled) surface consistent solutions are obtained. Each receiver only contributes to every fourth CMP, requiring filtering techniques to combine the solutions. The static solutions are obtained on NMO'ed data by correlating each input traces with some form of a smoothed brute section. The CSP gather, in contrast to a CMP gather, contains many contributions from all sources and all receivers within the prestack migration aperture. This greatly increases the number of correlations and ensures the coupling of all sources and receivers with all CSP's. The success may possibly depend on removing coherent noise to create a suitable CSP gather model for correlating input traces.

Many traces in a CSP gather are positioned with offsets close to the asymptote  $h_{ew}$  and are therefore independent of time and velocity. When CSP gathers are produced independent of time and velocity, the potential applications go beyond the limits of prestack time migration. Time migrations with complex moveout equations are possible, as well as approximate depth migrations.

This paper has used prestack migration to define CSP gathers based on equivalent offsets. Other criteria or restrictions for offsets may be defined. For example, azimuth restrictions may be applied to 2-D or 3-D data. Other applications allow the inclusion of converted wave velocity analysis and prestack migration.

### **SYNTHETIC MODEL EXAMPLE**

A synthetic model was created to evaluate the performance of the migration. The model consists of one scatter point and two linear reflectors as illustrated in Figure 12a. One linear reflector is short and horizontal, while the other dips steeply with one end meeting the horizontal reflector. The dipping reflector has a gap close to its middle. Simple source records were created from the model by estimating the travel times from each reflector and placing a wavelet at the travel times. No attempt was made to model amplitude variations. An example of one source record with the source point located directly above the scatter point is shown in Figure 12b.

The model was used to create 101 source records which were collected into CSP gathers spanning the reflectors. One of these CSP gathers is shown in Figure 12c at a location directly above the scatter point. The result of normal moveout applied to this CSP gather is shown in Figure 12d. Note the horizontal alignment of energy for the scatter point and the left end of the horizontal reflector. Stacking the NMO'ed CSP completes the prestack migration process. The stacked trace is the central trace in Figure 12e; the trace containing the scatter point. The result of equivalent offset migration compares favorably with conventional methods.

Figure 12f shows a part of a CSP gather created from one source record located directly above the scatter point. The location of the CSP gather is also above

Figure 12f shows a part of a CSP gather created from one source record located directly above the scatter point. The location of the CSP gather is also above the scatter point. A small offset interval helps to illustrate the time-varying equivalent offset variations that may occur for input traces. The variations are most apparent at shallow times on the traces on the right side of the figure. It is again emphasized that the time position of the samples on the input trace remains the same in the CSP gather.

## **REAL DATA EXAMPLES**

A real data example, acquired in the foothill west of Calgary, has been included. This line was conventionally processed using DMO and post stack migration to obtain the best section. A CMP super gather and semblance plot are shown in Figure 13a. A CSP gather and the corresponding semblance plot are shown in Figure 13b. The final DMO and migrated section is shown in Figure 13c, and the corresponding equivalent offset migration in Figure 13d.

Note the improvement in the semblance of the CSP gather over the CMP super gather. The peaks of the velocities are more accurately defined. This enabled rapid picking of the velocities which significantly reduced the processing time. It should be noted that the statics solution derived from conventional processing of CMP gathers was used to create the CSP gathers. Test of choosing the CSP velocities that are independent of the previous processing are under way.

## **MARINE DATA EXAMPLE**

An example of a super CMP gather and a CSP gather from a marine line is shown in Figure 14. The super CMP spanned seven CMP's to achieve full trace coverage. The CSP gather is displayed as a two sided plot and has an average fold of fifteen. The CSP gather was formed using  $h_{e\omega}$  at a constant equivalent offset for each input trace. Consequently, the CSP gather is independent of time and velocity, and thus not limited by the constraints of prestack time migration.

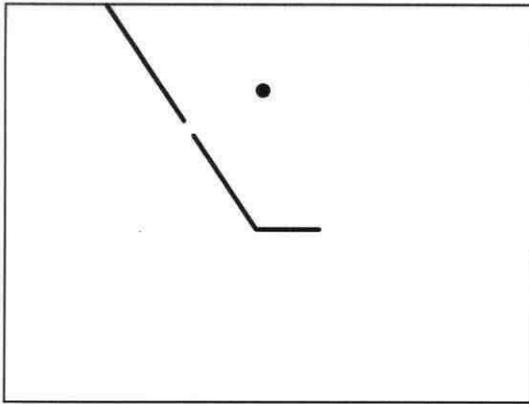
The CSP gather shows many coherent reflections that extend to far offsets, along with other reflections that become visible at offsets beyond the range of the CMP gather. High order NMO equations may be required to obtain optimum migration of this data.

## **CONCLUSIONS**

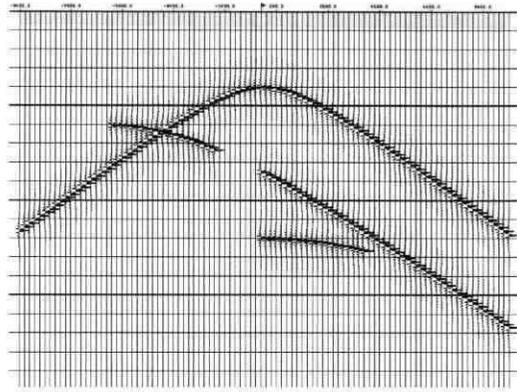
A robust method of prestack migration has been developed that is simpler and potentially much faster than conventional methods. It is based on Kirchhoff prestack time migration, but modifies the migration process to gathering, NMO, and stacking. The new method correctly maps energy from prestack traces to equivalent offsets in common scatter point (CSP) gathers. Conventional velocity analysis tools may be used on the CSP gathers to accurately determine RMS velocities. Other advantages may include coupled surface consistent statics, improved field design, converted wave processing, multiple evaluation and simplified processing.

## **ACKNOWLEDGMENTS**

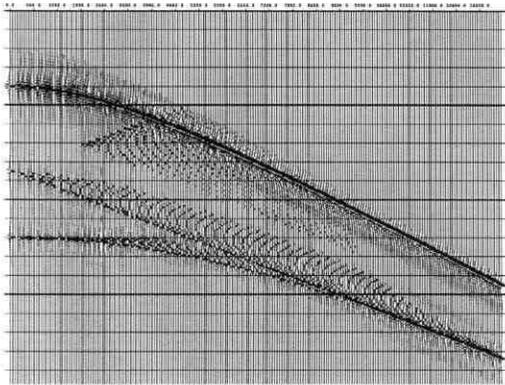
We wish to thank the sponsors of the CREWES Project for their support. The technical support from Henry Bland was greatly appreciated.



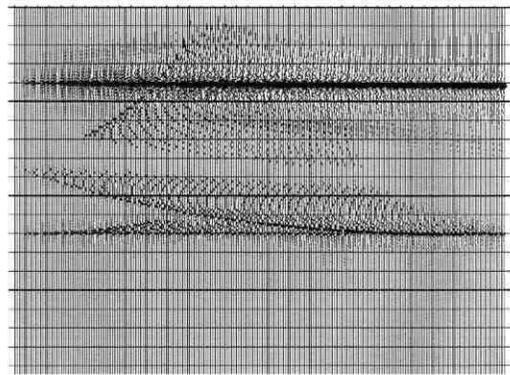
(a) Hockey stick model.



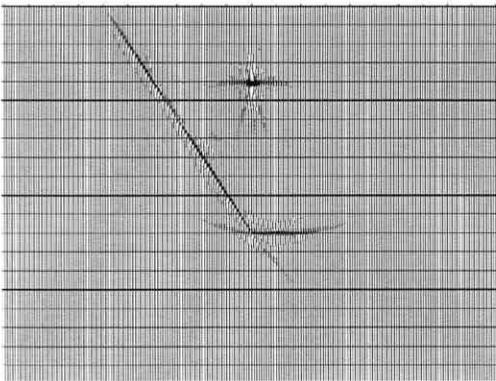
(b) Source record.



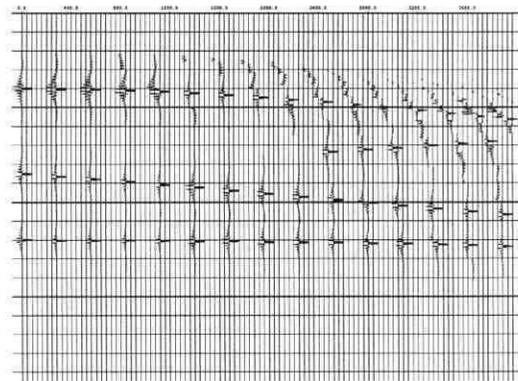
(c) CSP gather.



(d) CSP gather with NMO.

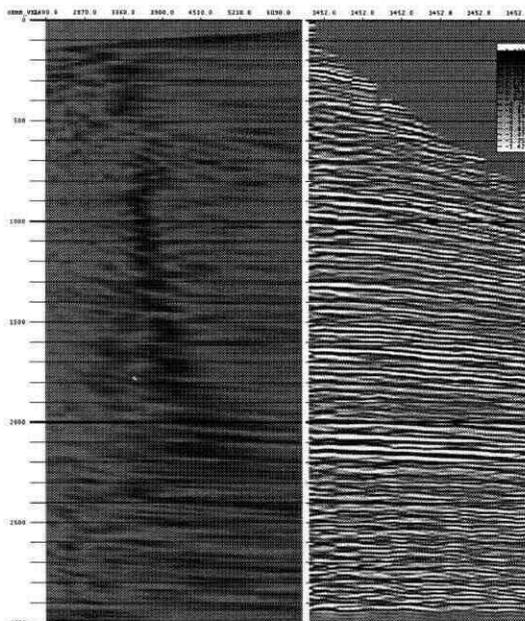


(e) Equivalent offset migration.

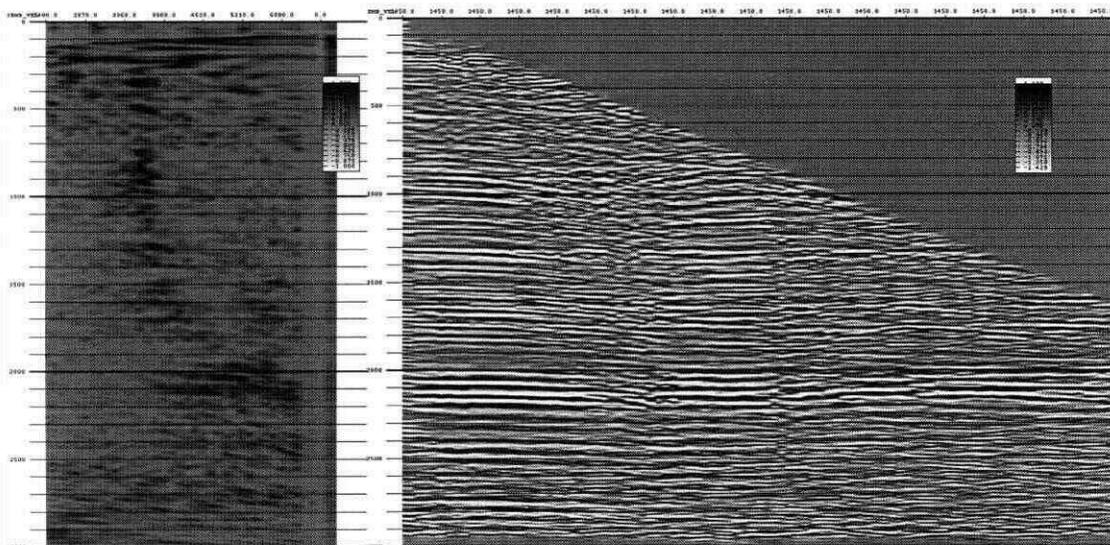


(f) Close up CSP gather.

FIG. 12 Synthetic model illustrating CSP gathers and equivalent offset migration.

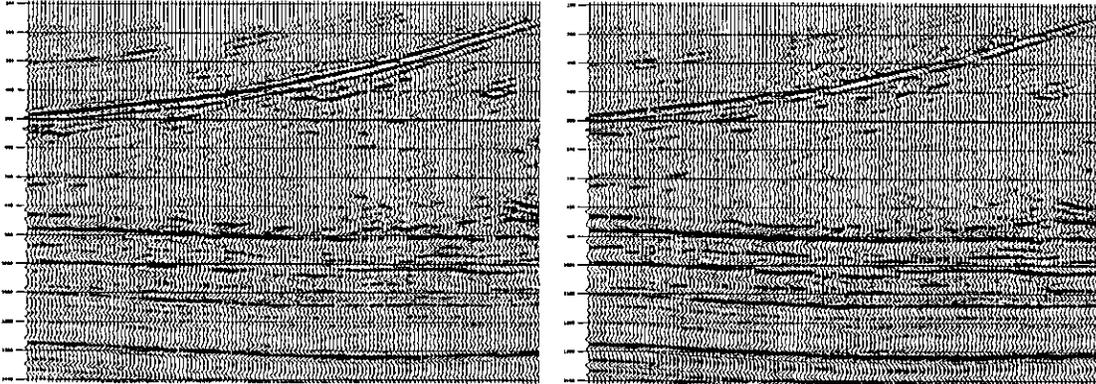


(a) CMP gather and semblance.



(b) CSP gather and semblance.

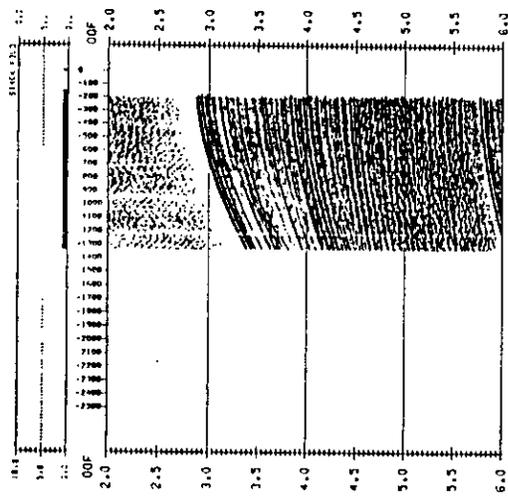
FIG. 13a, b Land example of CMP and CSP gathers, with semblance plots.



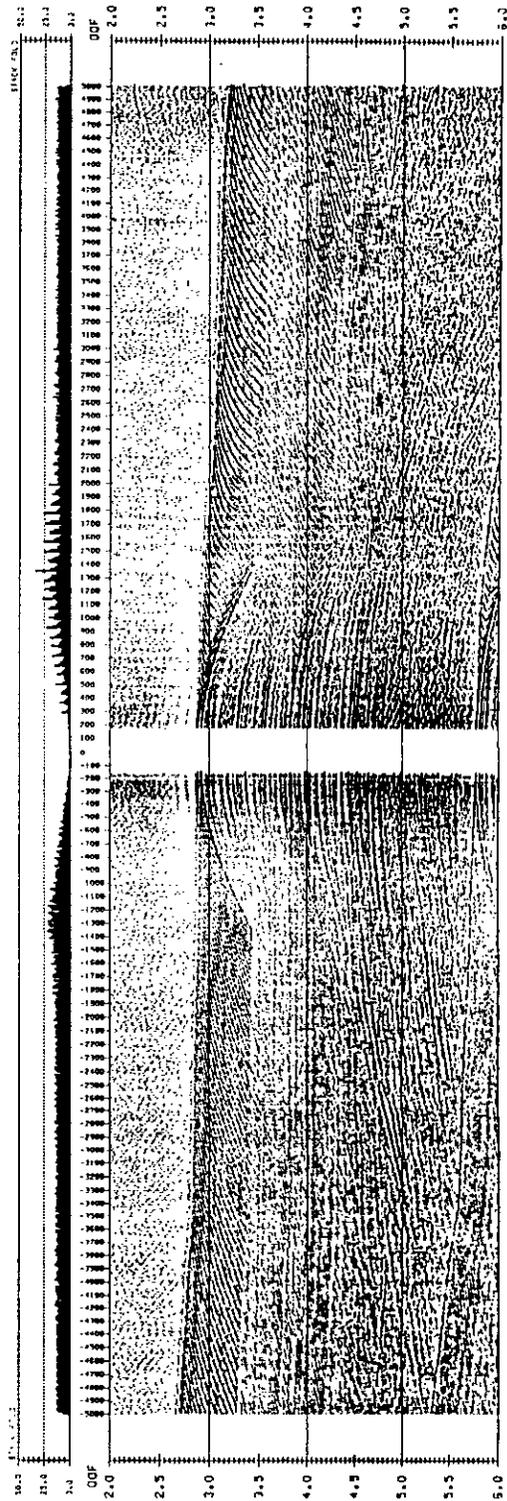
(c) DMO and post stack migration

(d) Equivalent offset migration.

FIG. 13 c, d Land examples continued.



An example of CMP gather.



An equivalent offset CRP gather.

FIG. 14. Comparison of CMP and CSP gathers, taken from a marine line.