Converted-wave prestack migration and velocity analysis by equivalent offsets and CCP gathers

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ABSTRACT

A theoretical method is presented to enable prestack migration and velocity analysis of converted wave data. It is based on a new method of P-P prestack migration using equivalents offsets and common scatter point (CSP) gathers. Equivalent offsets for converted wave data are defined for all samples that prestack migrate to a common conversion point (CCP). Samples from all the input traces are added to the appropriate offsets in the CCP gathers. Velocity analysis may be performed on the CCP gather using conventional velocity analysis tools such as semblance analysis. Migration is completed by NMO and stacking of the CCP gather.

INTRODUCTION

It is very difficult to estimate the velocities in P-S processing. The data is usually quite noisy and receiver statics may be quite large. Even when a reasonable static solution is found, the steps to estimate the velocities may be quite complex. A new method is presented that will allow an accurate estimate of the prestack migration velocities, and simplify the process of prestack migration.

Prestack migration of P-P data by equivalent offsets and common scatter point (CSP) gathers has been shown to be an alternative method to conventional prestack migration. This method forms output CSP gathers by assigning equivalent offsets for each sample in the input traces relative to the migrated output CSP trace locations. In the new P-P process, the travel time for rays from a given source-receiver pair shown in Figure 1 are equated with the travel times of a new co-located source-receiver pair shown in Figure 2. The position of the new co-located source and receiver defines the equivalent offset distance h_e from the CSP. Each input trace will contribute energy to all CSP gathers within the prestack migration aperture.

This method is ideally suited for converted wave processing. It is accomplished by replacing the P wave velocity of the receiver ray path with the S wave velocity. The equivalent offset for converted waves can be computed, and the CCP gather formed. The gather will have high fold and should enable the estimation of velocities with conventional methods such as semblance plots. Comparisons of the P wave velocity V_p with the semblance velocity should yield the shear wave velocity V_s or the velocity ratio γ, given by

$$\gamma = \frac{V_p}{V_s} . \tag{1}$$

The ratio γ may be time varying, though it is often assumed to be constant at values in the neighborhood of two.

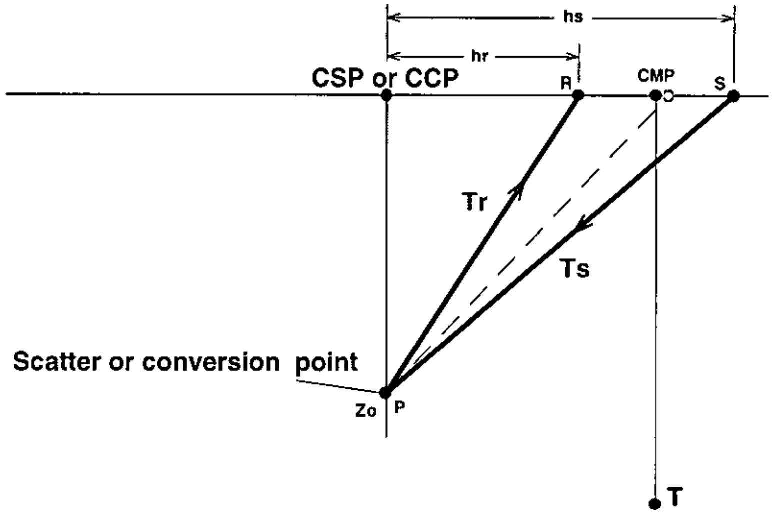

FIG. 1. The ray paths and travel times for a scatter or conversion point.

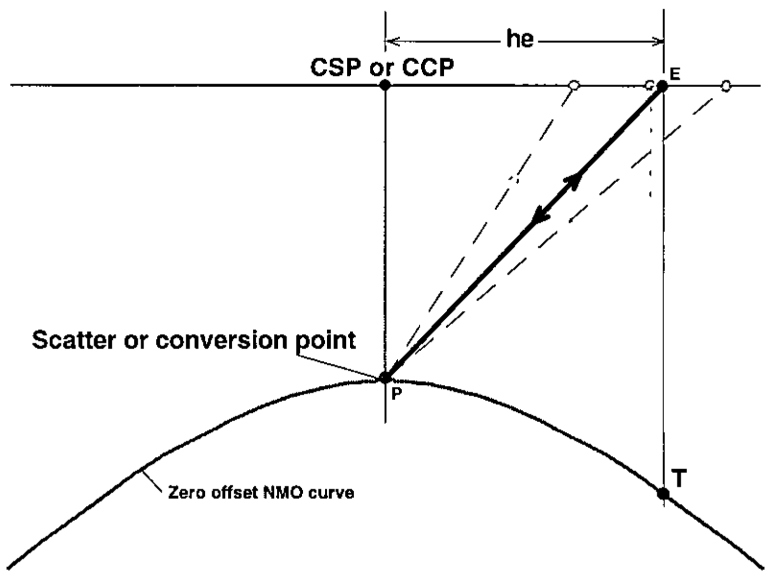

FIG. 2. The position of the equivalent offset ray paths.

THE EQUIVALENT OFFSET FOR CONVERTED WAVES

The equivalent offset for a converted wave is computed by equating the ray path travel times from the source T_s, and receiver T_r, with the travel times from a co-located source and receiver T_{es} and T_{er}, i.e.

$$T_{es} + T_{er} = T_s + T_r. \tag{2}$$

Using the concepts of prestack time migration and RMS velocities, the travel times may be replaced with

$$\left({}_pT_0^2+\frac{h_e^2}{{}_pV_{rms}^2}\right)^{1/2}+\left({}_sT_0^2+\frac{h_e^2}{{}_sV_{rms}^2}\right)^{1/2}=\left({}_pT_0^2+\frac{h_s^2}{{}_pV_{rms}^2}\right)^{1/2}+\left({}_sT_0^2+\frac{h_r^2}{{}_sV_{rms}^2}\right)^{1/2} \tag{3}$$

where ${}_pV_{rms}$ and ${}_sV_{rms}$ are the respective RMS velocities for P and S waves. The respective zero equivalent offset times are ${}_pT_0$ and ${}_sT_0$,. The distances h_e, h_s, and h_r are shown on Figures 1 and 2. Replacing the shear wave velocity with γ and the P wave velocity, and the T_0's with Z_0, we get

$$\frac{\left(Z_0^2+h_e^2\right)^{1/2}}{{}_pV_{rms}}+\frac{\gamma\left(Z_0^2+h_e^2\right)^{1/2}}{{}_pV_{rms}}=\frac{\left(Z_0^2+h_s^2\right)^{1/2}}{{}_pV_{rms}}+\frac{\gamma\left(Z_0^2+h_r^2\right)^{1/2}}{{}_pV_{rms}}\ . \tag{4}$$

When eliminating the velocity

$$\left(Z_0^2+h_e^2\right)^{1/2}+\gamma\left(Z_0^2+h_e^2\right)^{1/2}=\left(Z_0^2+h_s^2\right)^{1/2}+\gamma\left(Z_0^2+h_r^2\right)^{1/2} \tag{5}$$

or

$$\left(Z_0^2+h_e^2\right)^{1/2}=\frac{1}{(1+\gamma)}\left\{\left(Z_0^2+h_s^2\right)^{1/2}+\gamma\left(Z_0^2+h_r^2\right)^{1/2}\right\}. \tag{6}$$

Solving for the equivalent offset h_e gives

$$\boxed{h_e=\left[\frac{1}{(1+\gamma)^2}\left\{\left(Z_0^2+h_s^2\right)^{1/2}+\gamma\left(Z_0^2+h_r^2\right)^{1/2}\right\}^2-Z_0^2\right]^{1/2}}. \tag{7}$$

The asymptotes of h_e at the first usable sample is $h_{e\alpha}$. At larger times T tends to $h_{e\omega}$. The first asymptote $h_{e\alpha}$ may be found when Z_0 goes to zero, i.e.

$$h_{e\alpha}=\frac{\left(h_s+\gamma\, h_r\right)}{(1+\gamma)}. \tag{8}$$

The second asymptote $h_{e\omega}$, may be found by dividing equation (5) by Z_0, and approximating the square root by the first two terms in the Taylor series expansion with

$$(1+\gamma)\left(1+\frac{h_{e\omega}^2}{2Z_0^2}\right)=\left(1+\frac{h_s^2}{2Z_0^2}\right)+\gamma\left(1+\frac{h_r^2}{2Z_0^2}\right) \tag{9}$$

or

$$(1+\gamma)\frac{h_{e\omega}^2}{2Z_0^2} = \frac{h_s^2}{2Z_0^2} + \gamma\frac{h_r^2}{2Z_0^2} \tag{10}$$

which reduces to

$$h_{e\omega}^2 = \frac{h_s^2 + \gamma h_r^2}{1+\gamma}. \tag{11}$$

These two asymptotes function similarly to conventional P-P equivalent offset asymptotes, which define the range of offsets for the samples in the input trace.

PRACTICAL COMPUTATION OF THE EQUIVALENT OFFSET

The form of the equivalent offset defined above is not practical due to the inclusion of the variable Z_0. Practical applications require defining the equivalent offset with the actual times T from the input samples. A direct solution for h_e is complex, however a two step process appears to be practical.

The first step computes Z_0^2 from the total travel time T, the RMS velocity V_p, the input geometry from figure 1, and

$$TV_p = \left(Z_0^2 + h_s^2\right)^{1/2} + \gamma\left(Z_0^2 + h_r^2\right)^{1/2}. \tag{12}$$

Solving for Z_0 is still quite complex. The process defines intermediate values C_1 and C_2 as

$$C_1 = \frac{T^2V_p^2 + h_s^2 - \gamma^2 h_r^2}{1-\gamma^2}, \tag{13}$$

and

$$C_2 = \frac{2TV_p}{1-\gamma^2}. \tag{14}$$

The value of Z_0 may then be found from

$$Z_0^2 = \frac{C_2^2 - 2C_1 \pm C_2\left(C_2^2 + 4h_s^2 - 4C_1\right)^{1/2}}{2}, \tag{15}$$

by ensuring the value of Z_0^2 is real and positive.

Once the depth Z_0^2 is known, the second step computes the equivalent offset from either equation (7), or from a solution to the following equation

$$TV_p = \left(Z_0^2 + h_e^2\right)^{1/2} + \gamma\left(Z_0^2 + h_e^2\right)^{1/2}, \tag{16}$$

which gives

$$h_e = \left(\frac{T^2 V_p^2}{(1+\gamma)^2} - Z_0^2 \right)^{1/2}. \tag{17}$$

The above two steps may be used to compute the offset for each useful time sample of the input traces. This is an expensive and an unnecessary procedure as a number of samples will have offsets that fall in the same offset bin. An improved procedure starts by computing the first offset with the above procedure, but then computes the times T_n when the following samples will be located in the next offset bins. Simple loops will then move the input samples into the appropriate offset bins. The procedure requires computing Z_{0n}^2 from h_{en}, V_p, γ, h_s and h_r. Using intermediate values b_n from

$$b_n = \frac{(1+\gamma)^2 h_{en}^2 - h_s^2 - \gamma h_r^2}{2\gamma}, \tag{18}$$

we get

$$Z_{0n}^2 = \frac{h_s^2 h_r^2 - b_n^2}{2b_n - h_s^2 - h_r^2}. \tag{19}$$

The values of T_n are then computed from

$$T_n = \frac{(Z_{0n}^2 + h_s^2)^{1/2} + \gamma (Z_{0n}^2 + h_r^2)^{1/2}}{V_p}. \tag{20}$$

Samples from the input trace may now be moved in groups to the appropriate offset bins. The samples in a given input trace will be used for many CCP gathers. The actual samples used will depend on the distance from the input CMP location to the output CCP location. It is emphasized that **no time movement of the samples is required.**

Once the CCP gather has been formed, it can be modified with a number of conventional processes to reduce noise, estimate velocities, and complete the prestack migration process.

VELOCITY ANALYSIS

The data in the CCP gather assumes the traces come from co-located sources and receivers. If the V_p-V_s velocity ratio γ is assumed to be reasonably constant, the ray path from the source to the conversion point should be the same as the path from the conversion point to the receiver. Prestack time migration assumes these paths to be equal and linear when using RMS velocities. The travel two-way time T is found from

$$T = \frac{\left(Z_0^2 + h_e^2\right)^{1/2}}{V_p} + \frac{\left(Z_0^2 + h_e^2\right)^{1/2}}{V_s} = \frac{\left(Z_0^2 + h_e^2\right)^{1/2}}{V_p} + \frac{\gamma\left(Z_0^2 + h_e^2\right)^{1/2}}{V_p} \tag{21}$$

or

$$T = \frac{(1+\gamma)\left(Z_0^2 + h_e^2\right)^{1/2}}{V_p}, \tag{22}$$

which expands to

$$T = \left\{ \frac{(1+\gamma)^2 Z_0^2}{V_p^2} + \frac{(1+\gamma)^2 h_e^2}{V_p^2} \right\}^{1/2}. \tag{23}$$

This equation may be expressed in conventional NMO form as

$$T = \left\{ T_0^2 + \frac{h_e^2}{V_{sem}^2} \right\}^{1/2} \tag{24}$$

with zero equivalent offset two-way time T_0 and the velocity from a semblance analysis

$$V_{sem} = \frac{V_p}{1+\gamma}. \tag{25}$$

Equation (24) shows the conversion point reflections in the equivalent offset CCP gather to be hyperbolic with velocity V_{sem}.

The velocity ratio γ, or the shear wave velocity V_s may be computed from V_{sem} and the P wave velocity V_p (derived from P-P processing). The value of γ is therefore found from (25)

$$\gamma = \frac{V_p}{V_{sem}} - 1, \tag{26}$$

and the S wave velocity from (1) and (25)

$$V_s = \frac{V_p V_{sem}}{V_p - V_{sem}}. \tag{27}$$

PRESTACK MIGRATION

Once the equivalent offset CCP gathers have been formed, only three simple steps remain to complete the prestack migration process. First, the data in the CCP gathers require scaling and filtering to be compatible with the Kirchhoff migration. The remaining two steps are NMO, then stacking to complete the prestack migration process. This paragraph may appear too short, but the prestack migration is that simple.

CONCLUSIONS

A theoretical method to prestack migrate converted wave data has been presented. A major part of the process enables the velocities to be accurately determined. The method is based on the principles of Kirchhoff time migration. An equivalent offset was defined to allowed all energy within the migration aperture to be collected and placed into CCP gathers. The gather contains all the energy that would normally be prestack migrated into the trace at the CCP location. Standard processing of the CCP gather with NMO and stacking, completes the prestack migration process.

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