Numerical simulation of wave propagation in anisotropic media

Kangan Fang and R. James Brown

ABSTRACT

Numerical simulation or forward modelling of wave propagation is a cheap and effective way to produce test data and to examine the mechanism of wave propagation. Because of the complexities of anisotropy, forward modelling is a practical means to aid the anisotropy analysis. Although a lot of work has been done on the forward modelling in anisotropic media, it rarely reaches the level of practical use. In this paper, the pseudospectral method is proposed to do the forward modelling in anisotropic media.

INTRODUCTION

It is now commonly accepted that most upper-crustal rocks are anisotropic to some extent (Crampin, 1981). Anisotropy may be caused by fine layering in sedimentary rocks, by preferred orientation in crystalline solids, or by stress-aligned fractures or cracks (Crampin & Lovell, 1991). Shear-wave splitting is considered to be the most diagnostic phenomenon caused by anisotropy.

The detection of orientation and density of stress-aligned cracks in anisotropic media using multicomponent data is of much interests to petroleum explorationists. However, the analysis of data from anisotropic media is very difficult and uncertain due to the complexities of anisotropy. A lot of problems remain unsolved both in theory and in practice. It is also difficult to find test data for anisotropy research. It is very common to use physical modelling techniques to examine wave-propagation properties in anisotropic media and to produce test data (e.g., Cheadle et al., 1991; Ebrom et al., 1990). However, physical modelling also has its own limitations, such as costs and inflexibility to changes in model structure and parameters. Numerical modelling proves to be a good alternative in getting around some of these difficulties.

There are some analytical solutions that can be used for forward modelling (e.g., Daley and Hron, 1977; Guest et al., 1993). However, because of the complexities of anisotropy, analytical solution is not always available. Several papers have been published on the finite-difference modelling method in anisotropic media (e.g., Carcione, 1990; Dong and McMechan, 1995). But for large and complex models, finite-difference methods will not work well. This is because in anisotropic media, where there are at least three independent elastic parameters and usually at least five parameters (as for TI media), the finite-difference algorithm will become very complex and slow in implementation.

The pseudospectral method uses the Fourier transform to compute the spatial derivatives and finite differences to compute the time derivative, and hence is fast. The pseudospectral method has been successfully applied in the isotropic case (e.g., Kosloff et al., 1984). Lou and Rial (1995) used this method to compute wavefields in 2-D inhomogeneous anisotropic media. In this paper, we try to use the pseudospectral method to compute the wavefields in an azimuthally anisotropic medium.

PRINCIPLES

The wave equation governing wave propagation in elastic media is:

$$\rho \ddot{u}_i = C_{ijkl} u_{k,lj} + \rho g_i \tag{1}$$

where ρ is the density, u_i is the infinitesimal displacements vector, ",lj" denotes the partial derivatives with x_l and x_j, C_{ijkl} is the stiffness tensor, g_i is the body force per unit mass (Cheadle et al., 1991). And C_{ijkl} relate the stress tensor σ_{ij} and strain tensor ε_{kl} as Hooke's law:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \tag{2}$$

where

$$\varepsilon_{kl} = \frac{1}{2}\left(u_{l,k} + u_{k,l}\right) \tag{3}$$

The stiffness tensor has the following symmetries:

$$C_{ijkl} = C_{klij} = C_{jikl} \tag{4}$$

which reduces the number of independent elastic constants from 81 to 21.

Also the fourth-order stiffness tensor follows the transformation law:

$$C'_{ijkl} = \left(\frac{\partial x'_i}{\partial x_p}\right)\left(\frac{\partial x'_j}{\partial x_q}\right)\left(\frac{\partial x'_k}{\partial x_r}\right)\left(\frac{\partial x'_l}{\partial x_s}\right) C_{pqrs} \tag{5}$$

With the symmetry property of the stiffness tensor, one may apply the Voigt recipe (Thomsen, 1986), which uses two indices instead of four indices to represent the stiffness tensor as $C_{\alpha\beta}$: If $i = j$ (or $k = l$), $\alpha = i$ (or $\beta = k$); if $i \neq j$ (or $k \neq l$), $\alpha = 9 - (i + j)$ [or $\beta = 9 - (k + l)$].

The evaluation of the stiffness tensor determines the type of medium under consideration. For example, an isotropic medium has the following stiffness matrix:

$$\begin{bmatrix} C_{33} & (C_{33} - 2C_{44}) & (C_{33} - 2C_{44}) & & & \\ & C_{33} & (C_{33} - 2C_{44}) & & & \\ & & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{44} & \\ & & & & & C_{44} \end{bmatrix} \tag{6}$$

and a TIV (transverse isotropy with vertical symmetry) medium has the stiffness matrix:

$$\begin{bmatrix} C_{11} & (C_{11}-2C_{66}) & C_{13} & & & \\ & C_{11} & C_{13} & & & \\ & & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{44} & \\ & & & & & C_{66} \end{bmatrix} \tag{7}$$

To compute the spatial derivative by Fourier transformation, we forward-transform the displacement to the wavenumber domain, perform complex multiplication in the wavenumber domain and then reverse-transform it back to the space domain. For example, the derivative

$$\frac{\partial u_k}{\partial x_j \partial x_l} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(2\pi k_j\right)\left(2\pi k_l\right)U_k\left(k_i\right)\exp\left(-2\pi i k_i x_i\right)dx_i \tag{8}$$

where $U_k\left(k_i\right)$ is the Fourier transform of $u_k\left(x_i\right)$, which can be calculated by a 3-D FFT, and k_i is the wavenumber in the x_i direction.

Based on the second-order finite-differencing approximation of $\ddot{u}_i$ in equation (1), the displacement $u_i(t+\Delta t)$ can be expressed as:

$$u_i(t+\Delta t) = (\Delta t)^2\ddot{u}_i(t) + 2u_i(t) - u_i(t-\Delta t) \tag{9}$$

where Δt is the sampling interval in time.

THE EFFECTIVE STIFFNESS TENSOR

Suppose that we have an isotropic medium with Lame constants λ and μ. We introduce into it a weak distribution of parallel penny-shaped cracks to make it anisotropic. The crack is specified by the crack orientation and the crack density $\zeta = Na^3/v$ $(\zeta << 1)$, where N is the number of cracks of radius a in volume v. Hudson (1981, 1982) gave the expression for effective stiffness tensor in a cracked medium for long-wavelength seismic waves as:

$$C_{ijkl} = C^0_{ijkl} + C^1_{ijkl} + C^2_{ijkl} \tag{10}$$

where C^1_{ijkl} is the first-order and C^2_{ijkl} is the second-order perturbation of the isotropic elastic constants, C^0_{ijkl}, of the uncracked medium. The first-order and second-order perturbations are computed using the crack density and the Lame constants.

Using equation (10), we can determine an expression for an azimuthally anisotropic medium by, in effect, introducing a set of vertical cracks through the effective stiffness tensor. Together with equations (1), (2) and (3), the problem of wave propagation in a cracked anisotropic medium is then fully specified.

CONCLUSIONS

In this paper, the principle of forward modelling of wave propagation using the pseudospectral method in anisotropic media has been presented. The effective stiffness tensor can be computed via Hudson's (1981, 1982) method. The computation of synthetic data for anisotropic media is thus feasible and we propose to implement this forward modelling over the next year.

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