Relationships among elastic-wave values (R^{pp}, R^{ps}, R^{ss}, Vp, Vs, ρ, σ, κ)

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ABSTRACT

The average Vp/Vs value of a set of layers is a weighted sum of the interval velocity ratios. The average value is also bounded by the maximum and minimum interval values. The average value will change according to changes in the target layer. The thicker the layer, the greater its influence on the average value.

The approximate equations for converted-wave reflectivity R^{ps} and pure-shear reflectivity R^{ss} can be combined to give a simple relationship between the two: $R^{ps} \sim 4\sin(\theta) R^{ss}$, where θ is the reflection angle of the shear wave.

The normalized elastic parameters $(\frac{\Delta\alpha}{\alpha}, \frac{\Delta\beta}{\beta}, \frac{\Delta\rho}{\rho})$ are estimated from summed P-P and P-S reflection coefficients using a linear inversion method. Normalized Poisson's ratio $(\frac{\Delta\sigma}{\sigma})$ and Lamé parameters $(\frac{\Delta\lambda}{\lambda}, \frac{\Delta\kappa}{\kappa})$ can then be computed from the estimated velocity and density changes.

AVERAGE *Vp/Vs* VALUE OF MULTIPLE LAYERS

Often in seismic analysis we extract a low-resolution or macroscopic parameter, such as average velocity, which is dependent on higher resolution values such as interval velocities. We may thus be interested in understanding how the micro-values effect the macro-parameters. In this case, how do interval P- and S- velocity ratios affect the average velocity ratio. This is interesting for several reasons. One is when picking events and isochrons on P and S sections, we often take several cycles between picked events (Miller et al., 1995). This means that a series of layers are entering into the isochrons, isochron ratios and thus overal *Vp/Vs* calculation. The question is how does the overall or average *Vp/Vs* value relate to the interval *Vp/Vs* values?

Average *Vp/Vs* calculation

Suppose that we have a layered medium (with layers i=1, N) having P-wave and S-wave interval velocities (α_i, β_i). Each layer has thickness z_i and a set of transit times: t_i^p for one-way P waves and t_i^s for one-way S waves (Figure 1).

What is the average velocity ratio for the whole section? Let's first define an average *Vp/Vs* value as trhe ratio of average velocities (after Sheriff, 1984):.

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$$\gamma \equiv \frac{Z/T_p}{Z/T_s} \ , \tag{1}$$

where Z is the total depth travelled, T_p is the one-way P-wave travel time to depth Z, and T_S is the one-way S traveltime from Z to the surface, and then

$$\gamma = T_s/T_p \ . \tag{2}$$

But $t_i^s = \gamma_i t_i^p$, and

$$\gamma = \frac{\sum_{i=1}^{N} t_i^s}{T_p} = \frac{\sum_{i=1}^{N} \gamma_i t_i^p}{T_p} \tag{3}$$

$$\gamma = \sum_{i=1}^{N} \gamma_i r_i \tag{4}$$

where $r_i = t_i^p / T_p$ or the fractional transit time.

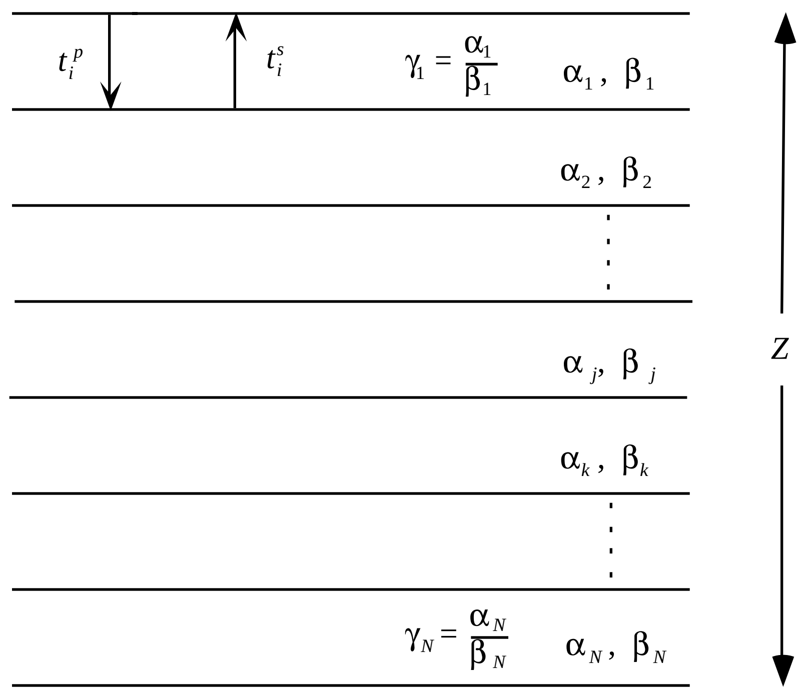

Fig. 1. Plane-layer elastic medium with N layers.

Thus, the average *Vp/Vs* value is the transit-time weighted sum of the interval velocity ratios. Furthermore, γ will be bounded by the minimum and maximum interval ratios (γ_i) as shown below:

$$\gamma = \sum_{i=1}^{N} \gamma_i r_i \geq \sum_{i=1}^{N} \min(\gamma_i) r_i = \min(\gamma_i) \sum_{i=1}^{N} r_i = \min(\gamma_i) \tag{5}$$

$$\gamma = \sum_{i=1}^{N} \gamma_i r_i \leq \sum_{i=1}^{N} \max(\gamma_i) r_i = \max(\gamma_i) \sum_{i=1}^{N} r_i = \max(\gamma_i) \tag{6}$$

Thus, $\min(\gamma_i) \leq \gamma \leq \max(\gamma_i)$.

Examples

We can take several examples to show the effect of a variable velocity layer on the average *Vp/Vs* value. The medium's velocities are given in Table 1, and Figure 2 shows the results. We see that for an isochron ratio or average *Vp/Vs* determination across a thick stack of layers, say 130 m, with only a 10 m target interval, there is little impact of that layer. On the other hand, a 100 m target layer has a large influence on the final *Vp/Vs* value.

Table 1. Five layer elastic model to compute the average *Vp/Vs* value.

Layer	Thickness (m)	*Vp* (m/s)	*Vs* (m/s)	*Vp/Vs*
1	30	2300	1100	1.77
2	30	3000	1800	1.67
3	10 - 100	3500	1000 - 3000	1.20 - 3.50
4	30	4500	2500	1.80
5	30	3750	2200	1.70

Two more examples, directly related to current field cases are shown. We observe the effects of altering the reservoir thicknesses and *Vp/Vs* values for the Lousana Nisku case and Blackfoot sand channel example.

Table 2. Average *Vp/Vs* values for Lousana Nisku case

Layer	Thickness (m)	*Vp* (m/s)	*Vs* (m/s)	*Vp/Vs*
Wabamun salt	25	4600	2300	2.00
Calmar shale	10	4300	2050	2.10
Nisku anhydrite	15	6100	3050	2.00
Nisku porous dolomite	5 - 40	4700 - 7000	3050 - 3950	1.55 - 1.77
Nisku tight dolomite	10	7000	3950	1.77

A thick porous dolomite influences the average *Vp/Vs* value significantly (Figure 3). A thin porous region will have an effect that is probably lost in the noise of real data.

Table 3. Average *Vp/Vs* values for Blackfoot sand channel

Layer	Thickness (m)	*Vp* (m/s)	*Vs* (m/s)	*Vp/Vs*
Mannville	20	4200	2330	1.80
Glauconitic channel	5 - 45	3700 - 4500	2300 - 2500	1.60 - 1.80
Basal quartz	10	4500	2500	1.80

Results from the model of Table 3 are shown in Figure 4. If we assume that we can pick real variations in *Vp/Vs* down to about 0.05, then a Glauconitic sand with thickness above about 10 m should produce an anomalous Vp/Vs value.

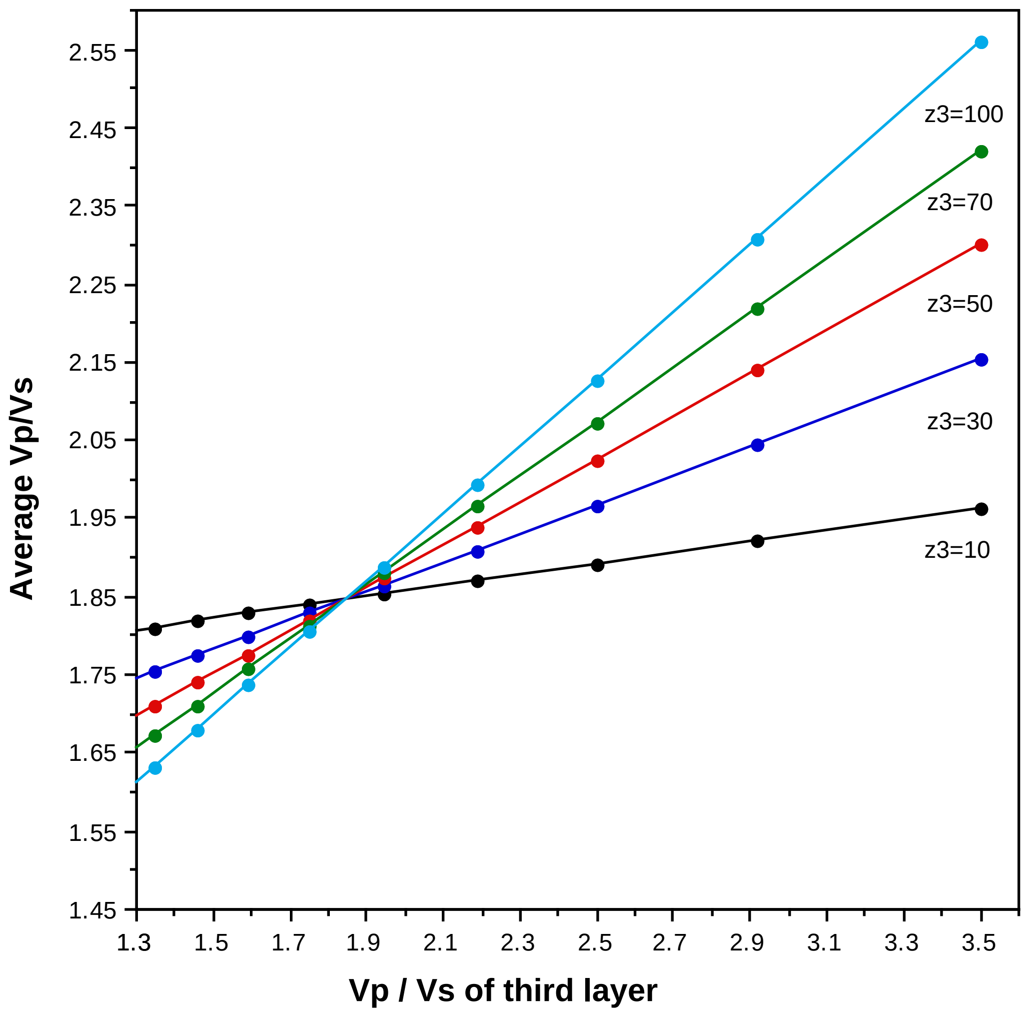

FIG. 2. Variation of the average V_p/V_S value over the 5 layer model (Table 1) with changes in the third layer V_p/V_S value.

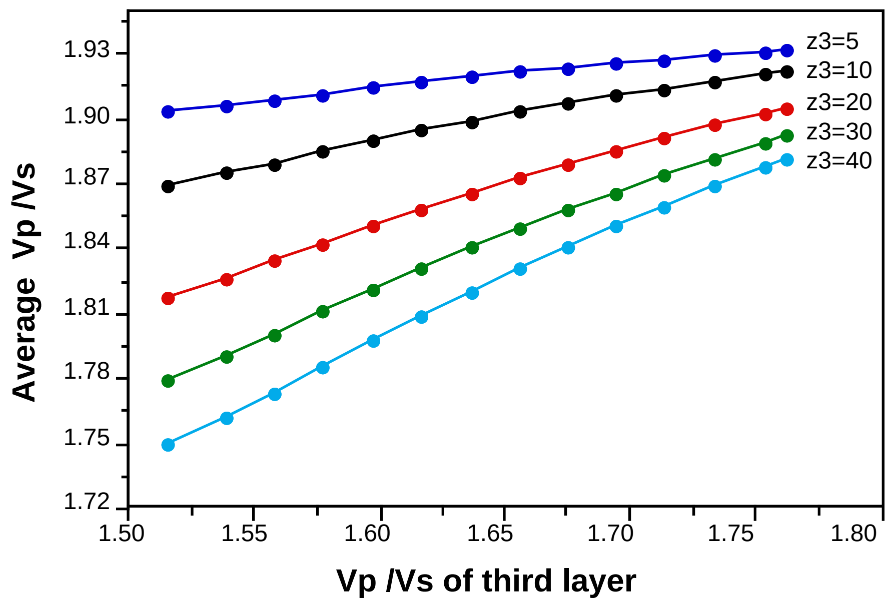

FIG. 3. Variation of the average *Vp/Vs* value from the Lousana Nisku model (Table 2).

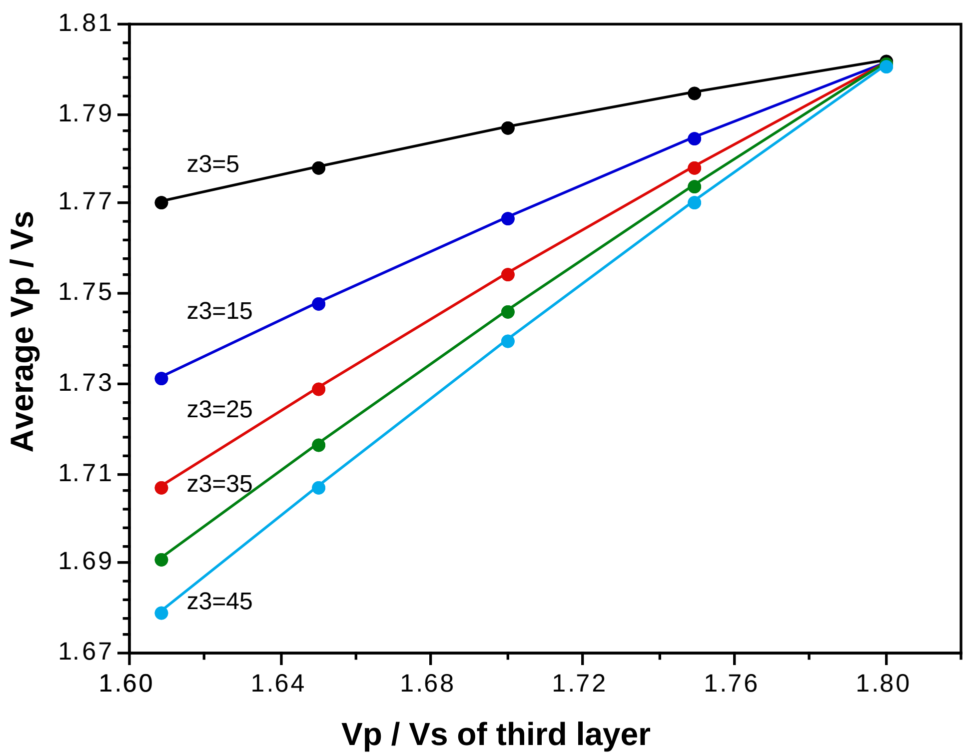

FIG. 4 Variation of the average *Vp/Vs* value across the Blackfoot sand channel model (Table 3).

AN APPROXIMATE RELATIONSHIP BETWEEN R^{ps} AND R^{ss}

We may have a converted-wave reflectivity section or a pure shear section, depending on what survey was conducted. Because both surveys relate to shear-wave properties it is natural to ask several questons: Which is better? Cheaper? How do they relate to each other? The first question is the subject of much current interest and study. More on it soon. The second question has an answer and it generally will be the P-S survey, as it is basically a standard P-P survey with just a different geophone. What the relationship is between R^{ps} and R^{ss} can be shown as below. The equations from Aki and Richards (1980) that approximate the converted-wave reflectivity R^{ps} and pure-S reflectivity R^{ss} are given as:

$$R^{ps}(\theta) = \frac{-p\alpha}{2\cos\theta}\Big[\Big(1 - 2\beta^2 p^2 + 2\beta^2 \frac{\cos\psi}{\alpha}\frac{\cos\theta}{\beta}\Big)\frac{\Delta\rho}{\rho} - \Big(4\beta^2 p^2 - 4\beta^2 \frac{\cos\psi}{\alpha}\frac{\cos\theta}{\beta}\Big)\frac{\Delta\beta}{\beta}\Big], \tag{7}$$

and

$$R^{ss}(\theta) = -\frac{1}{2}\left(1 - 4\beta^2 p^2\right)\frac{\Delta\rho}{\rho} - \left(\frac{1}{2\cos^2\theta} - 4\beta^2 p^2\right)\frac{\Delta\beta}{\beta} \tag{8}$$

Suppose that ψ, and thus θ and p are small then

$$R^{ss} \sim -\frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta\beta}{\beta}\right) \tag{9}$$

$$R^{ps} \sim -\frac{p\alpha}{2}\left[\left(1 + 2\frac{\beta}{\alpha}\right)\frac{\Delta\rho}{\rho} + 4\frac{\beta}{\alpha}\frac{\Delta\beta}{\beta}\right]$$

$$= -\frac{p\alpha}{2}\left[\frac{4\beta}{\alpha}\frac{\Delta\rho}{\rho} + \frac{4\beta}{\alpha}\frac{4\beta}{\alpha} + \left(1 - \frac{2\beta}{\alpha}\right)\frac{\Delta\rho}{\rho}\right]$$

$$= -\frac{p\alpha}{2}\left[-8R^{ss}\frac{\beta}{\alpha} + \left(1 - \frac{2\beta}{\alpha}\right)\frac{\Delta\rho}{\rho}\right]$$

Now as $\frac{\beta}{\alpha} \sim \frac{1}{2}$, the second term in the equation is very small. Thus

$$R^{ps}(\theta) \sim 4\sin\theta\, R^{ss}(\theta) \tag{10}$$

So the converted-wave reflectivity is approximately related to the pure-S reflectivity.

COUPLED P-P AND P-S INVERSION

Suppose that we are trying to estimate $\frac{\Delta\alpha}{\alpha}$ and $\frac{\Delta\beta}{\beta}$ from the Aki and Richards (1980) equations. We could first expand the equations in powers of the ray parameter *p*:

$$R^{pp} \sim R_o^{pp} - p^2\left(2\beta^2\frac{\Delta\rho}{\rho} - \frac{\alpha^2}{4}\frac{\Delta\alpha}{\alpha} + 4\beta^2\frac{\Delta\beta}{\beta}\right) \tag{11}$$

where $$R_o^{pp} = \frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta\alpha}{\alpha}\right) ,$$

and $$R^{ps} \sim \frac{-p\alpha}{2}\left(1+\frac{p^2}{2}\beta^2\right)\left[\left(1+\frac{2\beta}{\alpha}\right)\frac{\Delta\rho}{\rho} + \frac{4\beta}{\alpha}\frac{\Delta\beta}{\beta}\right]. \tag{12}$$

Then $$R^{pp} + R^{ps} = C_o + C_1 p + C_2 p^2 + C_3 p^3 + \ldots\ldots , \tag{13}$$

where $$C_o = R_o^{pp} ,$$

$$C_1 = -\frac{\alpha}{2}\left[\left(1+\frac{2\beta}{\alpha}\right)\frac{\Delta\rho}{\rho} + \frac{4\beta}{\alpha}\frac{\Delta\beta}{\beta}\right] ,$$

$$C_2 = -\left(2\beta^2\frac{\Delta\rho}{\rho} - \frac{\alpha^2}{4}\frac{\Delta\alpha}{\alpha} + 4\beta^2\frac{\Delta\beta}{\beta}\right) ,$$

$$C_3 = -\frac{\alpha\beta^2}{4}\left[\left(1+\frac{2\beta}{\alpha}\right)\frac{\Delta\rho}{\rho} + \frac{4\beta}{\alpha}\frac{\Delta\beta}{\beta}\right] .$$

This could be analysed now by a polynomial line fit up to the third power of the variation of $R^{pp} + R^{ps}$ with offset (*p*). Once we have estimates of $C_0,\ldots C_3$, then we can pose the problem as a matrix inverse. Note that C_3 is directly related to C_1 and will not constrain the problem. So:

$$\begin{pmatrix} C_0 \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ A & 0 & B \\ C & D & E \end{pmatrix} \begin{pmatrix} \frac{\Delta\rho}{\rho} \\ \frac{\Delta\alpha}{\alpha} \\ \frac{\Delta\beta}{\beta} \end{pmatrix} \tag{14}$$

$$C = GP \tag{15}$$

where A, B, C, D, E are functions of α,β,ρ.

We could estimate the scaled elastic changes by using a damped (with ε^2) least-square solution, for example:

$$\hat{P} = (G^T G)\left(G^T G + \varepsilon^2\right)^{-1} G^T C \qquad (16)$$

Once we have the normalized density and velocity changes from equation (16), we may be interested in finding other elastic parameter changes such as Poisson's ratio and the Lamé parameters.

POISSON'S RATIO AND LAMÉ PARAMETER VARIATION

Poisson's ratio σ can be written as a function of the P- and S-wave velocities as:

$$\sigma = \frac{\alpha^2 - 2\beta^2}{2(\alpha^2 - \beta^2)} = \frac{\gamma^2 - 2}{2(\gamma^2 - 1)} \qquad (17)$$

where

$$\gamma = \frac{\alpha}{\beta}$$

How do variations in α and β effect σ? Suppose that changes in the velocities $\Delta\alpha$ and $\Delta\beta$ are small then:

$$\Delta\sigma = \frac{\partial\sigma}{\partial\alpha}\Delta\alpha + \frac{\partial\sigma}{\partial\beta}\Delta\beta \qquad (18)$$

Using equations (17) and (18), the relative variation of σ is:

$$\frac{\Delta\sigma}{\sigma} = \frac{2\alpha^2\beta^2}{(\alpha^2 - \beta^2)(\alpha^2 - 2\beta^2)}\left(\frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta}\right) \cdot \qquad (19)$$

We note that if $\alpha = c\beta$, where c is a constant, then $\frac{\Delta\alpha}{\alpha} = \frac{\Delta\beta}{\beta}$ and changes in α and β produce no change in Poisson's ratio (γ is constant). If, however $\alpha = c\beta + d$, where c and d are constants, then:

$$\frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta} = \frac{\Delta\alpha}{\alpha}\left(\frac{-d}{c\beta}\right) \qquad (20)$$

and $\frac{\Delta\sigma}{\sigma}$ changes accordingly.

We recall that

$$\alpha^2 = (\lambda + 2\mu) / \rho\,, \qquad (21)$$

and

$$\beta^2 = \mu / \rho, \qquad (22)$$

where μ and λ are the Lamé parameters.

Now, it may be useful to isolate λ for petrophysical analysis as λ may depend more directly on pore fill.

$$\lambda = \rho\alpha^2 - 2\rho\beta^2 \tag{23}$$

Expanding equation (23) for small velocity and density changes gives

$$\frac{\Delta\lambda}{\lambda} = (\frac{2}{\alpha^2 - \beta^2})[\alpha^2(\frac{\Delta\alpha}{\alpha}) - 2\beta^2(\frac{\Delta\beta}{\beta})] + \frac{\Delta\rho}{\rho} \tag{24}$$

Again, a $\frac{\Delta\lambda}{\lambda}$ section might have less influence of lithology and highlight pore-fill changes.

For completeness, we can also estimate the incompressibility (bulk modulus) changes:

$$\kappa = \lambda + \frac{2}{3}\mu = \rho\alpha^2 - \frac{4}{3}\rho\beta^2 , \tag{25}$$

and

$$\frac{\Delta\kappa}{\kappa} = (\frac{2}{\alpha^2 - \beta^2})[\alpha^2(\frac{\Delta\alpha}{\alpha} - \frac{4}{3}\beta^2(\frac{\Delta\beta}{\beta})] + \frac{\Delta\rho}{\rho} \tag{26}$$

CONCLUSIONS

The average Vp/Vs value is a weighted sum of the interval velocity ratios. The average value is also bounded by the maximum and minimum interval values. It will change according to changes in the target layer. The thicker the layer, the greater its influence on the average value.

The shear reflectivities (R^{ps} and R^{ss}) are approximately related by a simple sine function. It should be possible to construct one section given the other.

A method is presented to calculate normalized velocity and density changes from P-P and P-S reflectivity coefficients. Using these changes we can also calculate changes in Poisson’s ratio, incompressibility, and the Lamé parameters.

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Dr. Clint W. Frasier of Chevron Petroleum Technology Corporation derived a similar result to equation (10) independently in an internal Chevron memo dated May 10, 1995.

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