Approximate parameters of anisotropy from reflection traveltime curves

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ABSTRACT

For elastic-wave propagation in a transversely isotropic medium, there are five elastic parameters, which may be expressed as the two vertical velocities and the Thomsen parameters (ε, δ, and γ). In this research, the discrete least-squares approximation will be used to fit the three coefficients (A_2, A_4 and A^*) of the non-hyperbolic P- and SV-wave traveltime curves. The coefficient A_2 determines the short-spread moveout velocity, A_4 gives the correction for nonhyperbolic moveout (in the case of strong anisotropy) and A^* is a parameter for correcting the behavior of moveout at large offset, which depends on A_2, A_4 and the horizontal velocity. For P-wave propagation, the coefficient A_2 depends on vertical velocity (V_{Po}) and Thomsen parameter δ, while the coefficient A_4 is controlled by V_{Po}, δ and ε. For SV-wave propagation, the coefficients A_2 and A_4 depend on the vertical velocity ratio $V_{\mathrm{Po}}/V_{\mathrm{So}}$, δ and ε. And for SH-wave propagation, the coefficient A_2 depends on the Thomsen parameter γ and the vertical velocity (V_{SHo}). In a homogeneous transversely isotropic medium, the wavefront of the SH wave is always elliptical, and the SH-moveout is hyperbolic, so that the coefficient A_4 for the SH wave vanishes. The three coefficients depend on the vertical velocities and Thomsen parameters (ε, δ, and γ). Therefore, by combining these coefficients, we will be able to recover the Thomsen parameters and the vertical velocities.

INTRODUCTION

It is well known that, in the presence of anisotropy, the traveltimes of waves reflected from a horizontal interface form a nonhyperbolic curve. That is, the short-spread moveout velocity is not equal to the vertical velocity, as in an isotropic medium. In conventional techniques, we ignore the difference between vertical rms velocities and moveout velocities. This may lead to unsatisfactory errors in interval velocities and in time-to-depth conversion. In the conventional case, the reflection moveout curves are approximated by the hyperbolic equation:

$$t^2 = t_v^2 + \frac{x^2}{V_{\mathrm{mo}}^2}, \qquad (1)$$

where t_v is the zero-offset arrival time, x is the source-receiver offset, and V_{mo} is the (short-spread) moveout velocity. In the presence of anisotropy, the reflection moveout curves are approximated by Tsvankin and Thomsen (1994) as:

$$t^2 = t_v^2 + A_2 x^2 + \frac{A_4 x^4}{1 + A^* x^2}, \qquad (2)$$

where

$$\left.\begin{aligned}
&\text{for a qP wave:} && A_2 = 1\Big/\left[V_{\mathrm{Po}}^2\left(1+2\delta\right)\right], \quad A_4 = \frac{-2\,(\varepsilon-\delta)}{t_{\mathrm{Po}}^2 V_{\mathrm{Po}}^4}\,\frac{\left[1+\dfrac{2\delta}{1-V_{\mathrm{So}}^2\big/V_{\mathrm{Po}}^2}\right]}{\left(1+2\,\delta\right)^4},\\
&\text{for a qSV wave:} && A_2 = 1\Big/\left[V_{\mathrm{So}}^2\left(1+2\sigma\right)\right], \quad A_4 = \frac{2\,\sigma}{t_{\mathrm{So}}^2 V_{\mathrm{So}}^4}\,\frac{\left[1+\dfrac{2\delta}{1-V_{\mathrm{So}}^2\big/V_{\mathrm{Po}}^2}\right]}{\left(1+2\,\sigma\right)^4},\\
&\text{for an SH wave:} && A_2 = 1\Big/\left[V_{\mathrm{SHo}}^2\left(1+2\,\gamma\right)\right], \quad A_4 = 0,\\
&\text{and} && A^* = \frac{A_4}{\dfrac{1}{V_{\mathrm{h}}^2} - A_2},
\end{aligned}\right\} \qquad (3)$$

where V_{h} is the horizontal velocity and $\sigma = \left(\dfrac{V_{\mathrm{Po}}^2}{V_{\mathrm{So}}^2}\right)\left(\varepsilon-\delta\right)$.

The coefficient A_2 is also identified as $1/(V_{\mathrm{mo}})^2$. In isotropic media, we assume that moveout velocities are identical to vertical velocities (in a single layer) but in anisotropic media, the moveout velocities depend on vertical velocities and the Thomsen parameters (ε, δ, and γ). If we know the three coefficients in equation (2), we will be able to calculate these parameters and the vertical velocities for each propagation mode (qP, qSV and SH waves).

THEORY

The least-squares approximation

To calculate the three coefficients in equation (2), we first try to linearize it by letting $y = t^2$ and $u = x^2$ and multiplying through by the denominator of the last term. Then equation (2) becomes:

$$y = C_o + C_1 u + C_2 u^2 + C_3\, u\, y \qquad (4)$$

where

$$C_{\mathrm{o}} = t_v^2, \quad C_1 = A^* t_v^2 + A_2, \quad C_2 = A^* A_2 + A_4, \quad \text{and} \;\; C_3 = -A^*.$$

Assume that $\tilde{y}_i$ are the regression estimates for the arguments u_i, that is:

$$\tilde{y}_i = C_o + C_1\, u_i + C_2\, u_i^2 + C_3\, u_i\, \tilde{y}_i, \qquad (5)$$

and regard y_i and u_i as observed values, corresponding to t_i and x_i, $i = 1,...,N$; N being the number of traces picked. The sum of the squares of differences between estimated and observed values is

$$E = \sum_i \left(\tilde{y}_i - y_i\right)^2$$

$$E = \sum_i \left(C_o + C_1 u_i + C_2 u_i^2 + C_3 u_i \tilde{y}_i - y_i\right)^2. \tag{6}$$

E can be seen as a function of the variables, C_o, C_1, C_2 and C_3. To minimize it, the necessary requirement are:

$$\frac{\partial E}{\partial C_j} = 2\sum_i \left(\tilde{y}_i - y_i\right)\frac{\partial \tilde{y}_i}{\partial C_j} = 0 . \qquad (j = 0,...,3) \tag{7}$$

Since $\frac{\partial \tilde{y}_i}{\partial C_o} = 1$, $\frac{\partial \tilde{y}_i}{\partial C_1} = u_i$, $\frac{\partial \tilde{y}_i}{\partial C_2} = u_i^2$, and $\frac{\partial \tilde{y}_i}{\partial C_3} = u_i \tilde{y}_i$,

equation (7) can be written into the four regression equations as:

$$NC_o + (\sum_i u_i)C_1 + (\sum_i u_i^2)C_2 + (\sum_i u_i \tilde{y}_i)C_3 = \sum_i y_i \tag{8}$$

$$(\sum_i u_i)C_o + (\sum_i u_i^2)C_1 + (\sum_i u_i^3)C_2 + (\sum_i u_i^2 \tilde{y}_i)C_3 = \sum_i u_i y_i \tag{9}$$

$$(\sum_i u_i^2)C_o + (\sum_i u_i^3)C_1 + (\sum_i u_i^4)C_2 + (\sum_i u_i^3 \tilde{y}_i)C_3 = \sum_i u_i^2 y_i \tag{10}$$

$$(\sum_i u_i \tilde{y}_i)C_o + (\sum_i u_i^2 \tilde{y}_i)C_1 + (\sum_i u_i^3 \tilde{y}_i)C_2 + (\sum_i u_i^2 \tilde{y}_i^2)C_3 = \sum_i u_i y_i \tilde{y}_i \tag{11}$$

where *N* is the number of observed values.

The equations (8) to (11) are a linear system of variables C_o, C_1, C_2 and C_3, and these could be solved very easily if we let $\tilde{y}_i \to y_i$. Then from equation (4), we could get the three coefficients and the two-way vertical traveltime as functions of C_o, C_1, C_2 and C_3, i.e.:

$$t_v^2 = C_o , \quad A^* = -C_3 , \quad A_2 = C_1 + C_3 C_o , \text{ and } A_4 = C_2 + C_3\left(C_1 + C_3 C_o\right) . \tag{12}$$

The iteration method

We need to iterate the procedure because of the nonlinear nature of the traveltime equation (2) which leads to the appearance of $\tilde{y}_i$ in the regression equations (8) to (11). In the first iteration, we use the observed values, y_i, for the estimated or calculated values, $\tilde{y}_i$, as input to the least-squares approximation [equations (4) to (12)]. The output of the least-squares approximation are coefficients, used to calculate the new values of $\tilde{y}_i$. These values are used as input for a second iteration, and so on. The procedure will iterate until the sum of the squares of differences between estimating and observed values, *E*, converges to a stable limit.

WORK PLAN

Data acquisition

The physical modelling of the Phenolic CE slab (Cheadle et al., 1991;Brown et al., 1991) will be used to test the algorithm. Since we have determined estimates of the Thomsen parameters of the Phenolic CE slab, we can estimate the velocities of this material as a function of angle of incidence. From these calculated velocity values, we can calculate traveltimes, which will be used to test the accuracy of the least-squares approximation. After that, the same approximation will be applied to the shot gathers recorded from the Phenolic CE slab.

Data processing

To apply the least-squares approximation, a Fortran program will be written to estimate the coefficients and then several calculations will be needed to solve for the anisotropy parameters. More generally, the approximation incorporating dipping events and azimuthally anisotropic media will be considered for the next step. We are also interested in the P-SV case, in which the traveltime equation is much more complicated than the P-P or S-S cases, as is the moveout velocity function.

REFERENCES

Brown, R.J., Lawton, D.C. and Cheadle, S.P., 1991, Scaled physical modelling of anisotropic wave propagation: multioffset profiles over an orthorhombic medium: Geophysical Journal International, **107**, 693-702.

Cheadle, S.P., Brown, R.J. and Lawton, D.C., 1991, Orthorhombic anisotropy: A physical seismic modeling study: Geophysics, **56**, 1603-1613.

Hake, H., Helbig, K. and Mesdag, C.S., 1984, Three-term Taylor series for t^2-x^2 curves of P- and S-waves over layered transversely isotropic ground: Geophysical Prospecting, **32**, 828-850.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954-1966.

Tsvankin, I., and Thomsen, L., 1994, Nonhyperbolic reflection moveout in anisotropic media: Geophysics, **59**, 1244-1258.

Tsvankin, I., 1995, Normal moveout from dipping reflectors in anisotropic media: Geophysics, **60**, 268-284.