Velocity sensitivity for equivalent offsets in CSP gathers

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ABSTRACT

Depth migration and prestack time migration require an accurate velocity model for optimum imaging. An important feature of Equivalent Offset prestack time migration is the convergence of velocities. The common scatter point (CSP) gathers may be formed from a gross velocity approximation, and then analyzed to produce a more accurate estimate of the velocity. The sensitivity of the input velocity, verses output velocity, is expressed as a ratio, and shows large variations on the input velocity have little effect on the output velocity.

Examples of semblance diagrams are shown which start with an infinite velocity, and converging to a reasonable estimate in a few iterations. A panel of one percent velocity increments is also included

CONVENTIONAL VELOCITY ANALYSIS

Introduction

Conventional processing based on common midpoint (CMP) gathers assume horizontal reflectors and that only traces in the CMP gather contain reflections from the subsurface below the CMP position. The inclusion of dip moveout (DMO) and migration allows the extension of conventional processing to include non-horizontal events. Velocity analysis however, even with DMO and migration, is still based on the CMP concept that limits the convergence and accuracy of reliable velocity solutions.

Figure 1 shows the ray paths for a source S and receiver R traveling to a horizontal reflector and to a scatter point. The corresponding travel times are plotted below the CMP location. The objective of seismic processing is to relocated the energy on the CMP trace back to the corresponding reflector and scatter with NMO and migration. The migration process may include DMO and post-stack migration, or a prestack migration. These migration processes move energy from the CMP trace to the prestack migration ellipse, and neighbouring traces reconstruct the energy at the reflector or scatter point. Accurate imaging occurs when exact velocities are used, and a major effort of processing is devoted to obtaining the best velocity models for the migrations.

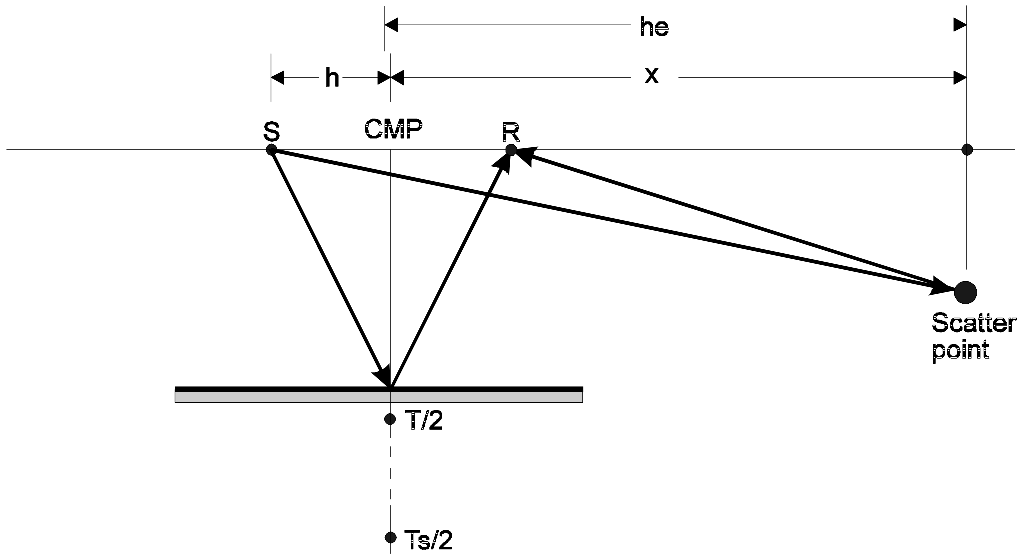


Fig. 1. Ray paths for a horizontal reflector and a scatter point.

NMO alone may be satisfactory in finding velocities for horizontal reflectors, but is fails to image correctly the energy of dipping events (or scatter points). The inclusion of migrations in the velocity analysis loop may aid in imaging the scatter points. After migration, the data is assigned an offset and then inverse normal moveout (INMO) applied with that offset. The assigned offset is somewhat arbitrary, and depends on the data configuration rather than on the geometry of the problem. For example, the diagram of Figure 1 may be considered a source record with the offset of the scatter point after migration at $(h + x)$ or, if the source and receiver are reversed, the offset would be $(h - x)$. If Figure 1 is viewed as a constant offset section, the offset of the scatter point would be forced to remain at h. Neither of these methods give an offset that represents the distance of the scatter point from the source and receiver. Even in the simplest case of zero offset and constant velocities, an error in the velocities will miss position the data in time and space.

Figure 2 illustrates this situation with a time section that has zero offset and constant velocity. The travel time of a scatter point on a dipping event is plotted below the CMP. The migration semi-circle becomes elliptical with a different velocity, shown in gray. Reconstruction of the reflection point will be difficult when using INMO at the same CMP. The real world further complicates the analysis by introducing variable velocities and assuming offsets are based only on the distance between the source and receiver.

It is proposed that the best offset is the equivalent offset used in prestack migration. This offset does represent the distance of the scatter point from the source and receiver, and provides excellent results.

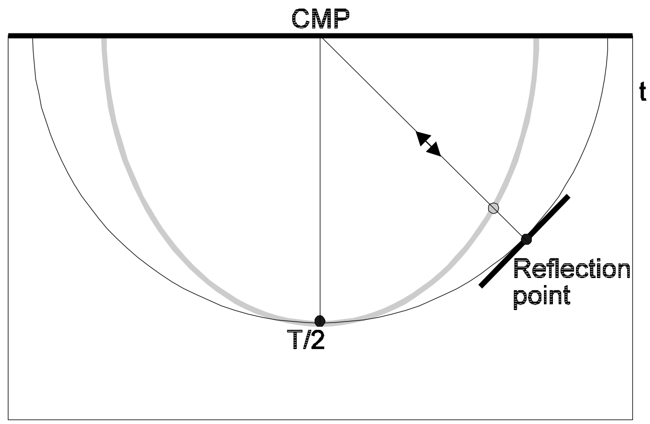


Fig. 2. Migration of the dipping reflection with the wrong velocity.

VELOCITY SENSITIVITY FOR PRESTACK MIGRATION

Use of the equivalent offset h_e, positions the input traces in a common scatter point (CSP) gather prior to any time shifting of the input data, (Bancroft 1994, 1995). Reflection events in a CSP gather are hyperbolic enabling a simplified velocity analysis. The equivalent offset is slightly dependent on the initial velocity guess, but the resulting analysis of the CSP gather yields a more accurate velocity. The sensitivity of the output velocity to the input velocity indicates the number of iteration that may be required for accurate migrations.

The computation of the equivalent offset for both 2-D and 3-D requires the vertical planes of the source and receiver be rotated to one common plane. The equation used to compute the equivalent offset is

$$h_e^2 = x^2 + h^2 - \frac{4x^2h^2}{T^2V_{in}^2}, \tag{1}$$

where x is the distance from the CSP to the rotated CMP, h is half the rotated source receiver offset, T is the time of the input sample, and V is the RMS velocity at the scatter point. For fixed values of x and h, equation (1) is differentiated with respect to V to give

$$\frac{dh_e}{dV} = \frac{4h^2x^2}{h_eT^2V_{in}^3}. \tag{2}$$

The sensitivity s_{in}, or change in equivalent offset h_e, with respect to change in velocity V_{in} is found from

$$s_{in} = \frac{\frac{\delta h_e}{h_e}}{\frac{\delta V_{in}}{V_{in}}}, \tag{3}$$

giving

$$s_{in} = \frac{4h^2 x^2}{h_e^2 T^2 V^2}. \tag{4}$$

The sensitivity of the output velocity V_{nmo} with respect to the equivalent offset h_e is found from the NMO equation with T and T_0 fixed, i.e.,

$$T^2 = T_0^2 + \frac{4h_e^2}{V_{nmo}^2}. \tag{5}$$

Differentiating with respect to h_e and inserting the appropriate values of h_e and V, the sensitivity s_{nmo} is found to be

$$s_{nmo} = \frac{\frac{\delta V_{nmo}}{V_{nmo}}}{\frac{\delta h_e}{h_e}} = 1. \tag{6}$$

The sensitivity S of the output velocity V_{out}, relative to the input velocity V_{in}, then becomes

$$S = \frac{\frac{\delta V_{nmo}}{V_{nmo}}}{\frac{\delta V_{in}}{V_{in}}} = \frac{4x^2 h^2}{h_e^2 T^2 V^2}. \tag{7}$$

When either x or h tends to zero, the sensitivity S also tends to zero, and the CSP gather is correctly formed independent of any input velocity. In addition, when $TV/2$ is large relative to either x or h, the sensitivity again tends to a small value. These conditions occur for a large portion of the data that form a CSP gather, and help to stabilize the estimated velocity. It should also be noted that the rotation of the source and receiver to a common plane reduces the original source and receiver offset to a smaller value of h.

The worst case sensitivity occurs when x equals h, when either the source or receiver is located above the scatter point. In this rare case, equation (7) may be reduced to a function of the geological dip β, or the pre-migration dip α, which is the same dip along the hyperbolas of the CSP gather, i.e.

$$S = \frac{(1-\cos\beta)^2}{1-(\cos\beta)^2} = \frac{4\sin^4\left(\frac{\sin^{-1}(\tan\alpha)}{2}\right)}{(\tan\alpha)}. \tag{8}$$

The following table lists this worst case sensitivity as a function of both α and β. Note that dip α is computed assuming linear rays and RMS velocities, and that the real geological dip will be greater (assuming an increase in velocity).

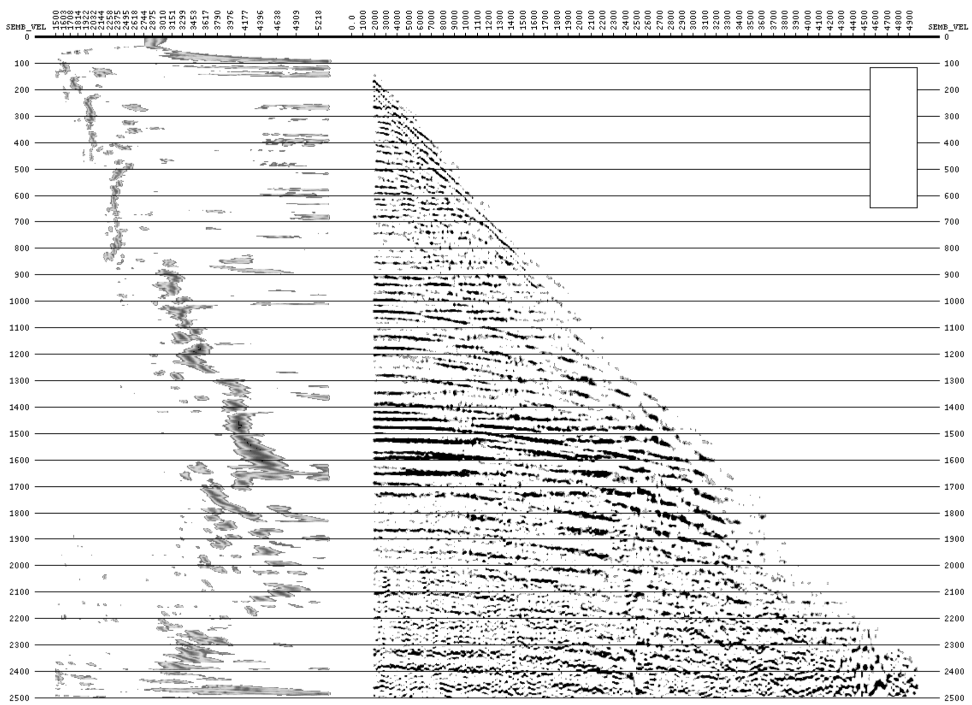
Table 1 Worst case velocity sensitivity as a function of dip.

Geological dip β	CSP gather dip α	Sensitivity S
10	9.9	0.01
20	18.8	0.03
30	26.6	0.07
40	32.7	0.13
50	37.5	0.22
60	40.9	0.33
70	43.2	0.49
80	44.6	0.70
90	45	1.00

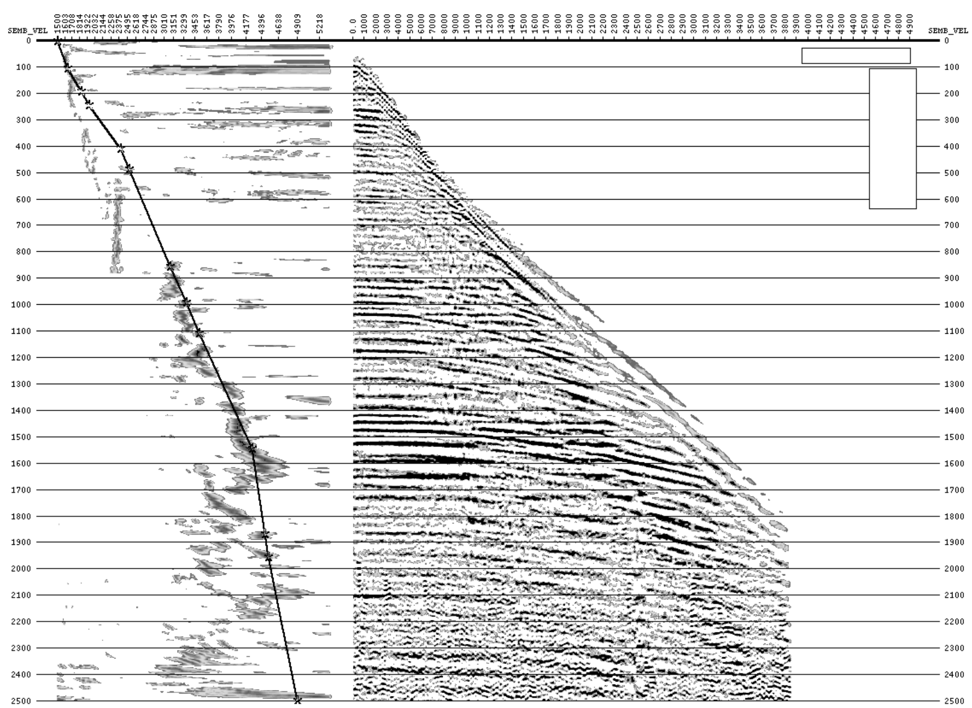
It is observed from the table that dips under 30 degrees have a worst case sensitivity less than 10 percent. An input velocity error of say 20% when creating the CSP gathers, will have less than a 2% error on the resulting velocity estimated from a semblance of the CSP gather. This is verified with shallow dipping planes type data where the number of iteration for velocity analysis may be as low as two. Highly structured data on the other hand with dips close to 70 degrees have a sensitivity of 50% so the number of iterations will be increased.

EXAMPLES OF REAL DATA

The first examples in Figure 3 are taken from the 2-D Blackfoot line, and shows two iterations of the semblance plots. The first CSP gather was formed with a velocity of 100,000 m/s to effectively remove the negative term of equation (1). A resulting CSP gather produced a well-focused semblance from which a first velocity pick was made. These velocities were then used to produce the second CSP gather and semblance. Very little difference in the semblance is observed. Some questionable picks are better resolved or moved in the second semblance to aid in their identification. A CMP gather and semblance has also been included in Figure 4 for a comparison, and also shows excellent results. It should be noted that the Blackfoot data was acquired with source interval equal to receiver intervals.



(a)



(b)

Fig. 3. Semblance and CSP gathers for Blackfoot data, a) with 100,000 m/s velocity used to create gather, and b) semblance and gather formed from picking the velocities in (a).

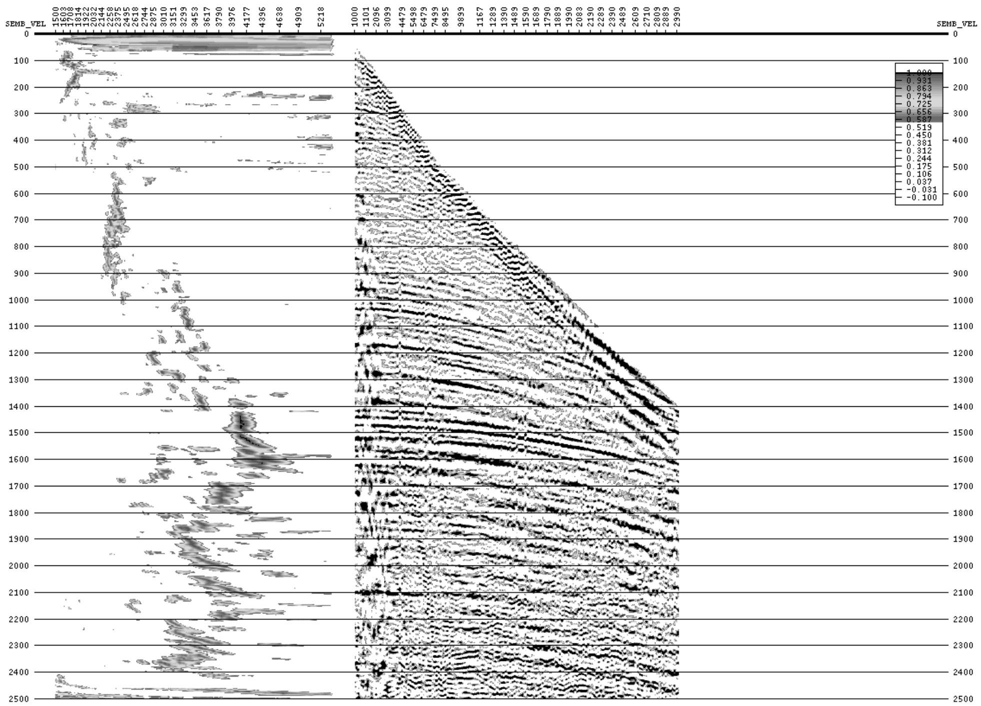


Fig. 4. A CMP gather and semblance of Blackfoot data for comparison with the CSP gathers.

Figure 5 contains multiple panels of a short piece of the Blackfoot line in which the velocity is incremented by one percent. These panels have been included to illustrate the sensitivity of prestack time migration, and that velocity tuning may be required after the semblance estimation is complete. The sensitivity of the velocities should not be considered a detriment to processing, as it is this sensitivity that helps eliminate multiples, and to obtain a superior image.

CONCLUSIONS

The estimation of prestack migration velocities is simplified with the Equivalent offset method of prestack migration. Data with little structure yields accurate velocities within two iterations. Structured data has more difficulty focusing, but will converge to a stable solution.

REFERENCES

Bancroft, J. C., and Geiger, H. D., 1994, Equivalent offsets and CRP gathers for prestack migration, Expanded Abstracts 1994 SEG International Convention, pp. 672-675.

Bancroft, J. C., 1995, Prestack migration, CREWES Research Report.

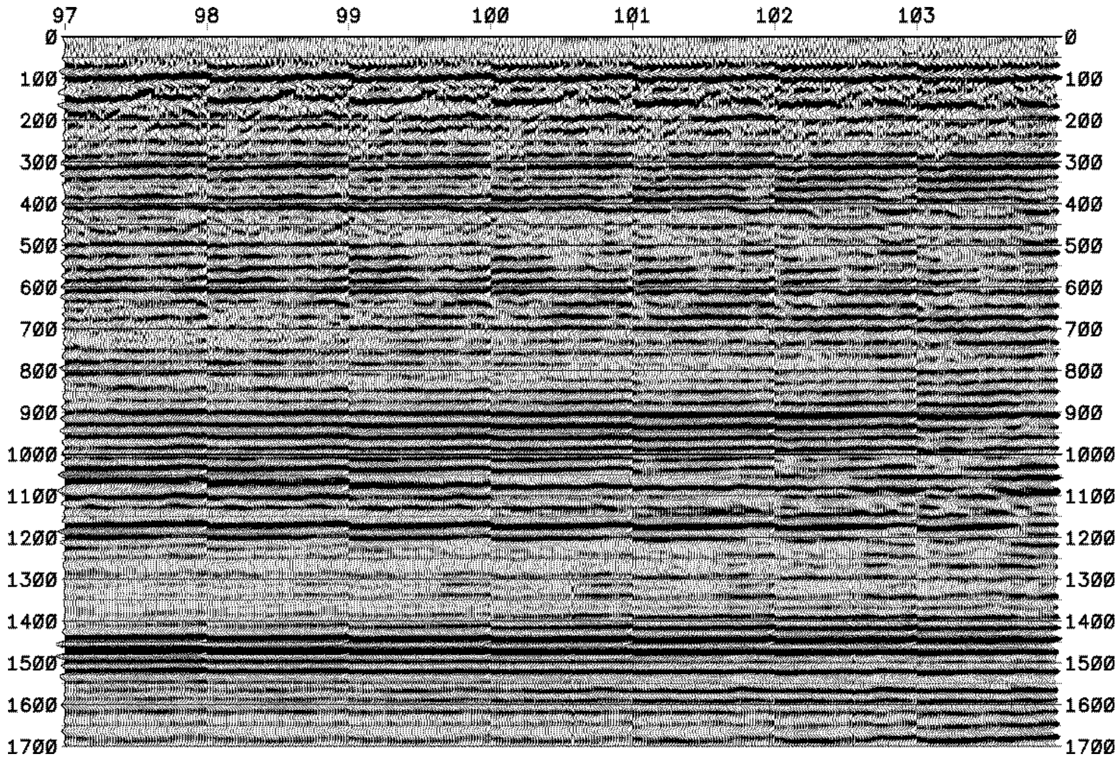


Fig. 5. Prestack migration panels of Blackfoot data, each panel at one percent increments.