# The relationship of Thomsen anisotropic parameters to the crack density of a cracked medium

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## ABSTRACT

In this paper, we first compute the effective stiffness tensor and Thomsen anisotropic parameters in the cracked media in which anisotropy is caused by stress-aligned fractures or cracks with a particular crack density. We then analyze the variations of velocities versus crack density and phase angle and we are able to relate velocities to the crack density directly.

## INTRODUCTION

Now it is commonly accepted that most upper-crustal rocks are anisotropic to some extent (Crampin, 1981). There are many factors that can cause anisotropy, among which stress-aligned fractures or cracks is a major one that is of importance to exploration geophysicists. The detection of the strike direction and density of the fractures or cracks, using multi-component data, is of great interest to the exploration industry. It would be helpful for anisotropy analysis and inversion if we could directly relate the degree of anisotropy to crack density.

Thomsen (1986) introduced three anisotropic parameters, $\varepsilon$, $\delta$ and $\gamma$, to describe weak anisotropy, which is believed to be the common case of anisotropy. These parameters can be computed with stiffness tensor of the anisotropic media. In turn, phase and group velocities can be determined with the Thomsen anisotropic parameters.

The difficulty for the computation of the three anisotropic parameters is the computation of the stiffness tensor. The determination of the stiffness tensor fully defines an elastic medium. Hudson (1981, 1982) investigated the stiffness tensor computation in a cracked elastic media with crack density for long-wavelength seismic waves and made it possible for us to compute Thomsen anisotropic parameters and velocities in cracked media.

Through the computation and analysis, we found that the weak anisotropy assumption is only valid for small crack density. Group velocity and phase velocity are almost equal and velocities decrease approximately linearly with crack density increasing in the weak anisotropy case. The analysis provides the possibility to derive crack density from velocities.

## THE EFFECTIVE STIFFNESS TENSOR IN CRACKED MEDIA

Suppose that we have an isotropic medium with Lame constants, $\lambda$ and $\mu$. We introduce into it a weak distribution of parallel penny-shaped cracks to make it anisotropic. The cracks are specified by the crack orientation and crack density, $\zeta = Na^3/v$ $(\zeta << 1)$, where N is the number of cracks of radius $a$ in volume $v$. Hudson (1981, 1982) presented an expression for the effective stiffness tensor in cracked media for long wave-length seismic waves as (Crampin, 1984):

$$C_{ijkl} = C^0_{ijkl} + C^1_{ijkl} + C^2_{ijkl} \tag{1}$$

where $C^1_{ijkl}$ is the first-order and $C^2_{ijkl}$ is the second-order perturbation of the isotropic elastic constants, $C^0_{ijkl}$, of the uncracked medium. The first-order and second-order perturbations are computed using the crack density and the Lame constants.

Using this equation, we can determine an expression for an anisotropic medium by, in effect, introducing a set of cracks through the effective stiffness tensor.

Expressed in two indices notation, stiffness tensor of an isotropic medium is:

$$C^0_{\alpha\beta} = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ & \lambda+2\mu & \lambda & & & \\ & & \lambda+2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{pmatrix} \tag{2}$$

Stiffness tensor of a transverse isotropic medium with vertical symmetry axis is:

$$C_{\alpha\beta} = \begin{pmatrix} C_{11} & (C_{11}-2C_{66}) & C_{13} & & & \\ & C_{11} & C_{13} & & & \\ & & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{44} & \\ & & & & & C_{66} \end{pmatrix} \tag{3}$$

and the first-order perturbation is:

$$C^1_{\alpha\beta} = \begin{pmatrix} \lambda^2 & \lambda^2 & \lambda(\lambda+2\mu) & & & \\ & \lambda^2 & \lambda(\lambda+2\mu) & & & \\ & & (\lambda+2\mu)^2 & & & \\ & & & \mu^2 & & \\ & & & & \mu^2 & \\ & & & & & 0 \end{pmatrix} \bullet D \bullet \left(-\frac{\zeta}{\mu}\right) \tag{4}$$

where $D$ = diag $(U_{11},U_{11},U_{11},U_{33},U_{33},0)$ and $U_{11}$ and $U_{33}$ can be computed from Lame constants of the uncracked medium and pore fluids (Crampin, 1984), for dry cracks:

$$U_{11} = \left(\frac{4}{3}\right) \bullet \frac{\lambda+2\mu}{\lambda+\mu}\ ,\ U_{33} = \left(\frac{16}{3}\right) \bullet \frac{\lambda+2\mu}{3\lambda+4\mu} \tag{5}$$

therefore, we are able to compute the effective stiffness tensor by the Lame constants of the uncracked medium and pore fluids in the cracks and the crack density.

## THOMSEN ANISOTROPIC PARAMETERS AND VELOCITIES

With the stiffness tensor obtained with Hudson's theory, we can compute the Thomsen anisotropic parameters and phase velocities. For the weak anisotropy case, the anisotropic parameter is (Thomsen, 1986):

$$\varepsilon = \frac{C_{11} - C_{33}}{2C_{33}}, \;\; \gamma = \frac{C_{66} - C_{44}}{2C_{44}}, \;\; \delta = \frac{\left(C_{13}+C_{44}\right)^2 - \left(C_{33}-C_{44}\right)^2}{2C_{33}\left(C_{33} - C_{44}\right)} \tag{6}$$

and phase velocities are:

$$V_P(\theta) = f_1(\theta,\varepsilon,\gamma,\delta), \;\; V_{SV} = f_2(\theta,\varepsilon,\gamma,\delta), \;\; V_{SH} = f_3(\theta,\varepsilon,\gamma,\delta) \tag{7}$$

where θ is the phase angle. Group angle and group velocities can be derived from phase angle and phase velocities (see Thomsen, 1986 for details).

## COMPUTATION RESULTS AND ANALYSIS

All the computations referred below are performed with P-wave velocity being 4000 m/s and $V_p/V_s$ ratio being 1.7.

Figure 1 shows the Thomsen anisotropic parameters versus crack density. Figure 1.b shows that all three parameters are nearly linear to crack density in the weak anisotropy case, where crack density ranges from 0. to 0.02.

Figure 2 shows the ratio of group to phase velocity and ratio of exact and approximate computations of P-wave velocity. We conclude that only in the small crack density case, is the weak anisotropy assumption valid and the group and phase velocity are the same (phase angle and group angle would be different).

Figure 3 shows SH-wave velocity versus crack density and phase angle. Also it is linear in the weak anisotropic case.

Figure 4 shows the phase velocities versus crack density and phase angle in 3D plotting, which enable us to examine the velocities variations more directly.

## CONCLUSIONS

Using Hudson’s theory, we can compute the stiffness tensor in cracked media and hence the Thomsen anisotropic parameters. We are able to directly relate the phase and group velocity directly to crack density in the cracked media. Theoretical computation shows that weak anisotropy assumption is valid at small crack density ranges and linearity should be a good approximation. We can also examine the velocity variations due to the inclusion of fluids in the cracks.


## ACKNOWLEDGMENTS

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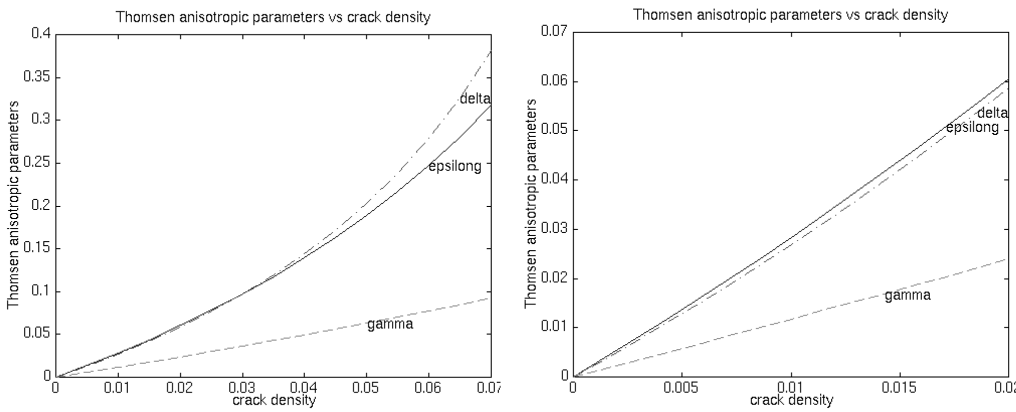
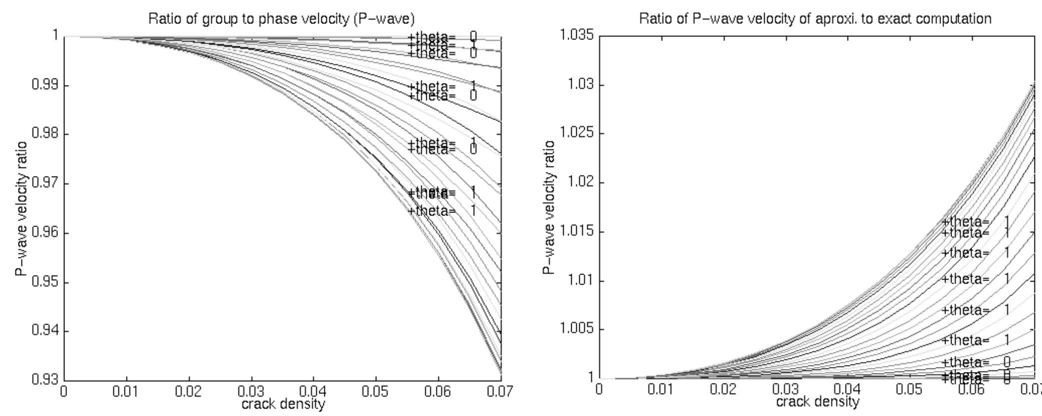


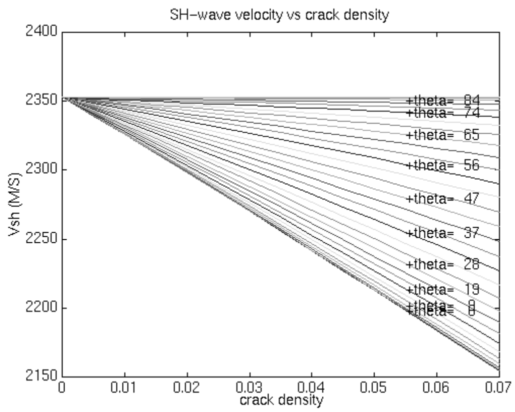

(1.a) (1.b)

Figure 1. Thomsen anisotropic parameters versus crack density; (a) crack density varies from 0. to 0.07; (b) crack density varies from 0. to 0.02.

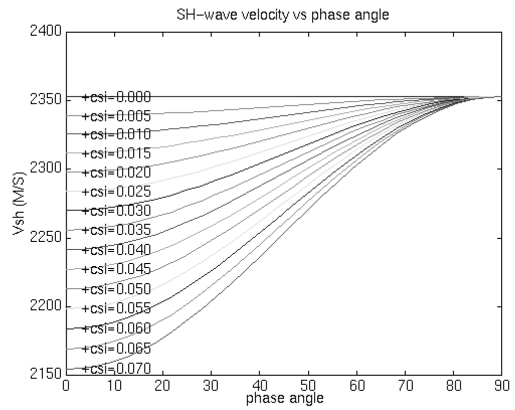



(2.a) (2.b)

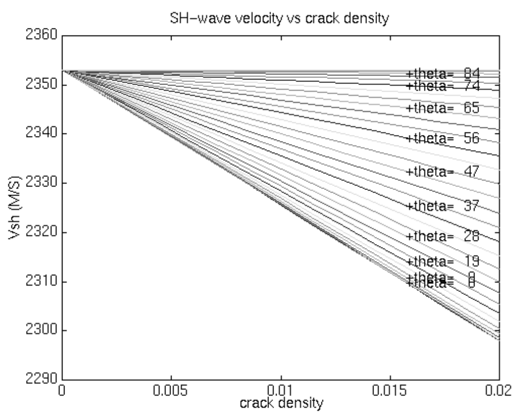
Figure 2. P-wave velocity ratio versus crack density; (a) group to phase; (b) aproximate to exact computation.




(3.a)




(3.b)

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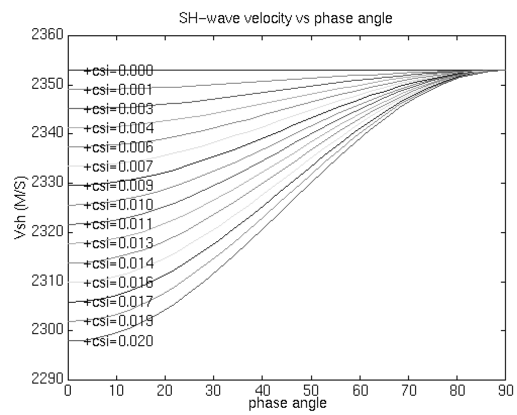


(3.d)

Figure 3. Phase velocities versus crack density and phase angle; (a) and (b) crack density if from 0. to 0.07; (c) and (d) crack density is from 0. to 0.02.

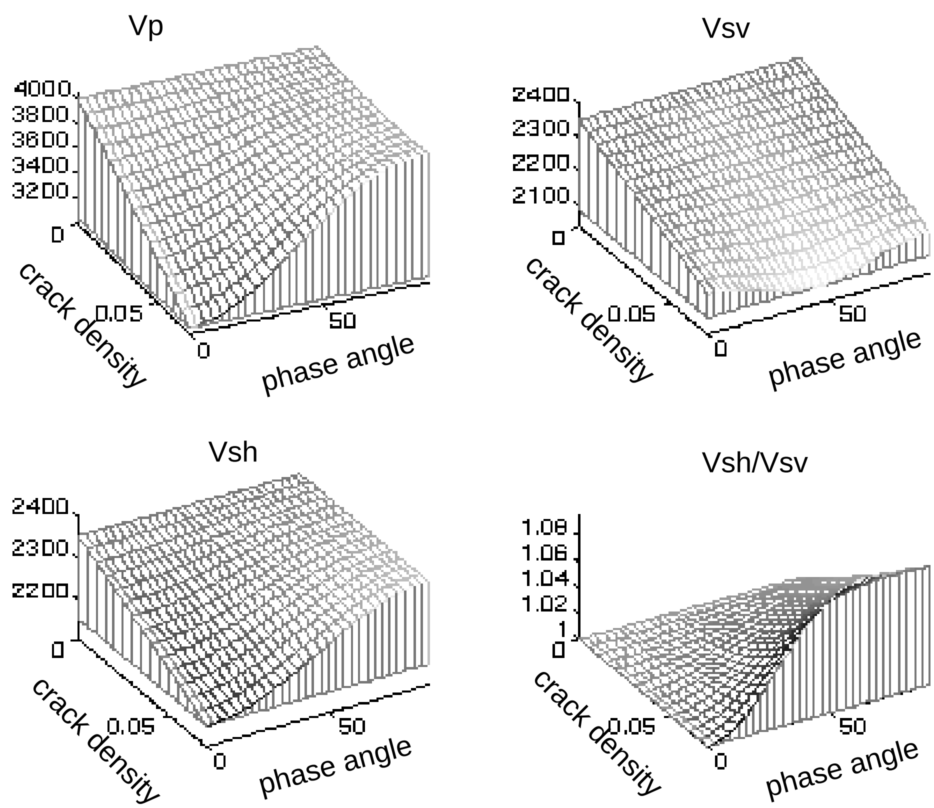


Figure 4. Phase velocities versus crack density and phase angle; (a) P-wave; (b) SV-wave; (c) SH-wave velocity; (d) ratio of $V_{sh}$ to $V_{sv}$.