

# The Plus-Minus time analysis method and its implementation

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## ABSTRACT

The Plus-Minus time analysis method, which is based on the plus-minus method of Hagedorn (1959), was developed to solve the time delays of reflection seismic data. The time delays are caused by low velocity near-surface deposits. The Plus-Minus time analysis method includes a Plus time analysis for depth analysis and a Minus time analysis for velocity determination. The method is based on the traveltimes reciprocity and the determination of the crossover point. The crossover point determination uses the traveltimes difference analysis of Lawton (1989) and the branch point analysis of Wang and Cheadle (1995). The use of all the possible overlapping shot spreads in the Plus time analysis allows overdetermination and statistical analysis. The Plus-Minus time analysis was coded in the Matlab language and tested with a synthetic dataset. The Plus-Minus time analysis method provides a reliable near-surface model with accurate depths and velocities.

## INTRODUCTION

### Purpose of the method

The Plus-Minus time analysis method has two objectives: to establish a near-surface model of the earth in terms of subweathering layer thicknesses and seismic wave velocities; and to compute static corrections to be applied to seismic data. The surface layers of the earth are often made of unconsolidated deposits or soil, which have low seismic wave velocities. For example in Western Canada the near-surface layers are generally composed of Quaternary glacial deposits (Clague, 1991), while in South America or Africa the surface layers are made of weathered soil materials. These low velocity layers will induce recording delays of seismic reflection data. The impact of these delays on the reflector seismic structures and continuities can be quite significant. The use of seismic refracted wave arrivals to establish the thicknesses of these layers and their seismic wave velocities represents one of the best ways to correct for these delays.

### Theory

The method used in this software to establish the first layer thickness is the plus-minus method of Hagedorn (1959), which includes the Plus time analysis for depth analysis and the Minus time analysis for velocity determination (van Overmeeran, 1987). The basis of the plus-minus method lies in the traveltimes reciprocity, i.e. the traveltimes of a seismic wave between two locations in one direction is equal to the traveltimes in the opposite direction. The Plus time analysis uses the concept of delay time analysis introduced by Gardner (1939, 1967) and further developed by Hawkins (1961) and Barry (1967). The Plus time analysis consists of a combination of two delay time analyses in opposite directions. In fact, the Plus Time value at a receiver inside the Plus-Minus time analysis window corresponds to the summation of the left and right delay times at the receiver (Figure 1).

To be able to use the Plus time analysis, reciprocal spread data are essential so that the forward arrival spread extends at least to the position of the reverse source ( $S_r$ ) and

the reverse arrival spread to the position of the forward source ( $S_f$ ) (Figure 1). The Plus-Minus time analysis window is defined by the two crossover points (forward spread ( $X_f$ ) and reverse spread ( $X_r$ )), which determine the limit between the first layer arrivals and the second layer arrivals.

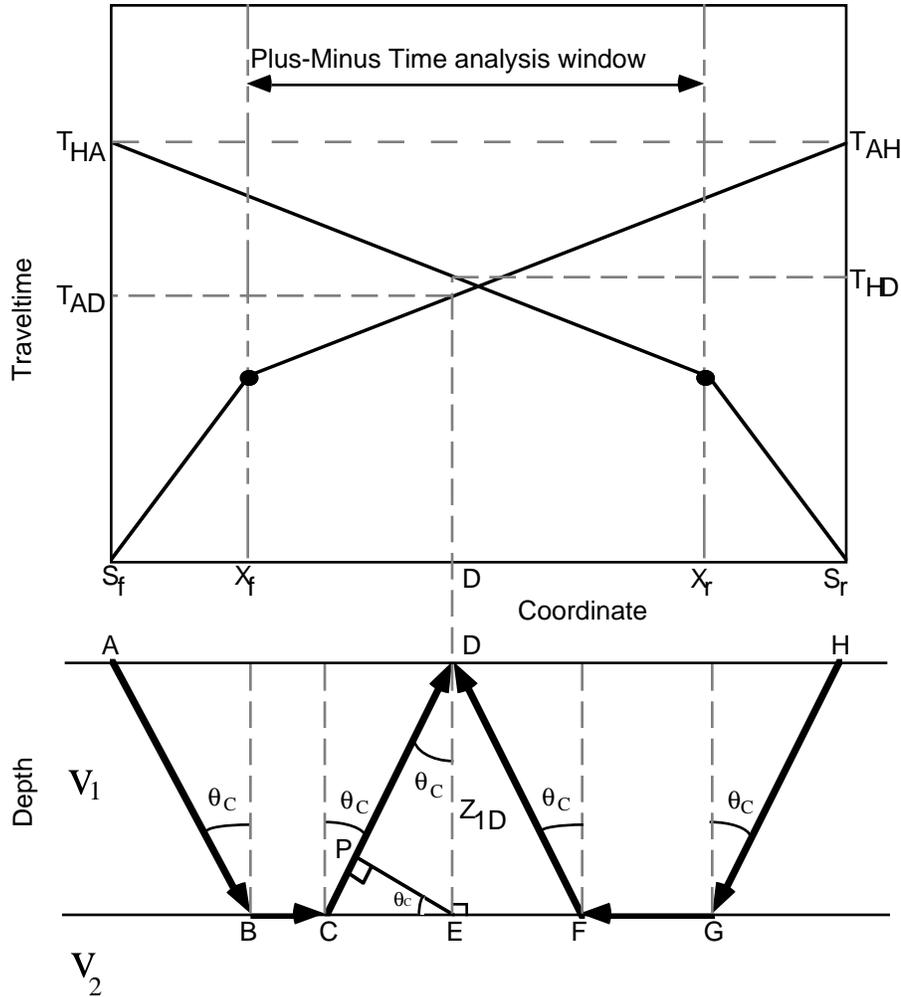


Figure 1. Plus time analysis according to the plus-minus method of Hagedorn (1959).

The Plus Time value ( $T^+$ ) can be evaluated for each of the receivers inside that window. The Plus Time value at a receiver ( $T^+D$ ) is defined as the sum of the traveltime at the receiver from a forward source ( $T_{AD}$ ) and the traveltime at the receiver from the reverse source ( $T_{HD}$ ), minus the traveltime between the two sources ( $T_{AH}$ ).

$$T^+D = T_{AD} + T_{HD} - T_{AH} \tag{1}$$

Each one of these raypath traveltimes can be replaced by smaller raypath traveltime segments:

$$T_{AD} = T_{AB} + T_{BC} + T_{CD} \tag{2}$$

$$T_{HD} = T_{HG} + T_{GF} + T_{FD} \tag{3}$$

$$T_{AH} = T_{HA} = T_{AB} + T_{BG} + T_{GH} \quad (4)$$

The raypath traveltime  $T_{BG}$  can also be replaced by smaller segments:

$$T_{BG} = T_{BC} + T_{CE} + T_{EF} + T_{FG} \quad (5)$$

The substitution of equation (5) into equation (4), and then of equations (2), (3), and (4) into equation (1), gives the following equation:

$$T^+_{D} = T_{CD} - T_{CE} + T_{FD} - T_{EF} \quad (6)$$

where  $T_{CD} - T_{CE}$  is the left delay time ( $\delta_{Dl}$ ) and  $T_{FD} - T_{EF}$  is the right delay time ( $\delta_{Dr}$ ) at receiver D. Now, if we replace the traveltime by the equivalent distance divided by the corresponding seismic velocity, equation (6) becomes:

$$T^+_{D} = CD/V_1 - CE/V_2 + FD/V_1 - EF/V_2 \quad (7)$$

where  $CD/V_1 - CE/V_2 = \delta_{Dl}$  and  $FD/V_1 - EF/V_2 = \delta_{Dr}$

The left delay time can be expressed as:

$$\delta_{Dl} = (CP + PD)/V_1 - CE/V_2 \quad (8)$$

From Snell's law, it can be shown that  $CP/V_1 = CE/V_2$ , so that the left delay time ( $\delta_{Dl}$ ) becomes  $PD/V_1$ .

From there, a link between the left delay time ( $\delta_{Dl}$ ) and the thickness of the first layer below receiver D ( $Z_{1D}$ ) can be established.

$$\delta_{Dl} = PD/V_1 = [Z_{1D} \cos(\theta_c)]/V_1 \quad (9)$$

where  $\theta_c = \sin^{-1}(V_1/V_2)$

Similarly, it can be shown that the right delay time ( $\delta_{Dr}$ ) is linked to the thickness of the first layer below receiver D ( $Z_{1D}$ )

$$\delta_{Dr} = [Z_{1D} \cos(\theta_c)]/V_1 \quad (10)$$

According to equations (9) and (10), the expression for the Plus Time ( $T^+_{D}$ ) can be expressed as:

$$T^+_{D} = 2[Z_{1D} \cos(\theta_c)]/V_1 \quad (11)$$

where  $T^+_{D}$  is equal to the sum of  $\delta_{Dr}$  and  $\delta_{Dl}$

Therefore, the thickness of the first layer at the receiver D can be found.

$$Z_{1D} = [(T^+D) \cdot (V_1)] / 2(\cos(\theta_c)) \quad (12)$$

The velocity of the first layer can be found using the inverse slope of the first layer arrivals ( $S_f$  to  $X_f$  and  $S_r$  to  $X_r$ ). From the second layer arrivals (after  $X_f$  for the forward spread and after  $X_r$  for the reverse spread), the second layer velocity can be derived using the Minus time analysis (Figure 2).

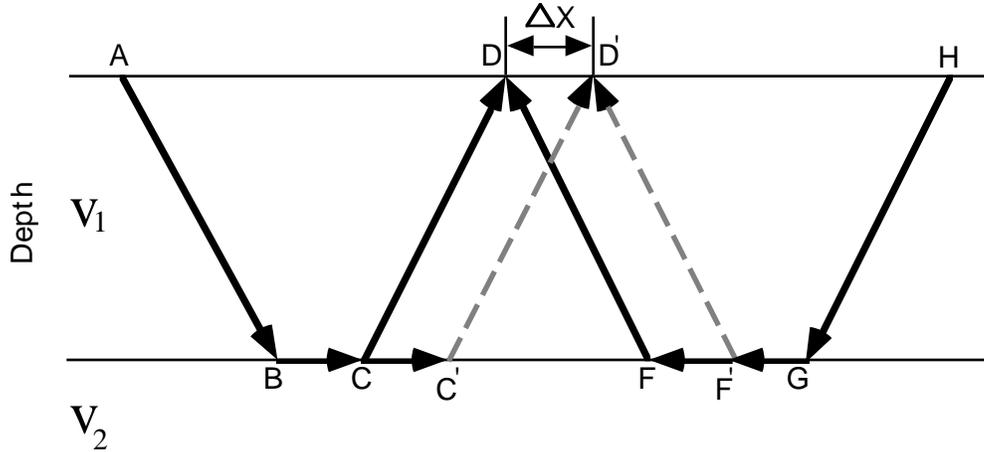


Figure 2. Minus time analysis according to the plus-minus method of Hagedorn (1959).

The definition of the Minus Time at a receiver ( $T^-D$ ) is the subtraction of the traveltime at the receiver from the reverse source ( $T_{HD}$ ) of the traveltime at the receiver from the forward source ( $T_{AD}$ ), minus the traveltime between the two sources ( $T_{AH}$ ).

$$T^-D = T_{AD} - T_{HD} - T_{AH} \quad (13)$$

A velocity analysis of the second layer can be undertaken using the Minus Time values at two receivers (D and  $D'$ ) with a specified separation distance ( $\Delta X$ ).

$$T^-D' = T_{AD'} - T_{HD'} - T_{AH} \quad (14)$$

The subtraction of equation (13) from equation (14) results in the following equation:

$$T^-D' - T^-D = T_{AD'} - T_{AD} + T_{HD} - T_{HD'} \quad (15)$$

where  $T_{AD'} - T_{AD}$  and  $T_{HD} - T_{HD'}$  are equal to  $\Delta X/V_2$

The velocity of the second layer ( $V_2$ ) is equal to twice the inverse slope of a best fit line through the Minus Time variations ( $\Delta T^-D$ ) calculated for each receivers inside the Plus-Minus time analysis window.

$$T-D' - T-D = \Delta T-D = 2(\Delta X)/V_2 \quad (16)$$

### *Summary*

Knowing the first and second layer velocities and the Plus Time values at each receiver, the thickness of the first layer below each receiver can be found according to the delay time analysis. The delay time represents the time to travel from the receiver to the refractor minus the time necessary to travel the normal projection of the raypath on the refractor (Figure 1 and equation 7). From Snell's law, a relation between the delay times (left and right) and the thickness of the first layer at the receiver can be established (equations 9 and 10). Finally, the link between the delay times and the Plus Times allows us to determine the thickness of the first layer below each receiver inside the Plus-Minus time analysis window (equation 12). This will allow the determination of the first layer thickness all along a seismic survey line, from which surface-consistent static corrections are extracted. Therefore, this analysis is restricted to a two layer case in two dimensions.

## **PROCESS**

### **General flow chart**

The Plus-Minus time analysis approach consists of seven major steps and four optional steps (Figure 3). As explained in the data file section, the refracted arrivals and the geometry files are required to properly initiate the processing. There are three final results of processing; a velocity model, a depth model, and the static corrections.

After the data are loaded, the inflection points on the refracted arrivals are identified. These are called crossover points (CVPs). These crossover points represent a change in the arrival times between the first layer arrivals and the second layer arrivals. Once the locations of the crossover points are known for each source arrival spread, the velocities of the first layer can be calculated using a polynomial fit of the direct arrivals. The second layer velocities are calculated using the Minus time analysis with the refracted arrivals.

Simultaneously, the time delays caused by the downgoing and upgoing refracted wave raypath through the first layer can be determined using Plus time and delay time analyses on the refracted arrivals. Once the Plus Times (or delay times) and the two layer velocities are known, the first layer thicknesses are computed in the depth calculation step. Finally, from the first layer thicknesses and velocities, the static corrections canceling the effect of that layer can be found using the static computation step.

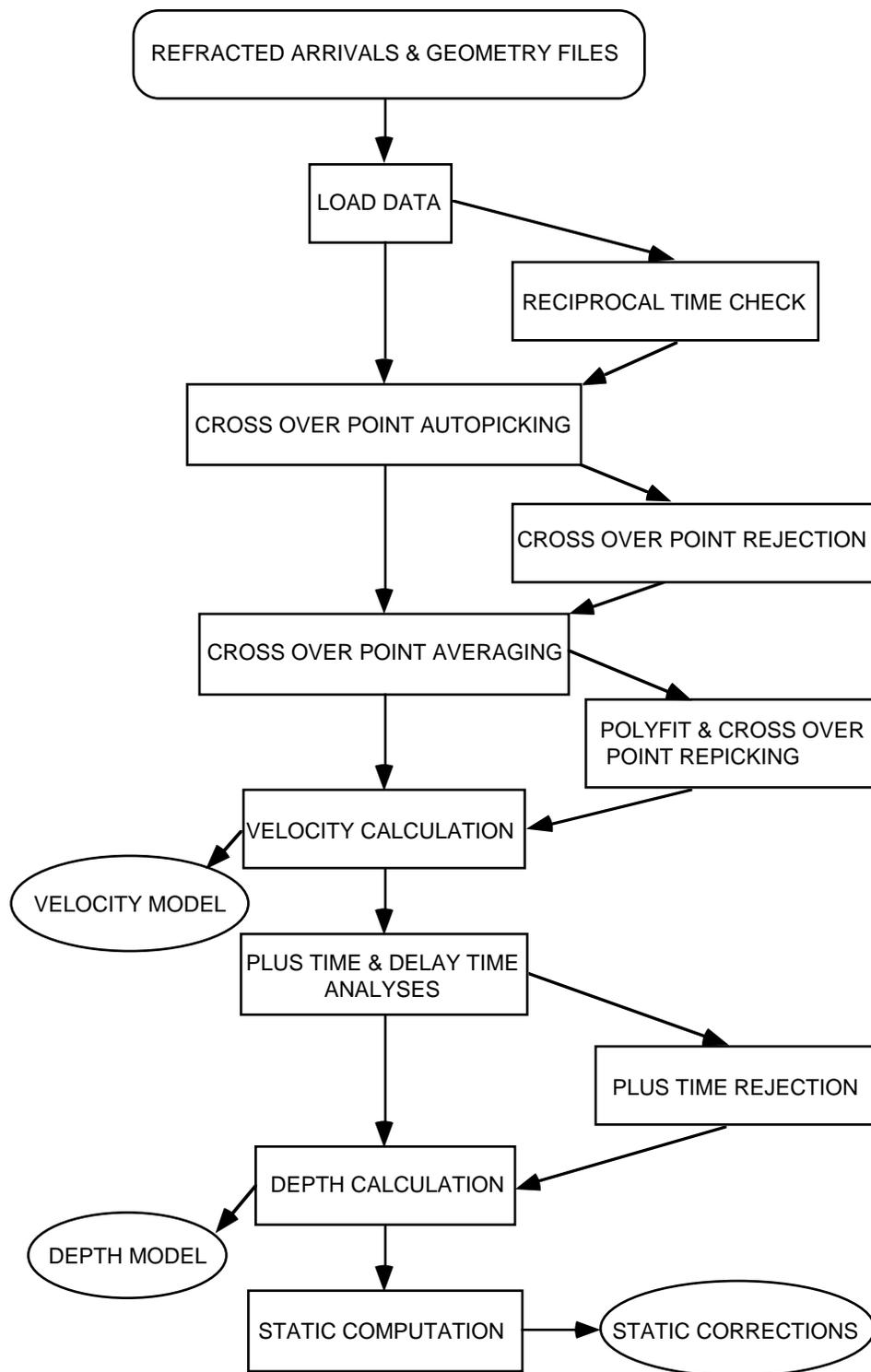


Figure 3. Processing flow chart.

### Reciprocal time check

As mentioned in the theory section, the basis of the Plus-Minus time analysis method lies in the traveltimes reciprocity, i.e. the traveltimes of a seismic wave between

two locations in one direction is equal to the traveltime in the opposite direction. For example, if seismic energy is sent from a shot and recorded at a certain distance by a receiver, the shot and receiver position may be switched, but the traveltime recorded in both cases should be equal.

The first step after loading the data is to check if the traveltime reciprocity is valid. The reciprocal time check process will determine the traveltime difference for all possible shot pairs, i.e. the two shot spreads have overlapping receivers at the pairing shot location. Figure 4 shows two generalized shot gathers where the receiver spread of the  $i$ th shot overlaps the receiver spread of the  $j$ th shot. It can be noted that the traveltime of the forward spread arrivals at the reverse shot location is equal to the traveltime of the reverse spread arrivals at the forward shot location. The subtraction of the two traveltimes equals zero and the traveltime reciprocity is correct.

With real data, however, the traveltime reciprocity is generally not respected due to the complexity of the raypath trajectory in the Earth's near-surface, and to arrival picking errors. Also, if dynamite is used, the source is often buried below the surface to variable depth. Therefore, the uphole time, which is the time taken by the seismic energy to travel back up to the surface, should be added before the reciprocity is verified. The reciprocal time check process allows the user to identify the shot pairs with reasonable reciprocal traveltime differences and the problematic shot pairs. There are several possibilities to solve the problem of shot pairs with significant reciprocal traveltime differences: first a review and correction of the arrival picking errors in the processing software used to create them; second the problematic shot pairs can be ignored in the subsequent process; third the reciprocal traveltime difference of a problematic shot pair can be used to adjust the arrival traveltimes of one of the shots according to the pairing shot (bulk shift).

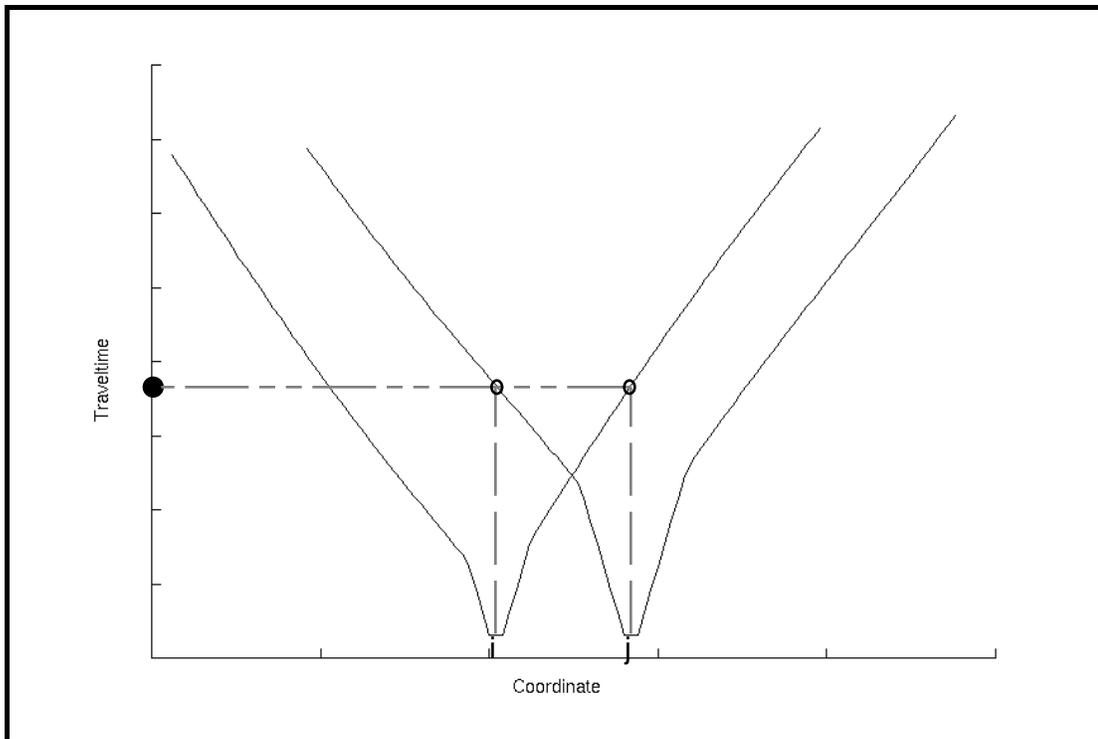


Figure 4. Reciprocal time check with shot gathers  $i$  and  $j$ .

### Crossover point autopicking

Crossover point picking represents the most important step of this program. The crossover points indicate a change in the arrivals between the first and second layers. At the crossover point, the traveltime of the second layer arrivals is equal to the traveltime of the first layer arrivals. Beyond the crossover point, the refracted arrivals from the second layer are recorded before the first layer arrivals. The first layer arrivals (also called direct arrivals) are the traveltimes of a seismic wave from the source to the receivers along the surface of the first layer. The second layer arrivals (also called refracted arrivals) are the traveltimes of a seismic wave from the source to the bottom of the first layer then along the first and second layer interface and back up through the first layer to the receivers (Figure 5).

The position of the crossover points on each shot spread is found using the traveltime difference analysis developed by Lawton (1989) and the branch point analysis of Wang and Cheadle (1995). The traveltime difference analysis consists of subtracting two adjacent shot arrival traveltimes at common receiver locations (Figure 6). The traveltime differences ( $\delta t$ ) remain constant when the arrivals from the two shots involve the same layer. The crossover point is located at the limit between a constant traveltime difference and an increasing or decreasing traveltime difference. For example, if we subtract the traveltime of two overlapping shot gathers, it can be noticed that the traveltime difference on the left of shot  $i$  location is constant until it reaches the left crossover point of the shot gather  $i$  and then the traveltime difference increases (Figure 6). On the other side (right of shot  $j$  location), the traveltime difference decreases until it reaches the right crossover point of shot gather  $j$  and then remains constant. Then, if we calculate the first and second derivatives of the traveltime differences, the crossover point locations correspond to the maximum value of the second derivative. This result is due to a maximum change of slope on the traveltime differences at the first and second layer inflection point and allows automatic crossover point location picking for each overlapping shot pair spread. The use of all possible overlapping shot pairs allows not only the identification of the crossover point (left and/or right) on each shot gather but also a statistically redundant crossover point location on the same shot spread (left or right).

Another important feature of the traveltime difference analysis is the cancellation of traveltime irregularities due to topographic features. However, in the real data case, traveltime irregularities can also be due to the complexity of the near-surface or to errors in the arrival picking and other instrumental factors, so that filtering is essential. A median filter is first applied to the traveltime difference to remove high frequency variation and keep the general trend of the traveltime difference, especially the step trend at the crossover point. Then after the first derivative is calculated a mean filter can be applied to smooth the short wavelength variations due to small steps in the median filtered traveltime difference curve, and keep the long wavelength variation due to major steps associated with the crossover point locations. The mean filter should be used with care and only if the first derivative curve appears noisy. The window length of these filters is determined by the user and should be tested on each dataset. Also, the first and second derivatives uses a parameter called differentiation separation length, which can be defined by the user. This parameter determines the number of samples in between the differentiation. For example if one is used the differentiation of the traveltime difference curve corresponds to its simple derivative, i.e. the sample interval is one.

$$\Delta t/\Delta x = [t(x + n\Delta s) - t(x)]/n\Delta s \quad (17)$$

where  $x$  and  $\Delta x$  are respectively the coordinate of the arrival and the coordinate variation;  $t$  and  $\Delta t$  are respectively the time of the arrival and the time variation; and  $\Delta s$  is the distance of a sample interval, while  $n$  is the differentiation separation length factor.

The window filter length and the differentiation separation length need to be tested on each different dataset to optimize the crossover point autopicking consistency. This will reduce the amount of manual editing of the crossover points.

Finally, the user can specify an offset range in which the crossover points can be picked. This feature should be used only if the user has sufficient knowledge of the data (near-surface geology, previous identification of some crossover points throughout the entire survey) and can confine the crossover points within a certain offset range from the shot locations. It can also be used if third layer arrivals are suspected at a certain offset, so that the crossover points might be divided between the first and second layer inflections and the second and third layer inflections. In both cases, the offset range limitation will enable consistent and logical crossover points picking and will save a lot of manual editing.

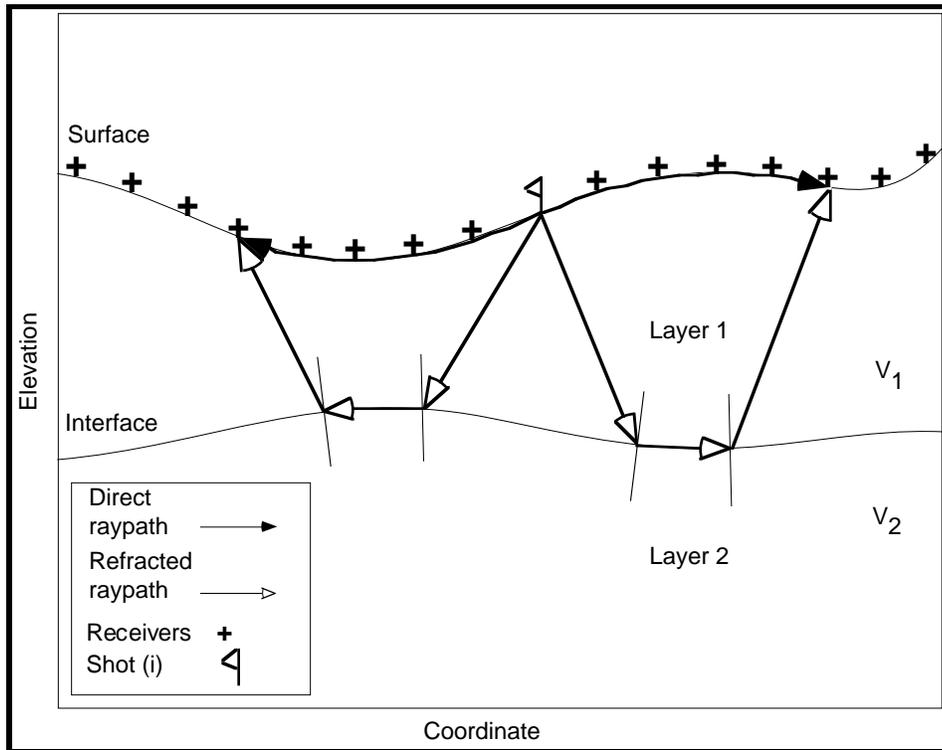


Figure 5. Direct and refracted raypath for shot gather i.

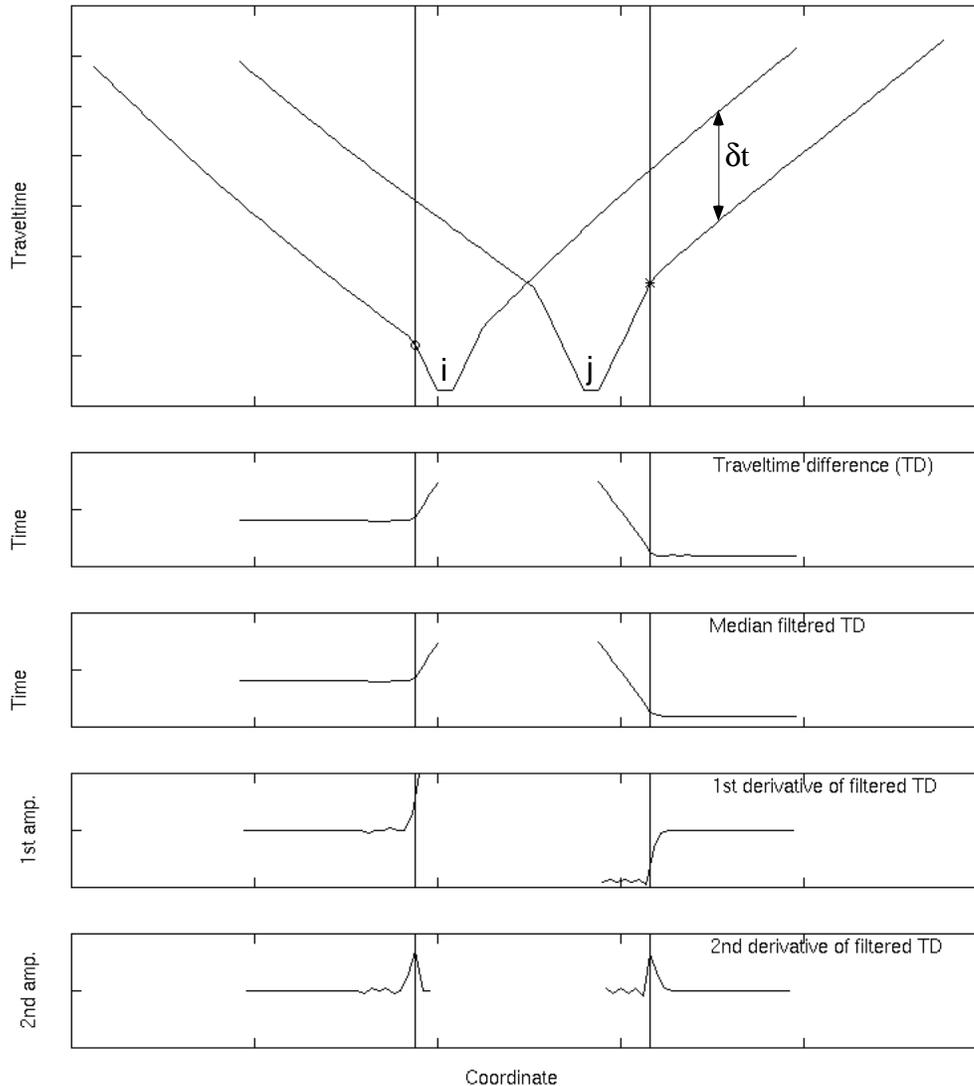


Figure 6. Crossover point autopicking of the left crossover point for shot gathers i (o) and of the right crossover point for shot gather j (\*), according to the traveltime difference analysis by Lawton (1989) and the branch point analysis of Wang and Cheadle (1995).

### Crossover point rejection

This optional process is based on the statistics of the crossover points previously picked. As mentioned in the above section, the use of all possible overlapping shot pairs brings an over determination of the crossover point on a shot spread. The next step is to average all the crossover points that belong to the same shot spread (forward or reverse) and use that value in the subsequent process. However, the traveltime subtraction of each possible overlapping shot spread with a common shot spread will result in a different traveltime difference, which can result in different crossover point locations. However, if the arrivals are consistent and not noisy, the crossover point picks for a shot spread side will be at the same surface location with any overlapping shot spreads. The utility of the crossover point rejection is to identify and remove the crossover point picks for the same shot spread that have a high deviation from the average crossover point value.

The rejection can be based on a factor of the standard deviation of the crossover point picks for each shot spread or on a constant deviation limit. The difference between the two rejection limits is that the factor of the standard deviation limit changes for each crossover point shot spread (right or left) and the constant deviation limit remains constant for all crossover point shot spreads. In both cases, the use of the crossover point statistics, which includes the standard deviation, the fold and the value of the crossover point averages, is recommended to determine the rejection limit. The user should look not only at the average crossover point values, but at the standard deviation and at the fold of each crossover point average. The combination of a high standard deviation with a low fold indicates a crossover point autopicking problem and manual editing might be required, while a low standard deviation with either a high or low fold generally indicates a successful crossover point autopicking. In the case of a high standard deviation and a high fold, the crossover point autopicking had some problem, and the crossover point rejection might help remove some bad crossover picks. One thing to keep in mind is that each time crossover point rejection is used, the redundancy (fold) of the crossover point averages is reduced.

**Crossover point averaging**

The next major step after the crossover point autopicking is the averaging of all the crossover points for a shot spread (left and/or right). For a shot with a leading (right) and a trailing (left) spread, two individual averaging processes are required; one for the left crossover points and one for the right crossover points. The left crossover points of shot gather *i* comes from the traveltime difference analysis of its left (trailing) spread with all overlapping left shot spreads to its right (Figure 7). The right crossover points of shot gather *i* come from the traveltime analysis of its right (leading) spread with all the overlapping right shot spreads to its left (Figure 8). The proper crossover point positioning is important for the velocity calculation and the Plus time analysis. The crossover points can be edited individually before the crossover point averages are computed (see editing section). The crossover point averages can also be edited.

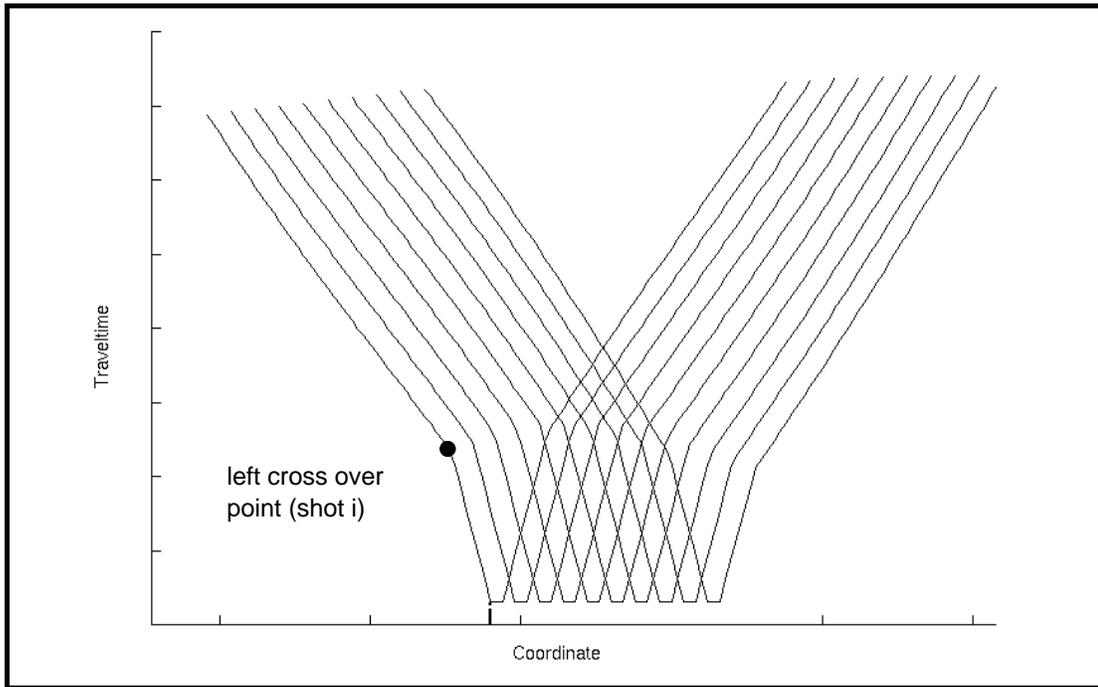


Figure 7. Left crossover point determination for shot gather *i*, established from the overlapping shot spread to the right .

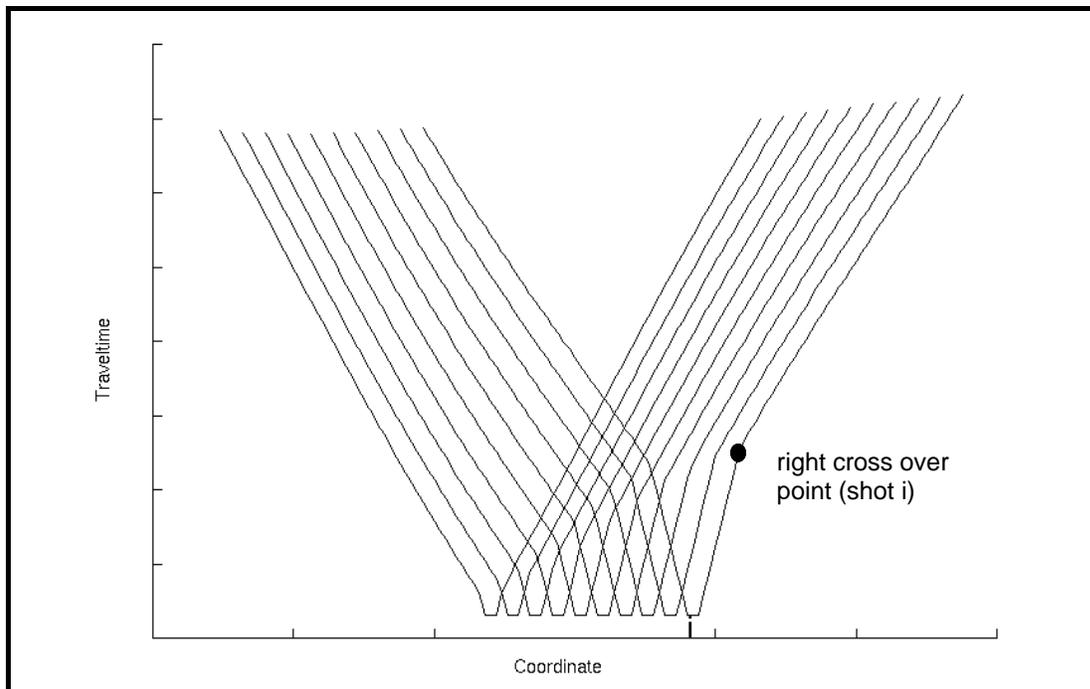


Figure 8. Right crossover point determination for shot gather *i*, established from the overlapping shot spread to the left.

### Polyfit and crossover point repicking

This optional process has the objective of obtaining a better crossover point consistency between shot spreads. In fact, the offset distance of the crossover point average of a shot spread from the shot location should not be too different than the offset distance of the crossover point average on the next shot spread from its location. The offset distances between the crossover point averages and their shot locations reveal the general shape of the first layer thickness. Generally, the offset varies proportionally with the variation of the first layer thickness and illustrates its long wavelength shape. Now abrupt crossover point average offset changes from adjacent shot spreads, depending on the distance between their shot locations, might indicate a crossover point picking problem. The user can solve the problem by editing each crossover point that enters into the crossover point average location or edit directly the crossover point average location (see editing section). Another way to resolve this problem is to fit a polynomial curve through the crossover point average offset values and repick all the crossover points entering in the average that sits outside a certain deviation from the polynomial curve. Two parameters are defined by the user: the first one is the polynomial degree for the polynomial curve (polyfit curve) and the second one is the deviation limit from the polyfit curve. The polynomial degree determines the amount of complexity in the polyfit curve, i.e. the higher the polynomial degree, the more complex the polyfit curve can be. This parameter should be tested first until the polyfit curve follows the general trend of the crossover point average offsets.

The deviation limit parameter is specified according to the tolerated distance between the crossover point average offsets and the polyfit curve. The deviation limit is determined according to the user interpretation. However, the user should know that the crossover point repicking process will be constrained within an offset range, which is the polyfit curve plus the deviation limit, and the polyfit curve minus the deviation limit. For a small deviation limit, the constraint on the crossover point repicking is important, so that the crossover point average offset curve will closely follow the

polyfit curve. The user should be concerned with influencing the crossover point repicking process too much, unless he or she has a sufficient knowledge of the data.

### **Velocity calculation**

Once the crossover point average locations are known, then the velocities of the first and second layer can be calculated. A polynomial fit of the direct arrivals enables the determination of the first layer velocity, while the Minus time analysis of the refracted arrivals will establish the second layer velocity. For a given shot gather, with a leading and a trailing spread, the velocity of the first layer at the shot location is given by the inverse slope of the two direct arrival segments (leading and trailing). The direct arrivals segments are limited by the shot location and the crossover point locations (Figure 9). The slope of the direct arrivals is found using a curve best fit of them (first degree polynomial fit). Then, the first layer velocity at the shot location corresponds to the average inverse slope of the two direct arrival segments. Finally, the first layer velocity values are interpolated to each surface station location.

The second layer velocity is found using Minus time analysis on the refracted arrivals within the Plus-Minus time analysis window (Figure 9 and equations 15 and 16). The Minus Time values are calculated for each receiver location inside the Plus-Minus time analysis window. Then the Minus Time variations ( $\Delta T_D$ ) are displayed as a function of twice the receiver separation ( $2\Delta x$ ) (Figure 10).

Finally, a first degree polynomial curve is fit through the Minus Time variation values, where the inverse slope of it corresponds to the second layer velocity. The second layer velocity value belongs at the half-way location in the Plus-Minus time analysis window, which is defined by the right crossover point of the shot gather  $i$  and the left crossoverpoint of the shot gather  $j$ . After all the possible Plus-Minus time analysis windows (different shot pairs) have been used, an average of all the second layer velocity values at the same location is calculated followed by an interpolation to each receiver location. Both first and second layer velocities can then be edited manually and median filtered to remove the high frequency variation and any bad velocity values (see editing section).

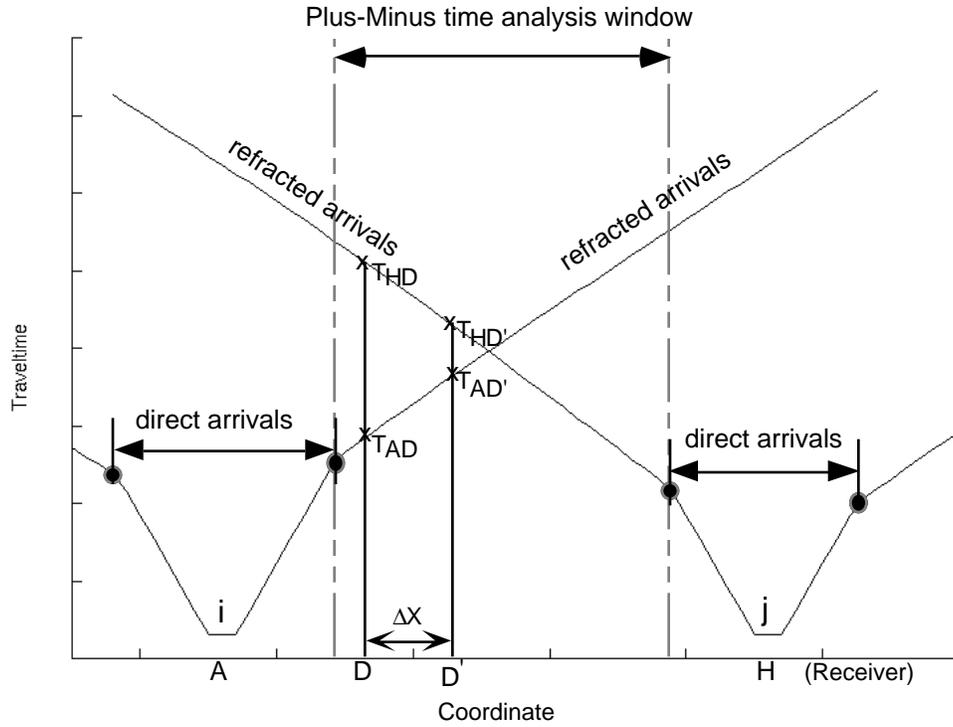


Figure 9. Shot gathers i and j with their left and right crossover points, and the Plus-Minus time analysis window .

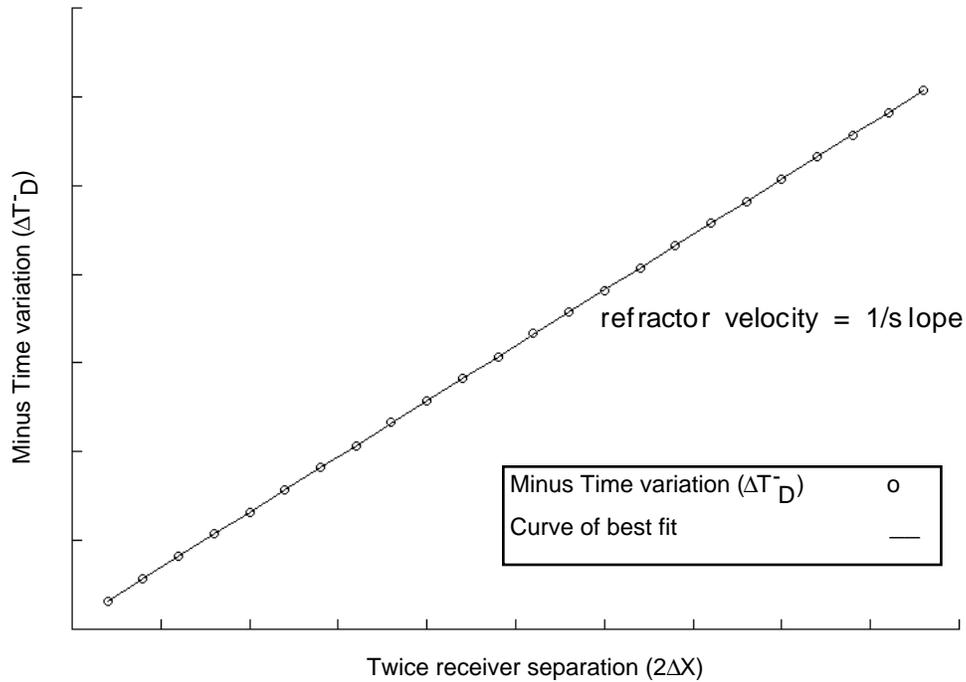


Figure 10. Minus Time variation displayed as a function of twice the receiver separation ( $2\Delta x$ ) and best fit curve.

### **Plus time and delay time analyses**

The Plus time analysis process, like the velocity calculation, requires the determination of the crossover point average locations, and can be run before or after the velocity calculation. As explained in the theory section, the Plus time analysis uses the delay time analysis concept to calculate the thickness of the first layer below each receiver. This process calculates the Plus Time values that are going to be used for the first layer thickness computation in the depth calculation process. The Plus time analysis computes the Plus Time values for each receiver within all the possible Plus-Minus time analysis windows (use of different shot pairs). All the Plus Time values are stored under the associated receiver locations. The use of all the possible Plus-Minus time analysis windows allows a redundant determination of the Plus Time values at each receiver location. This is important to reduce the effect of noisy data and bad arrival picks. Some of the Plus Time values can be rejected according to their deviation from the average Plus Time value at a receiver location (see Plus Time rejection section). As mentioned previously, the Plus time analysis process needs the crossover point average locations. Some of the crossover point averages may be missing, because they have been deleted or not computed. In this case, they need to be interpolated from the other crossover point average values or the shot spread without a crossover point average value will not be used in the Plus time analysis process. Also, if the presence of third layer arrivals is known, these arrivals can be excluded from the Plus time analysis computation by simply limiting the offset range.

The Plus time analysis cannot be used to determine the Plus Time values for the receivers outside the Plus-Minus time analysis window limit (Figure 11). Therefore, the delay time analysis is computed for these receivers (Figure 12) (Lawton, 1989). The delay time value at a receiver can be expressed as:

$$\delta_r = T_r - \delta_s - X/V_2 \quad (18)$$

where  $T_r$  is the refracted travelttime at the receiver,  $\delta_s$  is the shot delay time,  $X$  is the distance between the shot and the receiver, and  $V_2$  is the second layer velocity.

The shot delay time ( $\delta_s$ ) is equivalent to half the Plus Time value at that surface location and the receiver delay time can be multiplied by two to obtain the Plus Time value at this surface location. So, the Plus time analysis has to be computed prior to the use of the delay time analysis. Also, the second layer velocity has to be already calculated or known. The use of the delaytime analysis in conjunction with the Plus time analysis allows the determination of the Plus Time values at each surface location.

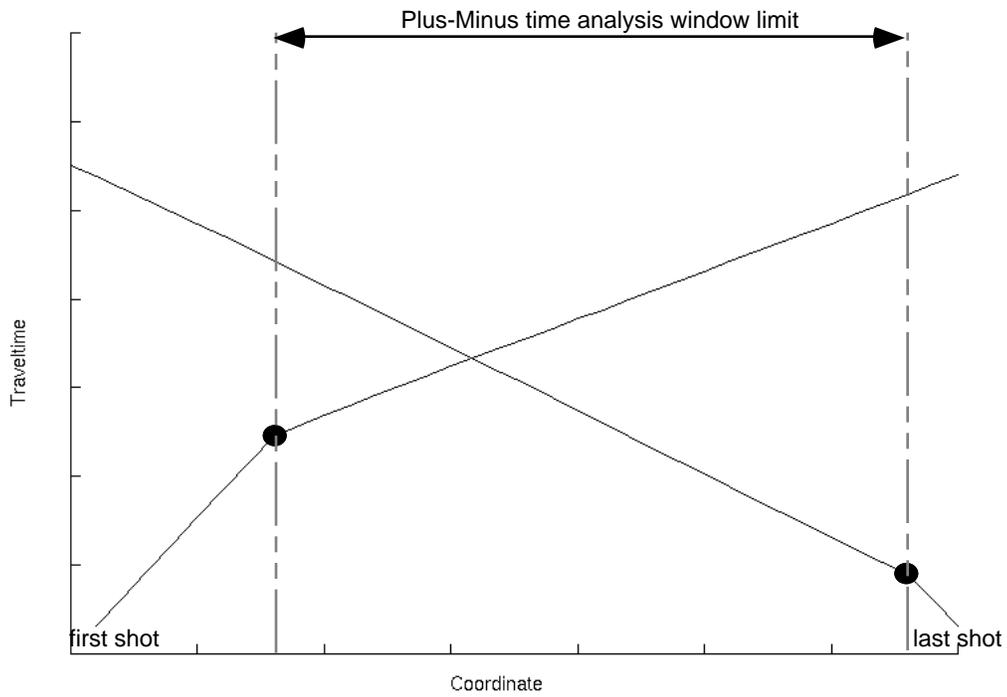


Figure 11. Plus time analysis limitation.

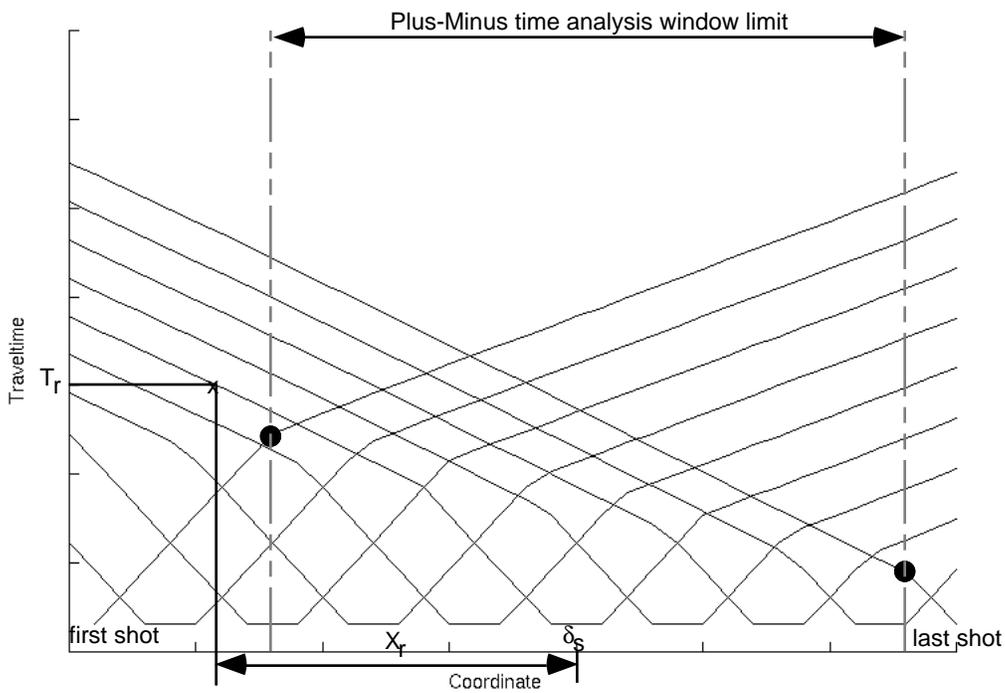


Figure 12. Delay time analysis for receiver locations outside the Plus-Minus time analysis window limit.

**Plus Time rejection**

The Plus Time rejection process is similar to the crossover point rejection in terms of the statistical rejection concepts. The rejection can be done according to a constant deviation limit or according to a factor of the standard deviation limit. The constant

deviation limit remains unchanged for all the receiver locations, while the standard deviation changes for each receiver location. At each receiver location, the Plus Time average value, the standard deviation and the fold are known and can be examined. The Plus Time average curve should have a reasonable long wavelength trend combined with short wavelength variations. If a Plus Time average value is completely out of the range of the other Plus Time average values and the standard deviation for this Plus Time average value is high, it reveals that there are some bad Plus Time values entering into the Plus Time averaging at this receiver location. In general, high standard deviation indicates the presence of bad Plus Time values, which can be rejected using the Plus Time rejection process. Again, like the crossover point rejection, the Plus Time rejection process will reduce the redundancy (fold) of the Plus Time averages. The Plus Time rejection process should be used before the delay time analysis computation.

### **Depth calculation**

After the Plus time analysis process is completed, the Plus Time average values at each receiver are calculated. At this stage, the first and second layer velocity at each receiver as well as the Plus Time average should be known. The calculation of the first layer thickness below each receiver is based on the delay time analysis concept (equations 9 and 10). Before the depths are calculated, the user has to make sure that the Plus Time average values are consistent with the interpretation and the two layer velocities are reasonable, i.e. no noisy short wavelength variations and the first layer velocity is not greater than the second layer velocity. The Plus Time average values and the first and second layer velocities are needed to evaluate the thickness of the first layer below each receiver (Figure 13 and equation 19). The first layer thickness can be manually edited or median filtered before the static corrections are computed to reduce the high frequency noise (see editing section).

$$Z_1 = [(T^+)*(V_1)]/2(\cos(\theta_c)) \quad (19)$$

$$\text{where } \theta_c = \sin^{-1}(V_1/V_2)$$

### **Static computation**

The final process computes the static corrections to be applied to the seismic data. The corrections are for the weathering layer (first layer) and for the surface elevation variation (topography). The weathering correction will cancel the time delays produced by the first low velocity layer. The elevation correction will bring the shots and the receivers to a common flat datum. The static computation process is based on the depth and velocity model and will calculate the receiver static corrections and shot static corrections based on surface consistent static corrections. There are three type of static corrections, the weathering correction, the elevation correction and the total correction, where the later is the summation of the two other corrections.

The weathering correction corresponds to the time needed to bring the receiver or the shot (according to the surface station) to the base of the first layer using the thickness and the velocity of the first layer (Figure 13).

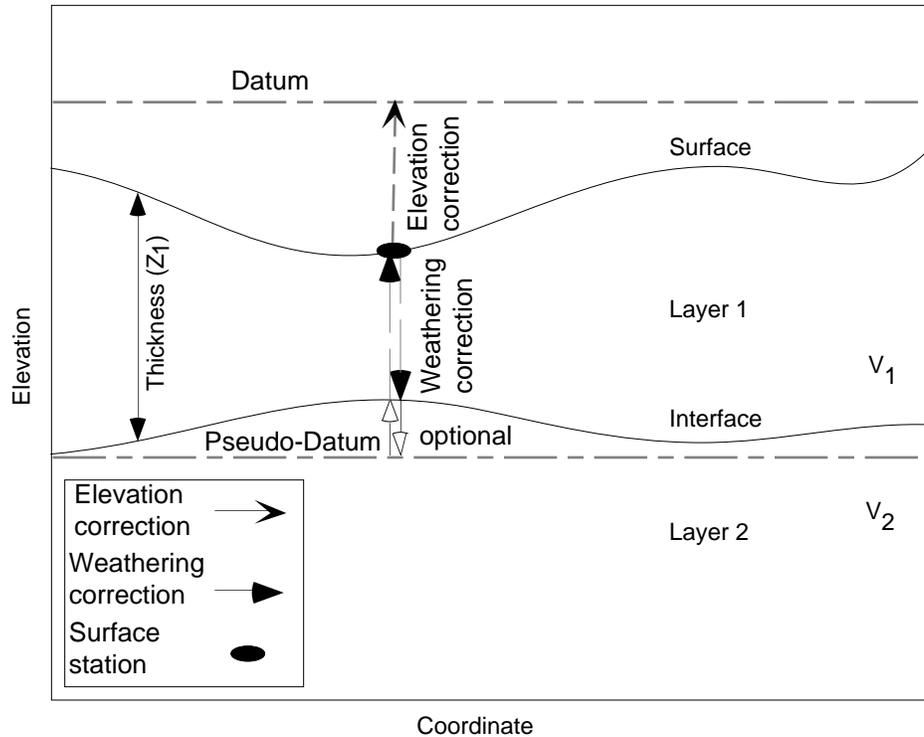


Figure 13. Surface consistent static corrections including elevation and weathering static corrections for a 2 layer model.

If the shot is buried below the surface, the uphole time is used to bring the shot back to the surface (before the weathering correction is applied). Then, the shot or the receiver is brought back to the surface using the replacement velocity. The replacement velocity is normally choose in function of the second layer velocity, either and the second layer velocity itself or an average of it.

*Weathering correction*

The weathering static correction for a shot or a receiver can be expressed as follow:

$$\text{Weathering static correction} = -Z_1/V_1 + Z_1/V_{rep} \text{ (+ uphole time, if necessary)} \quad (20)$$

where  $Z_1$  is the first layer thickness,  $V_1$  is the first layer velocity, and  $V_{rep}$  is the replacement velocity used

Optionally, a pseudo-datum can be used as an intermediate step after the shot or the receiver has been taken to the base of the first layer (Figure 13). The pseudo-datum corresponds to the deepest first and second layer interface elevation. The shot or the receiver is taken to the pseudo-datum using the second layer velocity (equation 21).

$$\text{Weathering static correction} = -Z_1/V_1 - (\text{Int} - \text{Psd})/V_2 + (\text{Surf} - \text{Psd})/V_{rep} \quad (21)$$

where Int is the first and second layer interface, Psd is the pseudo-datum elevation, and Surf is the surface elevation

### *Elevation correction*

The elevation correction corresponds to the time needed to bring back the receiver or the shot from the surface to a flat datum using the replacement velocity (both flat datum and replacement velocity are user defined).

$$\text{Elevation static correction} = (\text{Dat} - \text{Surf})/V_{\text{rep}} \quad (22)$$

where Dat is the final flat datum

### *Total correction*

Finally, the total correction is a summation of the weathering and elevation static corrections. The total static correction for a shot or a receiver can be expressed as follow:

$$\text{Total static correction} = \text{weathering static correction} + \text{elevation static correction} \quad (23)$$

$$\text{Total static correction} = -Z_1/V_1 + Z_1/V_{\text{rep}} + (\text{Dat} - \text{Surf})/V_{\text{rep}} \quad (24)$$

If a pseudo-datum is used, equation (24) becomes:

$$\text{Total correction} = -Z_1/V_1 - (\text{Int} - \text{Psd})/V_2 + (\text{Surf} - \text{Psd})/V_{\text{rep}} + (\text{Dat} - \text{Surf})/V_{\text{rep}} \quad (25)$$

Which can then be simplified as:

$$\text{Total correction} = -Z_1/V_1 - (\text{Int} - \text{Psd})/V_2 + (\text{Dat} - \text{Psd})/V_{\text{rep}} \quad (26)$$

The advantage of using a pseudo-datum is that the relative total static correction will not be dependent on the choice of replacement velocity. A change in the replacement velocity will result only in a bulk shift of the total static corrections.

## **IMPLEMENTATION**

### **Program overview**

The Plus-Minus time analysis method has been implemented as a interactive, menu driven application in the Matlab computing environment. The program, called PMT, takes at its input first break picks and geometry from a 2D seismic survey. PMT is designed so that the user is guided throught the processing flow shown in figure 3. In addition, to the automated processing, manual, interactive editing may be performed on the following intermediate results: crossover points, crossover point averages, the velocity model, and the depth model. When satisfactory depth and velocity models are built, the user may output the depth and velocity models and the static corrections. At any point in the processing flow, the project may be saved to disk.

### **Hardware / software requirement**

The Plus-Minus time analysis software is written in the Matlab language. Matlab is a commercial package which provides a technical computing and visualization environment. A Matlab license is required to run the Plus-Minus time analysis software. Currently, arrival times and geometry are input from ProMax exported database files. The refracted arrival times are previously picked in the processing software (currently ProMax) and then input to the Plus-Minus time analysis software.

The geometry files include the receiver coordinates and elevations, and the shot coordinates and elevations, as well as the uphole time (see data files section). The software can be run on any platform for which Matlab is available, including UNIX, MS-Windows, and Macintosh.

**Data files**

*Files required for import*

The Plus-Minus time analysis method requires the refracted arrival traveltimes and survey geometry. The import of this information is currently supported using ProMax ASCII database export files.

All the necessary information is in the ProMAX database under the trace order (TRC), surface station (SRF) and source index number (SIN) (Table 1). The picked refracted arrivals are found in the database under trace order with the name given when saved (GEOMETRY FB\_PICK) (Table 2). Then go to ASCII/SAVE and save the files with a specified name and the extension ".a\_db".

The same process is repeated for all the geometry files. The receiver coordinates and elevations should be found in the database under surface station (SRF) as receiver x coordinates (GEOMETRY X\_COORD) and receiver y-coordinates (GEOMETRY Y\_COORD), and as receiver elevation (GEOMETRY ELEV) (Table 3). The source elevations and coordinates can be found under source index number (SIN) as source x-coordinates (GEOMETRY X\_COORD) and source y-coordinates (GEOMETRY Y\_COORD), and as source elevation (GEOMETRY ELEV) (Table 4).

If available, the uphole time file is found under source index number (SIN) as source uphole time (GEOMETRY UPHOLE). Finally to create a link between the refracted arrivals and the surface station and the source index number, two more files must be exported. These are found under trace number (TRC) as corresponding surface station for each trace number (GEOMETRY SRF) and as trace numbers for each source index number (GEOMETRY SIN) (Table 2).

Areas	Lines	Orders
Name	Name	TRC SRF SIN

Table 1. Dataset line Orders.

Orders	Attributes
TRC	GEOMETRY SIN GEOMETRY SRF GEOMETRY FB_PICK

Table 2. Trace order attributes.

Orders	Attributes
SRF	GEOMETRY X_COORD GEOMETRY Y_COORD GEOMETRY ELEV

Table 3. Surface Station order attributes.

Orders	Attributes
SIN	GEOMETRY X_COORD GEOMETRY Y_COORD GEOMETRY ELEV GEOMETRY UPHOLE

Table 4. Source Index Number order attributes.

*Exporting files from PMT*

The output of the PMT method analysis is a depth model, a velocity model and static corrections. The depth model contains the interface elevation between the first and second layer at each receiver location (surface station) (Table 5). This model is exported into a file with the extension “.dpt”. The velocity model contains the wave velocity of the first and second layer at each receiver location (Table 6). This model is exported into a file with the extension “.vel”. The static corrections contains the source and receiver statics (Table 7). Each of these static corrections include the weathering correction, the elevation correction and the total correction at the corresponding coordinate location. The exported static files have the extension ".sta" added. All the output files are in ASCII format and may be read by most processing software packages.

Receiver #	Coordinate	Surface elevation	Layer1-Layer2 interface elevation	Layer1 thickness
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Table 5. Depth model.

Receiver #	Coordinate	Layer1 velocity	Layer2 velocity
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Table 6. Velocity model.

Receiver #	Coordinate	Weathering	Elevation	Total
Shot #	Coordinate	Weathering	Elevation	Total

Table 7. Static corrections.

**Interactive editing**

During the processing, the following intermediate results may be manually edited. Many of the processing steps use parameterized algorithms which may not produce optimal results with noisy data.

*Crossover points*

If there is a second near-surface layer there will be two crossover points, and the crossover point autopicking routine will sometimes pick the deeper one. This can usually be controlled by specifying the maximum offset during the autopicking process.

The crossover point edit module allows the user to modify each individual crossover point location manually. To enable the "crossover point editing", the "crossover point autopicking process" must be completed. Crossover point editing uses two overlapping shot gathers and the corresponding crossover point locations, i.e. the left cross over point of the left shot and the right cross over point of the right shot. To help edit the crossover point, the arrival picks and the traveltimes difference with the median filtered equivalent, as well as the first and second derivative of the filtered traveltimes difference are displayed in different windows (Figure 6). If the crossover point

locations have not been edited before, their locations should correspond to the maximum of the second derivative curves. The options are to move the crossover point locations, to delete the crossover points or to add a crossover point. Now, only one cross over point is allowed on each traveltime difference segment curve and should belong to the smaller traveltime arrival spread. Mouse button one is used to move a cross over point by clicking anywhere inside the valid traveltime difference segment in any of the five curve windows.

### *Crossover point averages*

The crossover point average editing module can be used after the crossover points have been picked and averaged. All the crossover points (CVPs) at a given location are averaged in the crossover averaging module. The CVP average editing module allows for the precise control of this averaging process. This editing module looks at one shot gather (left and/or right spread(s)) and at the corresponding left and/or right crossover point average(s). Figure 14 shows, in the top right corner, the arrivals with the crossover point averages. Below the arrivals window, all the possible traveltime difference (TD) curves with the shot gather are being aligned to their flat portions and then stacked. Stacking the TD curves results in noise reduction. Then, the median filtered equivalent of the stacked TD curves as well as the first and second derivatives are shown. Now the locations of the maximum second derivative might not correspond to locations of the crossover point averages due to some remaining bad crossover point picks in the averages or to some stacking problem and noise in the data. The main point of this editing function is to let the user interpret which location for the crossover point average is more valid.

The stacking process of all the traveltime difference curves should reduce the level of noise in the data by canceling the inconsistent traveltime differences due to bad arrival picks. The process of crossover point averaging and rejection is also statistically reliable, so that the final decision on crossover point location should be made as an interpretation by the user.

Crossover point rejection can be accomplished interactively in this function or in the crossover point rejection process. The advantage of doing the crossover point rejection in this function is that the standard deviation limit can be modified and the resulting crossover point average observed interactively. Also, instead of stacking all the possible TD curve, the user can constrain the range of the overlapping shots that are going to be used. Now the cross over point averages saved in the database can be moved to the maximums of the second derivative of the stack traveltime difference (STD) or to the crossover point average locations according to the rejection or anywhere else along the traveltime difference segments.

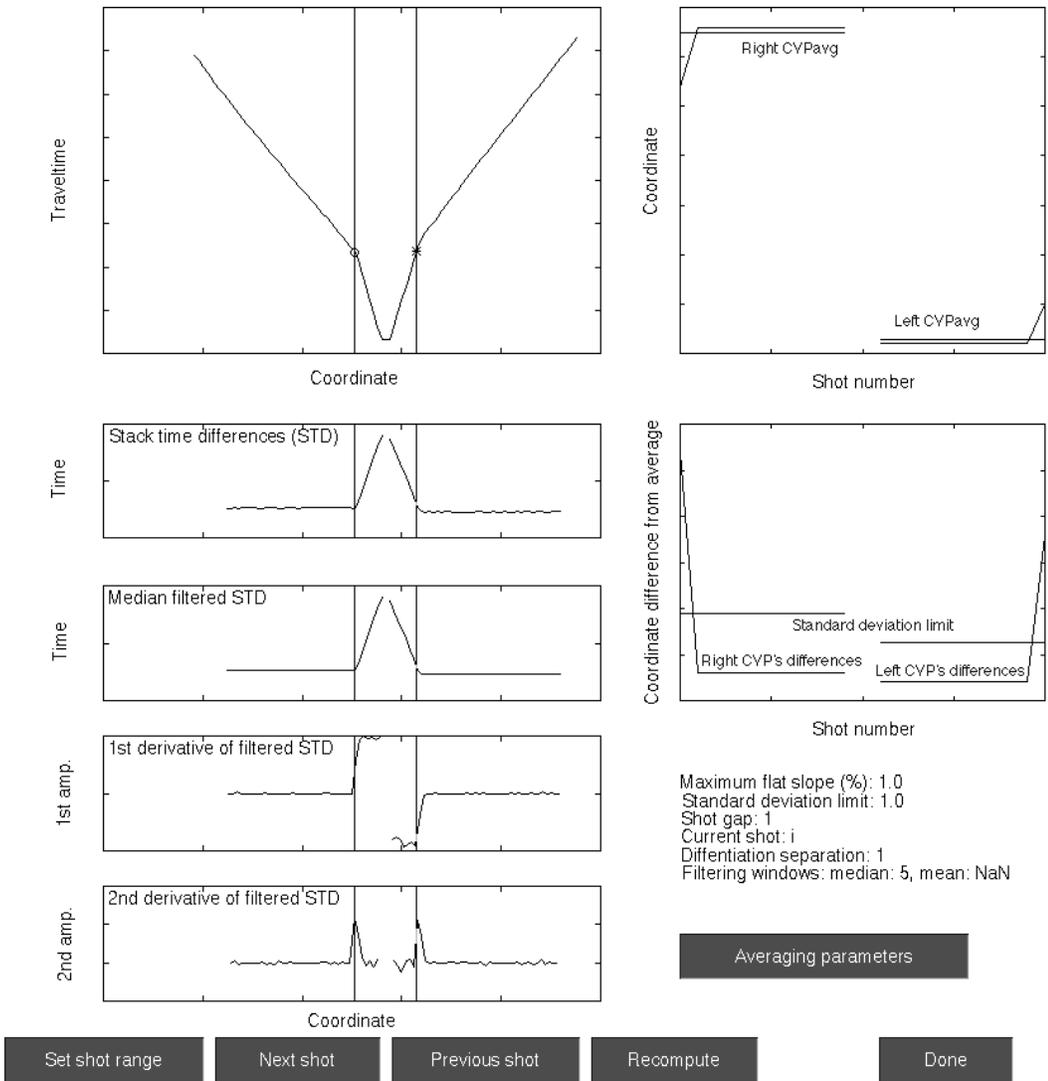


Figure 14. Crossover point averaging editing.

*Velocity model*

The first and second layer velocities can be median filtered and manually edited using the edit velocity model function. The median filter is applied to the first or/and the second layer velocity with a specified window length. The window length is determined according to the degree of smoothness wanted. The velocities should not be changed drastically from one receiver to the next one, according to a realistic near-surface earth model. Also, the first layer velocity should not be greater than the second layer velocity. The use of the "velocity model display" prior to editing the velocity model should help the user identify the modification needed. The velocity changes are allowed only along the velocity axis, so that the velocity values remain at the same receiver locations.

*Depth model*

Finally, the depth model editing function allows the filtering and the manual editing of the interface between the first and second layers. The filtering of the first and second

layer interface using a median filter with a specified window length will smooth the interface and reduced its frequency content. As the window length increases, the frequency content of the interface decreases. The use of the median filter is generally not recommended in order to keep all the details in the depth model, which determine the short wavelength static corrections. The median filter should be used; if necessary, or if only the long wavelength component is wanted. The first and second layer interface can also be manually edited in the same way. The depth model manual editing should be used only to modify unreasonable irregularities along the interface. The depth model depends on the velocity model and the Plus Time values, so that irregularities in the depth model are due to the velocity model or/and to the Plus Time values. It is better to correct the velocity model and check the Plus Time values than edit the depth model. Remember that changes in the depth model will directly change the static corrections, so that only necessary modifications should be undertaken.

## SYNTHETIC DATASET

### Initial model

A synthetic model has been designed to test the Plus-Minus time analysis method developed. The model consists of two layers with a first layer velocity of 650 m/s ( $V_1$ ) and a second layer velocity of 1600 m/s ( $V_2$ ) (Figure 15). Only the first layer thickness is defined. It varies due to surface topography and the second layer elevation variation.

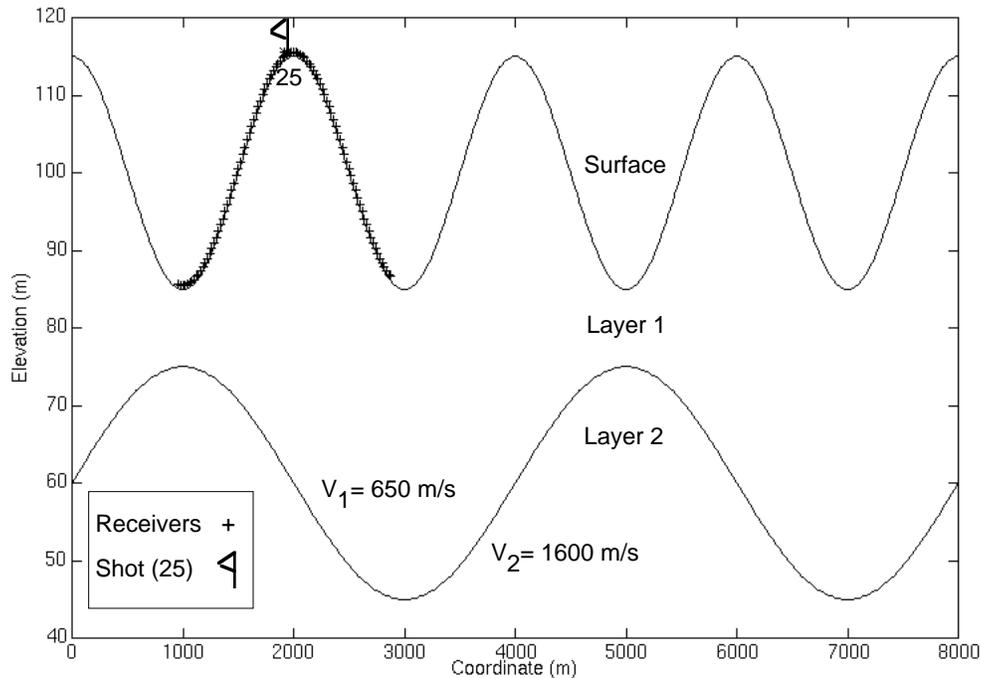


Figure 15. Synthetic initial model showing the location of shot gather (25) (vertical exaggeration = 67).

### Refracted arrivals

From the initial model, direct and refracted arrival traveltimes have been calculated according to the delay time analysis (Figure 16 and equation 27).

$$\text{Refracted traveltimes} = h_s \cdot \cos(\theta_c) / V_1 + h_r \cdot \cos(\theta_c) / V_1 + X / V_2 \quad (27)$$

where  $h_s$  is the thickness of the first layer below the shot,  $h_r$  is the thickness of the first layer below the receiver,  $\theta_c$  is the refracted critical angle ( $\theta_c = \sin^{-1}(V_1/V_2)$ ), and  $X$  is the distance between the shot and the receiver.

$$\text{Direct arrival traveltimes} = X / V_1 \quad (28)$$

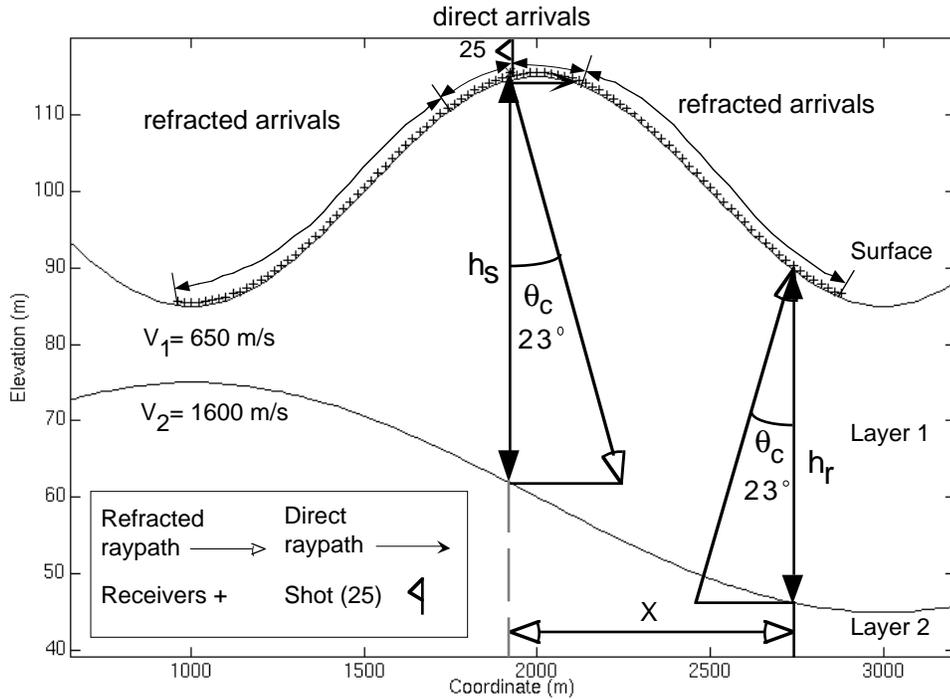


Figure 16. Direct and refracted traveltime calculation for shot gather 25 of the synthetic model.

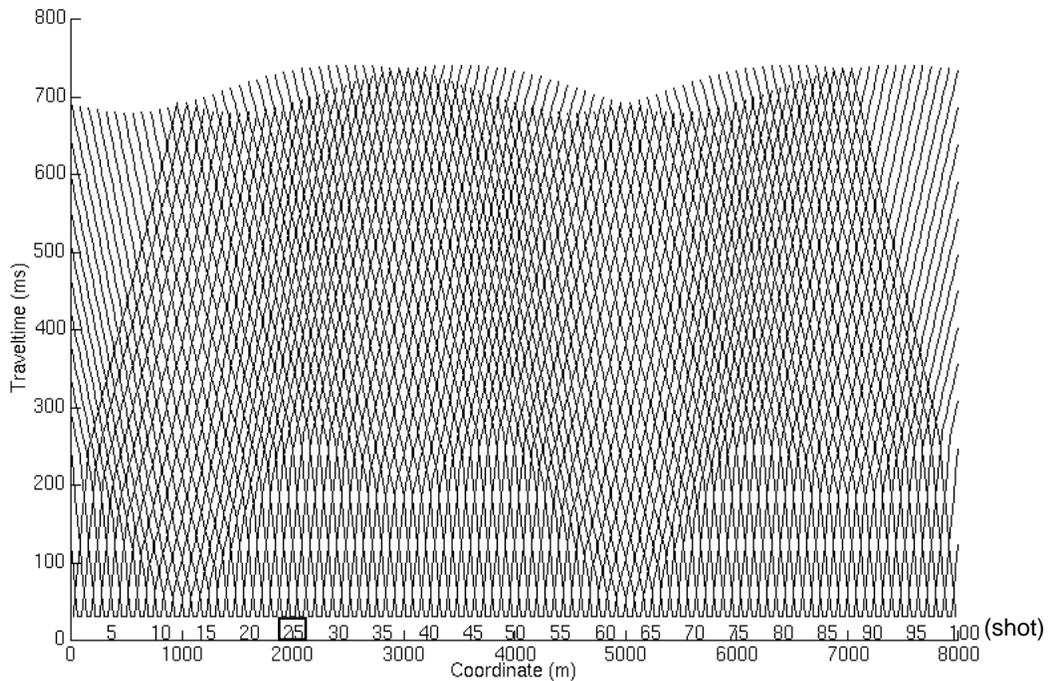


Figure 17. Direct and refracted arrival traveltimes resulting from the synthetic model.

### Reciprocal time differences

The reciprocal time check process calculates all possible shot pair reciprocal traveltimes differences. The reciprocal traveltimes differences of shot gather 25 with all the possible overlapping shot gathers is shown in (Figure 18). Note that reciprocal traveltimes differences are equal to zero, so that traveltimes reciprocity is valid. This result was expected because synthetic calculated traveltimes are used.

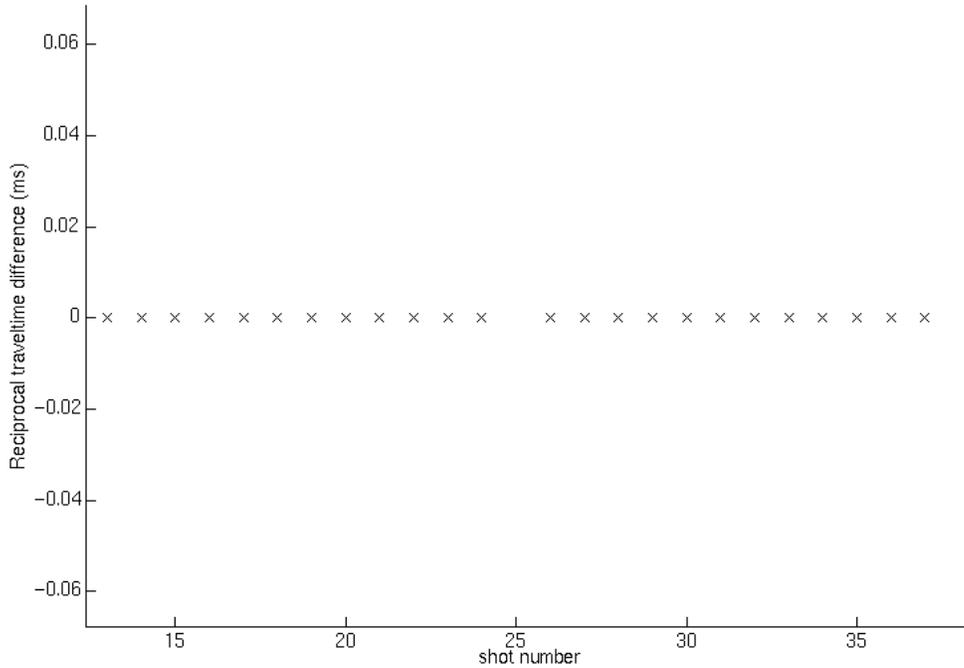


Figure 18. Reciprocal traveltimes differences for the overlapping shot gathers with the shot gather 25 of the synthetic data.

### Crossover points

The synthetic calculated traveltimes being free of noise provide an excellent example showing the success of the CVP autopicking process. Figure 19 shows the autopicking process of the left crossover point for shot gather 20 and of the right crossover point for shot 25. The crossover point autopicking process was undertaken using a short median filter on the traveltimes difference curve (window length of 5 samples).

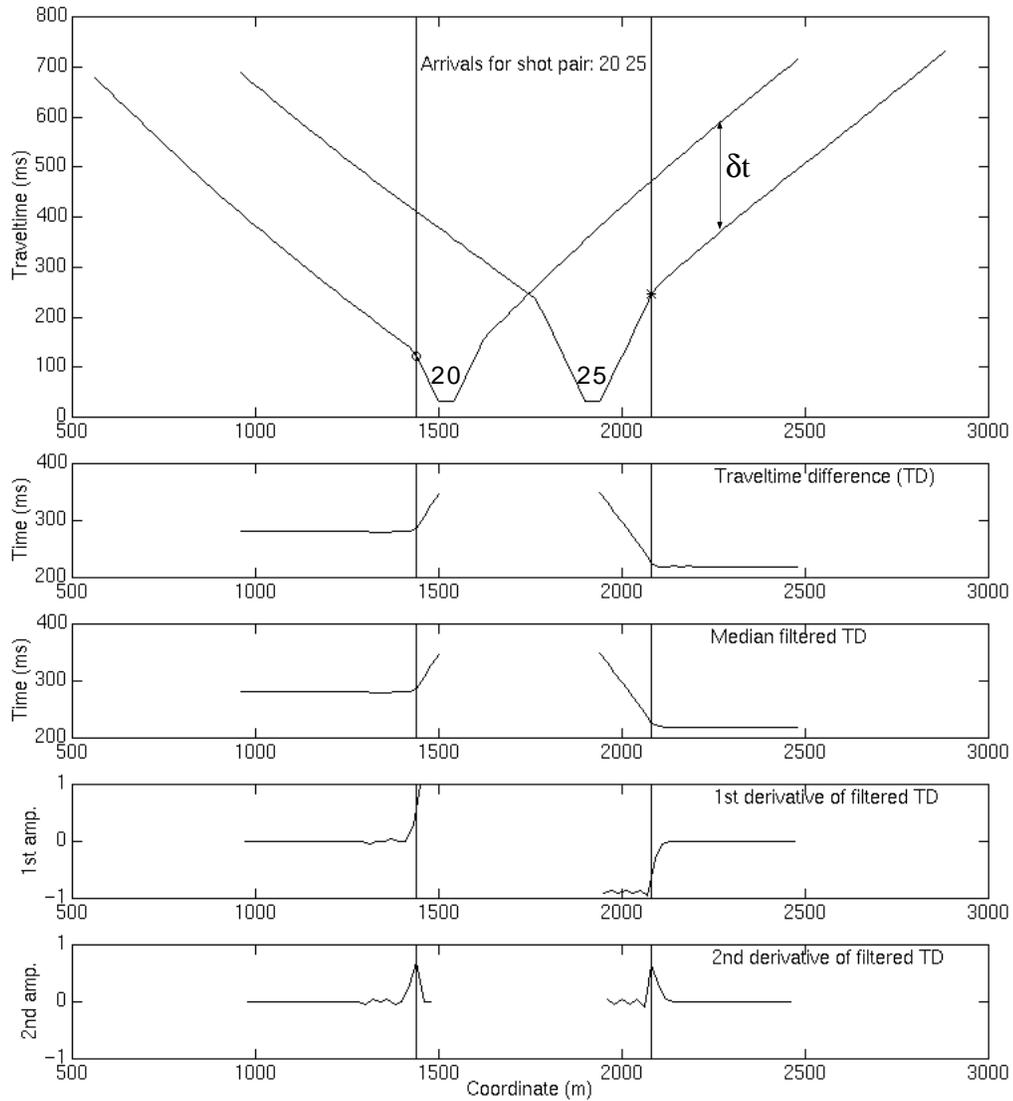


Figure 19. Crossover autopicking process for the left crossover point of shot gather 20 (o) and for the right crossover point of shot gather 25 (\*).

Once the crossover point autopicking process is completed for all the possible overlapping shot gathers, the crossover point averages are computed. Figure 20 shows all the crossover point averages with the arrival traveltimes in a time versus coordinate graph. From the crossover point averages, it can be noticed that the crossover point autopicking process was successful. The statistic results, including the crossover point average offset (from the corresponding shot location), the standard deviations, and the fold, are displayed in figure 21. One thing to remark is that the crossover point average offsets are proportional to the first layer thickness (Figure 15). The standard deviation varies from zero meters to about 35 meters. The standard deviation seems to increase with offset for unknown reasons. Finally, the fold for the right crossover point averages decreases on the left extremity of the survey due to fewer overlapping shot gathers available, while the fold of the left crossover point average decreases on the right extremity of the survey for the same reason. The use of the crossover point statistics display is to help indicate inconsistency in the crossover point average locations and identify region in the survey that need to be worked on by using the

crossover point rejection process, the crossover point repicking process or the crossover point editing process.

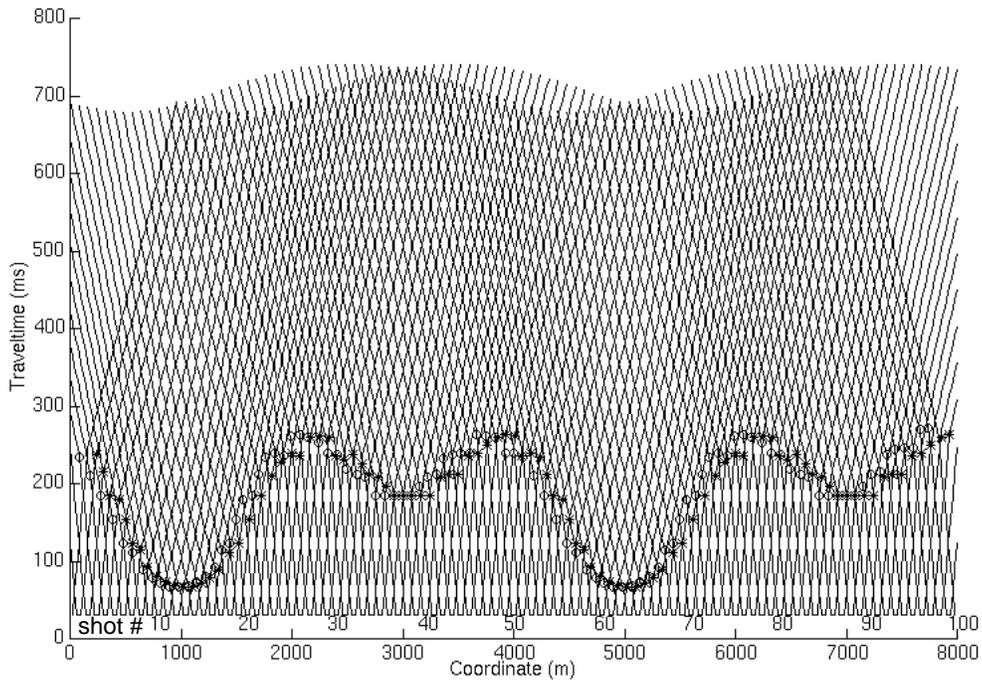


Figure 20. Crossover point averages of the synthetic dataset for the left trailing (circle) and right leading (star) shot spreads with all the arrival traveltimes.

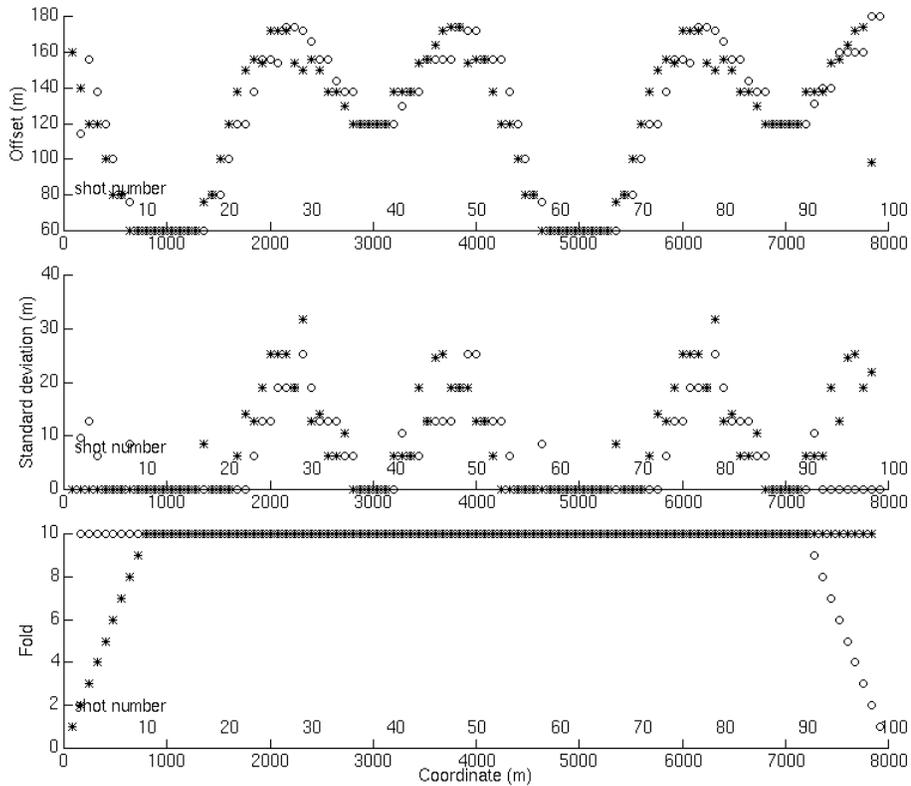


Figure 21. Crossover point statistic display for the synthetic dataset.

## Velocity model

The velocity of the first and second layer can be computed after the crossover point averages are known. The velocity calculation process uses the direct arrival traveltime to find the velocity of the first layer and the refracted arrival traveltimes to establish the second layer velocity. Figure 22 shows the velocity model calculated from the synthetic arrival traveltimes for each surface station location. This display will allow the user to determine if manual editing or filtering are needed before the depth calculation process is undertaken (see editing section). In this case, the first and second layer velocities are in agreement with the initial synthetic model, which demonstrates that the velocity calculation is reliable.

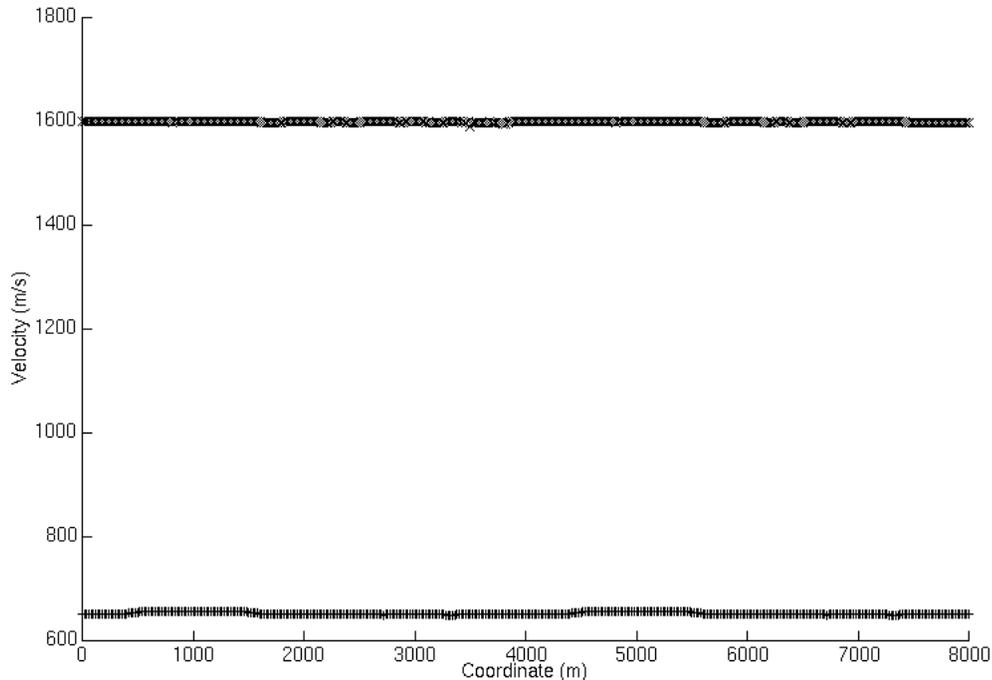


Figure 22. Velocity model calculated from the synthetic arrival traveltimes, including the first layer velocity (+) and second layer velocity (x). Both velocities were median filtered with a window length of 7 samples.

## Plus Time and delay time

Once the crossover point averages are computed the Plus time analysis process can also be undertaken. Figure 23 displays the Plus Time average values at each receiver with the corresponding statistics (standard deviation and fold). It plays the same role as the crossover point statistics display, which is to identify crossover point picking problem areas. The Plus Time statistics display will help determine the parameters for the Plus Time rejection process. The presence of bad arrival traveltimes will introduce bad Plus Time values. Hopefully, the ratio of bad to good Plus Time values for a receiver location is low, so that the Plus Time rejection process will exclude the bad Plus Time values and keep the good ones. In this case, there are no bad arrival traveltimes so that the Plus time rejection process is not needed.

Also, it can be noticed that there is a Plus Time value at each receiver location even at the two line extremities. These Plus Time values come from the use of the delay time analysis for the surface location outside the Plus-Minus time analysis window limit. The fold at these surface locations is less than at the other surface locations because

delay time analysis uses only one shot spread at a time while Plus time analysis uses a combination of two shot spreads.

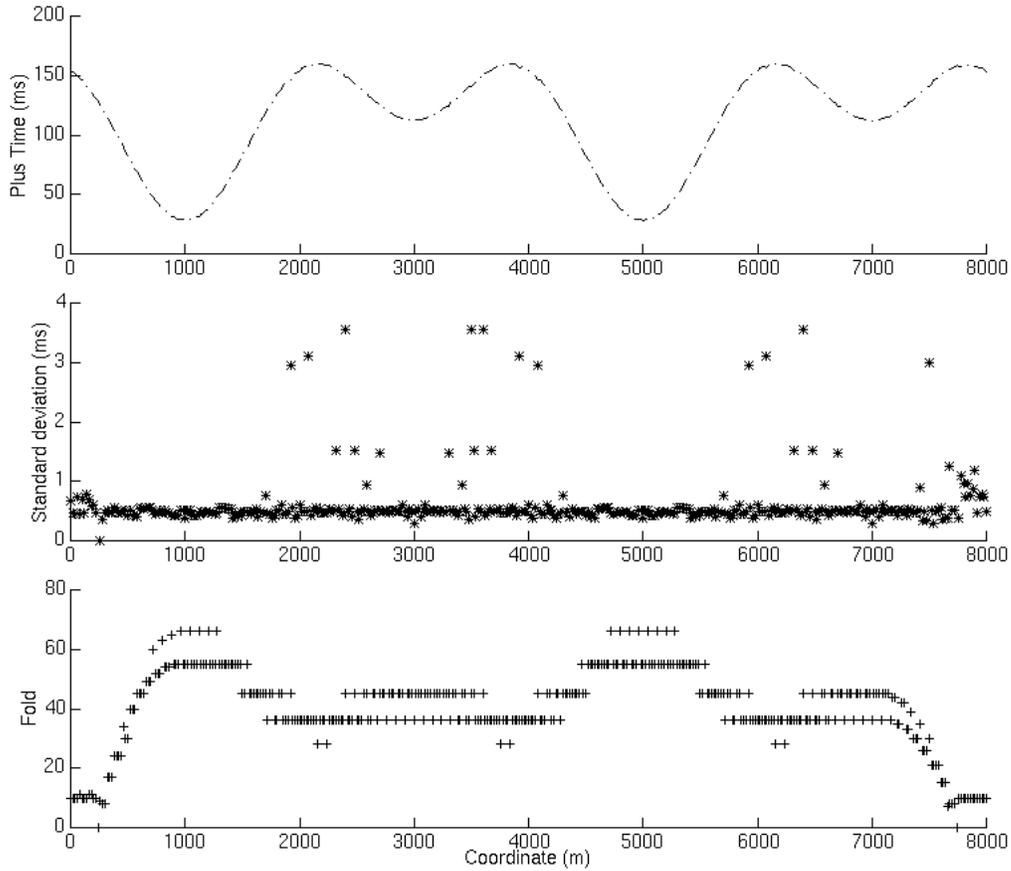


Figure 23. Plus Time values and statistics for the synthetic dataset.

**Depth model**

Knowing the velocities and the Plus Time values, the calculation of the first layer thicknesses were undertaken. Figure 24 shows the results of that calculation by displaying the surface elevation with the first and second layer interface (the first and second layer interface elevation is found by subtracting the calculated first layer thickness from the surface elevation). The calculated depth model closely matches the initial depth model (Figure 15). Figure 25 displays the initial first layer thickness with the calculated one. The difference between the two thicknesses is very small (Figure 26). The root mean square error of the calculated first layer thickness with the initial one is of 0.19 m and the maximum thickness divergence is of 0.57 m.

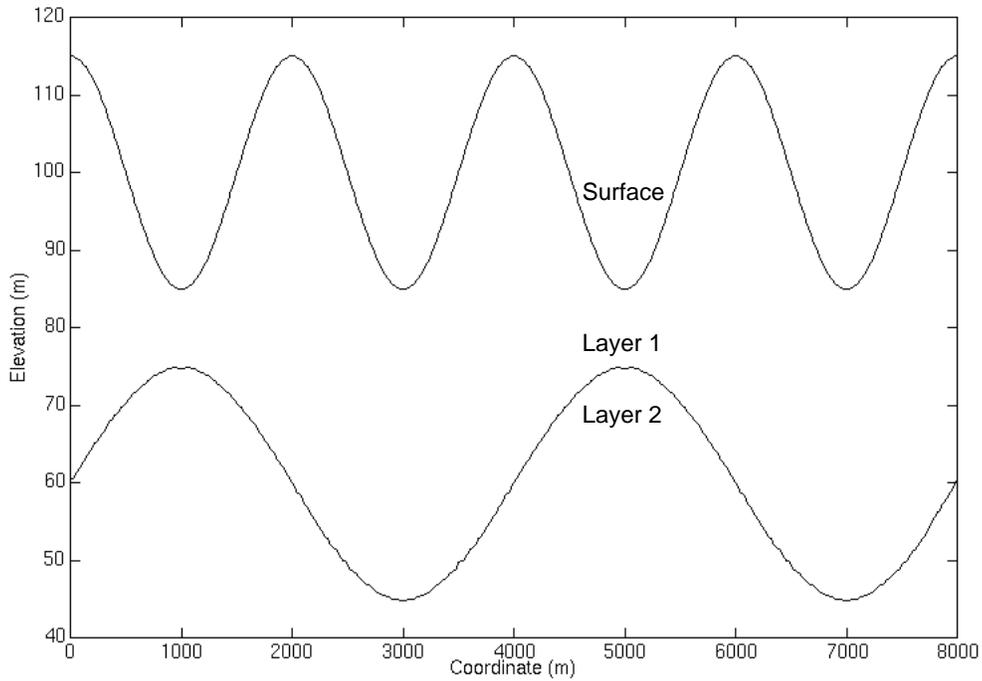


Figure 24. Calculated depth model from the synthetic arrival traveltimes.

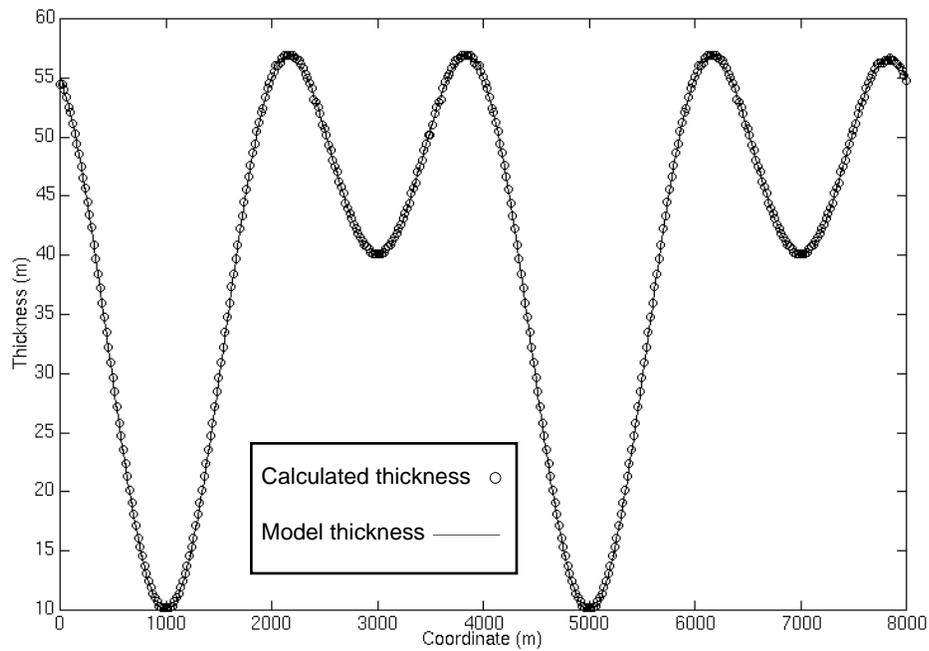


Figure 25. First layer thickness comparison between the initial model and the calculated model.

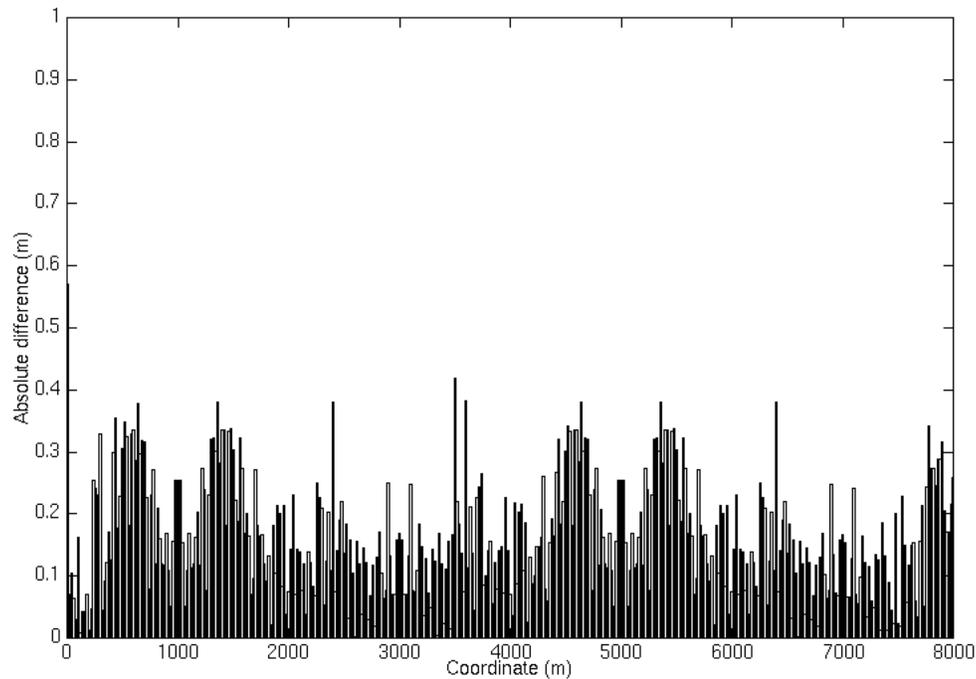


Figure 26. First layer thickness absolute difference between the initial model and the calculated model.

### Static corrections

Once the depth model and first and second layer velocities are known, the static corrections can be computed. Figure 27 shows the surface consistent elevation correction, weathering correction and total correction for a datum elevation of 100 m and a replacement velocity of 1600 m/s (according to the second layer velocity). Considering the root mean square error of the calculated depth model with the initial depth model (0.19m), this error will translate to a 0.17 ms root mean square static error. The maximum divergence error of 0.57 m will result in 0.52 ms static error.

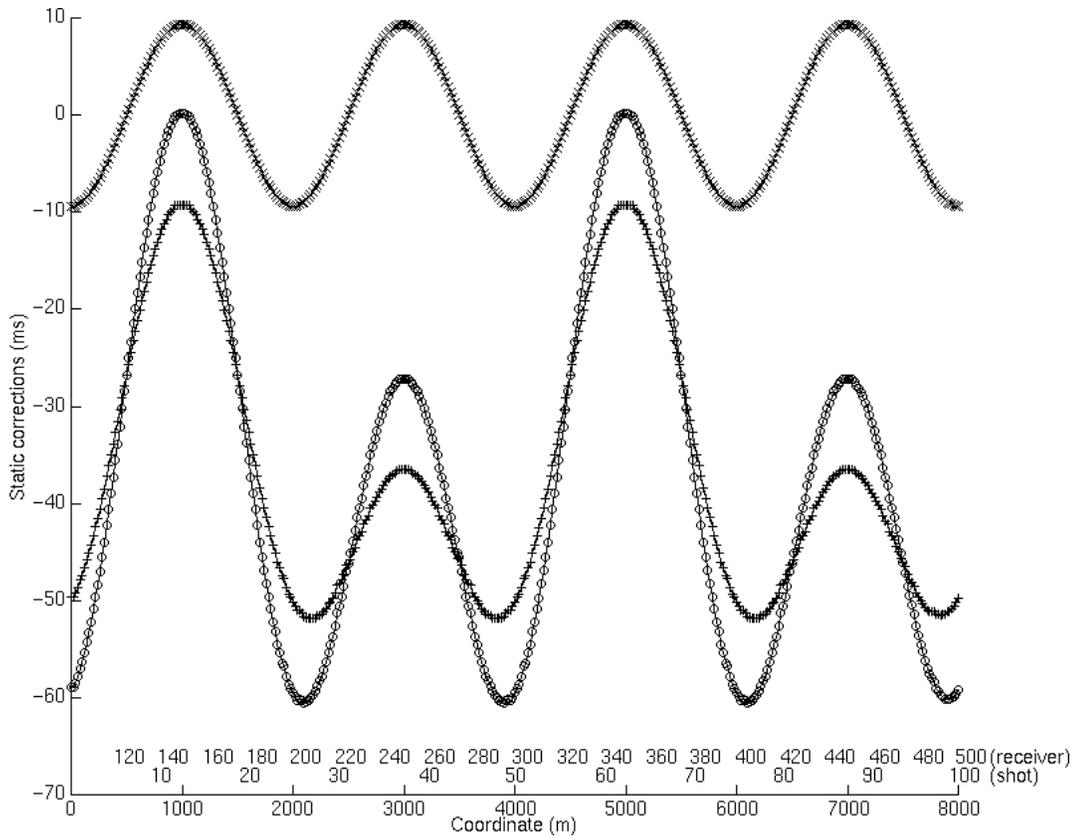


Figure 27. Surface consistent static corrections established from the calculated depth model: elevation correction (\*), weathering correction (+), and total correction (o).

## CONCLUSION

The Plus-Minus time analysis provides a reliable near-surface model when tested with a synthetic dataset. The method gives accurate depths and velocities. The strength of this method is an overdetermination of the solution and available statistical analysis. The crossover point determination uses a combination of the traveltime difference and branch point analyses. This allows for automatic crossover point picking. The Plus-Minus time analysis can be fully automatic and no input near-surface model is needed. Therefore, the resulting near-surface model is strictly based on the refracted arrivals. The Plus-Minus time analysis method is simple and fast to use. The option of user interaction is omnipresent allowing a more consistent interpretation. This is facilitated by a user friendly menu interface.

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