An efficient and accurate algorithm for constructing common scatter point gathers

Xinxiang Li and John C. Bancroft

ABSTRACT

The equivalent offset migration (EOM) technique has been successfully used for field data from different areas. As an efficient alternate approach of pre-stack Kirchhoff time migration, it involves a procedure of constructing common scatter point (CSP) gathers. An efficient and accurate algorithm for constructing CSP gathers is presented. This method will improve the accuracy, quality and the speed in forming the accurate CSP gathers as well as benefits the poststack image section.

INTRODUCTION

Bancroft and Geiger(1994) and Bancroft et al. (1995) introduced a new approach of pre-stack time migration, which is called equivalent offset migration (EOM). As an alternative method to fulfill Kirchhoff migration, EOM is also based on the subsurface scatter point model. With the new concept of equivalent offset, pre-stack seismic dataset is sorted into a new dataset named common scatter point (CSP) gathers. The whole procedure of EOM is then completed by NMO correction and stacking applied on CSP gathers.

Kirchhoff migration principle

For a subsurface scatter point with RMS velocity V_{rms} located at depth z_0 and surface location x_0, as shown in Figure 1, two experiments can be done.

The first, theoretically put a source and a receiver at the same surface location x as in Figure 1a, the recorded travel time T_1 will be

$$T_1 = 2\sqrt{{}_{\frac{1}{2}}T_0^2 + \left(\frac{x - x_0}{Vrms}\right)^2} \tag{1}$$

where ${}_{½}T_0 = z_0/V_{ave}$ is the one-way travel time vertically from surface to the scatter point or reverse. When the source-receiver location x moves along a surface line, it is easy to see that T_1 changes along a hyperbola in $T - x$ coordinates.

The second, the sources and the receivers are separated on the earth surface. Assume that a source locate at x_s and a receiver locate at x_r as shown in Figure 1b, the recorded travel time T_2 will be

$$T_2 = \sqrt{{}_{\frac{1}{2}}T_0^2 + \left(\frac{x_s - x_0}{V_{rms}}\right)^2} + \sqrt{{}_{\frac{1}{2}}T_0^2 + \left(\frac{x_r - x_0}{V_{rms}}\right)^2} \tag{2}$$

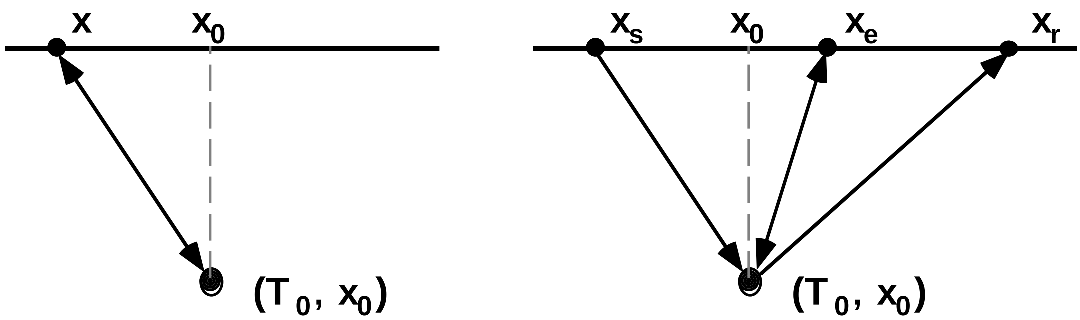

Figure 1. Surface scatter point model. (a) the left, co-located source and receiver assumption, it is the basic model for poststack Kirchhoff time migration. (b) the right, separated source and receiver, prestack Kirchhoff time migration methods starts with this model. EOM technique goes further with a new concept of equivalent offset.

Equation (2) is called double square root (DSR) equation, which is the basic property of the scatter point model in arbitrary offset case. It is the fundament equation for pre-stack Kirchhoff migration.

Equations (1) and (2) can be written in CMP location $x_{cmp} = (x_s + x_r)/2$ and half source-receiver offset $h = (x_s - x_r)/2$ coordinates as

$$T_1 = 2\sqrt{\tfrac{1}{2}T_0^2 + \left(\frac{x_{off}}{V_{rms}}\right)^2} \qquad (1')$$

$$T_2 = \sqrt{\tfrac{1}{2}T_0^2 + \left(\frac{x_{off} - h}{V_{rms}}\right)^2} + \sqrt{\tfrac{1}{2}T_0^2 + \left(\frac{x_{off} + h}{V_{rms}}\right)^2} \qquad (2')$$

where $x_{off} = x_{cmp} - x_0$ is the distance between CMP location and the scatter point surface location.

Equivalent offset and CSP gather

An essential characteristic of EOM method is that it connects equation (1) and (2) with the concept of equivalent offset. Any source location x_s and receiver location x_r, or equivalently any CMP location x_{cmp} and half source-receiver offset h, there is a surface point (circle for 3-D line) x_e such that $T_1(x_e) = T_2(x_{cmp}, h)$, i.e.,

$$2\sqrt{\tfrac{1}{2}T_0^2 + \left(\frac{x_e - x_0}{V_{rms}}\right)^2} = \sqrt{\tfrac{1}{2}T_0^2 + \left(\frac{x_{off} + h}{V_{rms}}\right)^2} + \sqrt{\tfrac{1}{2}T_0^2 + \left(\frac{x_{off} - h}{V_{rms}}\right)^2}\ . \qquad (3)$$

The distance from x_e to x_0 on surface is defined as the equivalent offset h_e associated with the given scatter point (its position and velocity), and the sample time $T = T_1 = T_2$. The value of h_e can be expressed as (from Bancroft et al, 1995)

$$h_e^2 = x_{off}^2 + h^2 - \left(\frac{2 \cdot x_{off} \cdot h}{V_{rms}(T_0)T}\right)^2 . \tag{4}$$

Writing V_{rms} as $V_{rms}(T_0)$ is for emphasizing the value of velocity field is given at the scatter point location instead of the recorded travel time T.

The surface position of a vertical array of scatter points is referred to the common scatter point (CSP) location. CSP gathers are formed by summing input traces into offset bins of the gather. The energy of a sample on an input trace (a trace sorted into conventional coordinates such as CMP gathers) may come from scatter points under all different CSP locations, and different samples on a trace may be associated with different equivalent offsets even for a same CSP location.

Suppose the CSP gather at location x_0 is constructed traces with equivalent offsets $\{h_e(l) = l \cdot \Delta h_e : l = 0,1,\cdots,N_h\}$. Each input trace with CMP location x_{cmp} and half source-receiver offset h contributes its energy to this CSP gather according to equation (4). Specifically, if a sample with travel time T has a h_e by equation (4) falls between $h_e(l-1/2)$ and $h_e(l+1/2)$, it will be distributed to the trace with equivalent offset $h_e(l)$.

Discussions in the following sections will make sure that, for each equivalent offset bin boundary $h_e(l-1/2)$, there is a determined value of T (denoted by $T(l)$) satisfying

$$T(l) = \frac{2 \cdot x_{off} \cdot h}{V_{rms}(T_0)\sqrt{x_{off}^2 + h^2 - h_e^2(l-1/2)}} . \tag{5}$$

All samples with travel time between $T(l)$ and $T(l+1)$ have equivalent offset by equation (4) falling between $h_e(l-1/2)$ and $h_e(l+1/2)$. That means these samples will all be distributed to the trace with equivalent offset $h_e(l)$.

As being emphasized above, $V_{rms}(T_0)$ usually changes with travel time $T(l)$, so $T(l)$ can not be directly computed by equation (5). The algorithm may be expressed as:

1. Estimate T_0 from preliminary knowledge;

2. Get velocity $V_{rms}(T_0)$ by known velocity function at present CSP location;

3. Calculate $T(l)$ by equation (5), or by

$$T^2(l) = T_0^2 + {4h_e^2(l-1/2)} \Big/ {V_{rms}^2(T_0)} . \tag{6}$$

Step **2** and **3** are easy to understand, the main concern is the step **1**. First, if we can get accurate T_0, the result $T(l)$ will be accurate (based on accurate velocity). The main difference between our new algorithm and old algorithm is that we try to cancel the direct influence of $T(l)$ on T_0 according to the connection between equations (5) and

(6), then get an accurate solution of T_0 by using a special property of RMS velocity. While, the old method try to get an approximate solution of T_0 by a loop over the above three steps.

THE ALGORITHMS

The old algorithm first gets an approximate of T_0 by equation (6) using the preliminary knowledge about the first useful energy in the input trace for given CSP location at initial time(Bancroft et al. 1995)

$$T(l) = {}^{2\cdot x_{off}}\!\big/_{V_{rms}(0)}$$

or previous $T(l)$ and V_{rms} at previous T_0 (step **1**), then by equation (5) get an new approximate of $T(l)$ (step **3**). Figure 2 may be helpful for understanding the iterative mechanism of the method.

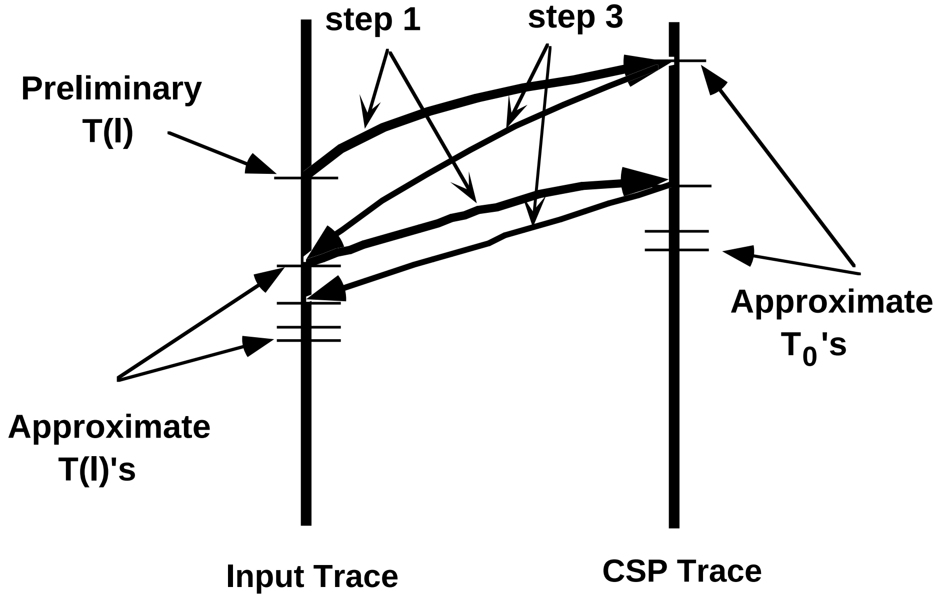

Figure 2. Iterative approach to get $T(l)$ by loop over the three steps.

Now, let's see our new algorithm. From equation (4), we can get

$$[T_0V]^2 = \frac{4x_{off}^2 \cdot h^2}{x_{off}^2 + h^2 - h_e^2(l-1/2)}, \quad (7)$$

where $V_{rms}(T_0)$ is simplified as V. Revising equation (6), we have

$$[T_0V]^2 = [T_0V]^2 + 4h_e^2(l-1/2). \quad (8)$$

Combine (7) and (8),

$$\left[T_0 V\right]^2 = \frac{4 \cdot x_{off}^2 \cdot h^2}{x_{off}^2 + h^2 - h_e^2(l-1/2)} - 4h_e^2(l-1/2). \tag{9}$$

This is the basic equation our new method uses. It is more convenient because the both sides of the equation can be accurately computed by known quantities. The right hand side is a real number while the left hand side is a function of T_0, this function is a known continuous function because V_{rms} is always continuous with T_0 no matter what the interval velocity is. However, for getting T_0 from (9), it is required that the function $\left[VT_0\right]^2$ has a inverse function. We fortunately found that for RMS velocity, this function is always invertable, the proof is in next section.

VALIDITY AND EFFICIENCY

There are still two questions to answer in our algorithm: when we construct the trace at a CSP gather equivalent offset bin location $h_e(l)$, the boundaries of this bin are $h_e(l-1/2)$ and $h_e(l+1/2)$. Using the algorithm above we can get $T(l)$ and $T(l+1)$. The first question is: If all the samples with travel time between $T(l)$ and $T(l+1)$ really have the equivalent offsets between $h_e(l-1/2)$ and $h_e(l+1/2)$? The second question is: If $\left[VT_0\right]^2$ is invertable as a function of T_0 ?

In this section, we will give positive answers to these two questions, thus the validity of our algorithm is provided.

$\left[VT_0\right]^2$ is an absolutely increasing function of T_0

Assume T_0 is sampled by arbitrary time interval δt, i.e. $T_0 = n \cdot \delta t \; (n = 0,1,2,\ldots)$. For a given CSP location, the RMS velocity field is a function of n and δt

$$\left[V_{rms}(n \cdot \delta t)\right]^2 = \frac{V_1^2 \cdot \delta t + V_2^2 \cdot \delta t + \ldots + V_n^2 \cdot \delta t}{n \cdot \delta t}$$
$$= \frac{1}{n}\sum_{k=1}^{n} V_k^2$$

where V_k is the interval velocity (positive real number) in $\left[(k-1)\cdot \delta t,\ k \cdot \delta t\right]$. Then

$$\left[V_{rms}(T_0)T_0\right]^2 = \left[V_{rms}(n \cdot \delta t)\cdot n \cdot \delta t\right]^2 = n(\delta t)^2 \sum_{k=1}^{n} V_k^2$$

and

$$\left[V_{rms}\left[(n+1)\cdot \delta t\right]\cdot (n+1)\cdot \delta t\right]^2 - \left[V_{rms}(n \cdot \delta t)\cdot n \cdot \delta t\right]^2$$
$$= (n+1)(\delta t)^2 \sum_{k=1}^{n+1} V_k^2 - n \cdot (\delta t)^2 \sum_{k=1}^{n} V_k^2$$

$$= (\delta t)^2 \sum_{k=1}^{n} V_k^2 + V_{n+1}^2 (n+1) \cdot \delta t > 0 .$$

Validity of calculating T only at the equivalent offset bin boundary

By the physical property of scatter point model, when the scatter point depth z_0 or T_0 changes at a CSP location, the travel time T on any recorded trace should be a continuous function of T_0. In addition, because $[VT_0]^2$ is a continuos function of h_e and is invertable as function of T_0, so T_0 is also a continuous function of h_e. Finally, T is a continuous function of h_e. Assume this function is f, i.e. $T = f(h_e)$, we have

$$T(l) = f\left(h_e(l-1/2)\right),\ T(l+1) = f\left(h_e(l-1/2)\right),$$

then because of the continuity of f, for any T^* between $T(l)$ and $T(l+1)$, there must be a $h_e^* \in [h_e(l-1/2), h_e(l+1/2)\ ]$ such that $f\left(h_e^*\right) = T^*$.This is to say, any sample with travel time between $T_{k,l-}$ and $T_{k,l+}$ can be assigned an equivalent offset in the bin $h_e(l)$ (i.e. between $h_e(l)$ and $h\ (l+1)$.

By now, we have answered the two questions opened above.

Efficiency of the new algorithm

Comparing the two methods mentioned above, it is easy to see that the main differences from the old algorithm is :

1. Calculate the right hand side of equation (9): usually, $h_e(l-1/2)$ can be considered as constant because it is independent of both CSP location and input trace; x_{off} and h are independent of $\boldsymbol{l}$, they just need to be computed once for fixed CSP gather and input trace.

2. Construct a vector $[VT_0]^2$ for time samples: there is such a vector for every CSP gather, so we can construct these vectors as a matrix just when we get the velocity function for all CSP locations. It dose not increase much computation cost.

3. Get T_0 from equation (9): for this process, no more summation or multiplication operations are in need, we can get very accurate (the error less than one sample interval) T_0 only by comparing the value of right-hand-side term with a series of $[VT_0]^2$.

The main advantage of our new method is its very high accuracy, especially when high quality imaging is required.

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