Energy concentration as a function of dip on Cheop’s pyramid and CSP gathers

John C. Bancroft and Hugh D. Geiger

ABSTRACT

Kirchhoff prestack time migration assumes 2-D energy scattered from a point is spread evenly on a surface known as Cheop’s pyramid in the prestack data volume. The scatter points may be combined to construct a reflector, and the Cheop’s pyramids from each scatter point reconstruct the reflection energy. The reconstructed energy is no longer distributed evenly across the surface of a given Cheop’s pyramid but is instead concentrated in a localized band. When collected into common scatter point (CSP) gathers, concentrated data from a scatter point with a given dip will be distributed along a hyperbola defined by the migration imaging velocity.

The concentration of energy on Cheop’s pyramid and the distribution of energy in the corresponding CSP gather will be presented for horizontal and dipping data.

INTRODUCTION

A scatter point disperses energy from any source to any receiver within the migration aperture. Travel times T for the scattered energy are defined by the double square root (DSR) equation. Defining $T_{0\beta}$ as the vertical zero-offset two-way travel time (and ${}_{1/2}T_{0\beta}$ as the one-way travel time) the 2-D DSR equation is given by:

$$T = \sqrt{{}_{1/2}T_{0\beta}^2 + \frac{\left((x - x_{CSP}) - h\right)^2}{V_{RMS}^2}} + \sqrt{{}_{1/2}T_{0\beta}^2 + \frac{\left((x - x_{CSP}) + h\right)^2}{V_{RMS}^2}}\ , \tag{1}$$

where x is the common midpoint (CMP) position, x_{CSP} the surface location of the image ray passing through the scatter point, h the source-receiver half offset and V_{RMS} the migration imaging velocity.

When plotted in a 3-D prestack volume (x, h, t), the 2D travel times define a surface known as Cheop’s pyramid (Claerbout, 1985), as illustrated in Figure 1. The apex of Cheop’s pyramid is the two-way time position of the scatter point within the prestack volume $(x=x_{CSP}, h=0, t=T_{0\beta})$. Energy from the scatter point is distributed smoothly over the entire surface with amplitudes approximately proportional to $T_{0\beta}/T$.

Reflecting surfaces can be considered as an organized collection of scatter points. Energy on the surfaces of the corresponding collection of Cheop’s pyramids interfere constructively and destructively to form reflection events within the prestack volume. Kirchhoff prestack time migration sums energy in the prestack volume (see Figure 1) over the surface of Cheop’s pyramid for every sample location in the zero-offset plane $(x, h=0, t)$. The output zero-offset plane is the prestack migrated time image of the subsurface. Conventional processing sums (stacks) energy in CMP gather planes

($x=x_{CMP}$, h, t) over normal moveout (NMO) hyperbolas for every sample location in the zero-offset plane *(x, $h=0$, t)*. The output zero-offset plane is an unmigrated stacked section that can be imaged using post stack migration. After NMO and stacking, only the energy from scatter points where the apex of Cheop's pyramid lies in the CMP gather at *($x_{CSP}=x_{CMP}$, $h=0$, t)* are correctly imaged. Scatter points away from the location of the CMP gather *($x_{CSP}\neq x_{CMP}$, $h=0$, t)* contribute non-hyperbolic energy that cannot be collapsed by NMO. Dip moveout (DMO) corrects the non-hyperbolic moveout to hyperbolic, but is only valid for constant velocity media. In contrast, prestack migration is valid for vertical velocity variations and can produce excellent imaging in cases where mild lateral velocity variations exist.

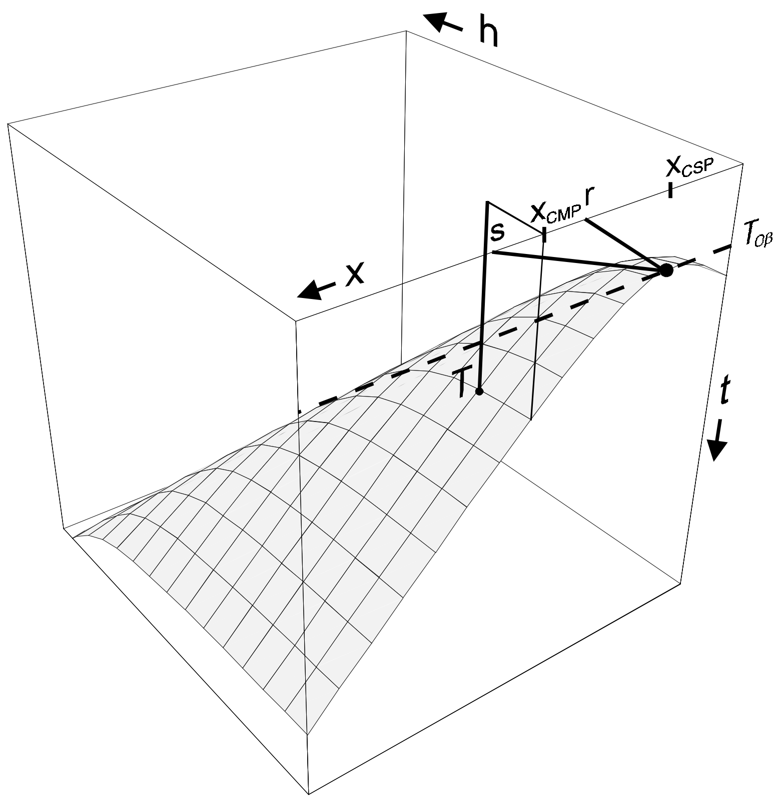

FIG. 1. Prestack volume for 2-D seismic line with axis of CMP *x*, source-receiver half-offset *h* and two-way time *t*. The gray surface is a portion of Cheop's pyramid for a scatter point with surface location x_{CSP} and travel time $T_{0\beta}$. The total travel time *T* from an arbitrary source *s* and receiver r with CMP surface location x_{CMP} is shown as the thick vertical line. The horizontal dashed line is the position of scatter points for a horizontal reflector.

Equivalent offset prestack migration (EOM) introduces an intermediate step in prestack migration by forming common scatter point (CSP) gathers (Bancroft and Geiger 1994, Bancroft et al 1996). CSP gathers are formed at any desired CSP surface location x_{CSP} by assuming an imaginary collocated source and receiver at equivalent offset h_e relative to x_{CSP}. It has the same two-way travel time to a subsurface scatter point as the original source and receiver. For each CSP surface location, an equivalent offset can be assigned to every sample in the migration aperture. In practice it is sufficient to create offset bins that collect samples that lie within a specified range of equivalent offsets. The time of each sample remains the same during the gathering process. All energy in the prestack volume (within the migration aperture) will sum into a CSP gather with constructive and destructive interference to create hyperbolic moveout curves for each scatter point in the gather.

ENERGY CONCENTRATION FOR A HORIZONTAL REFLECTOR

Energy from a 2-D horizontal reflector forms a hyperbolic cylinder within the prestack volume *(x, h, t)* as illustrated in Figure 2a. A CMP plane that intersects the cylinder contains energy in a hyperbolic curve (Figure 2b). NMO and stacking reconstruct the energy to the reflector position on the zero offset plane ($x=x_{CMP}$, $h=0$, $t=T_{0\beta}$). For a horizontal reflector, post stack time migration produces no additional change to the position of data in the zero offset plane.

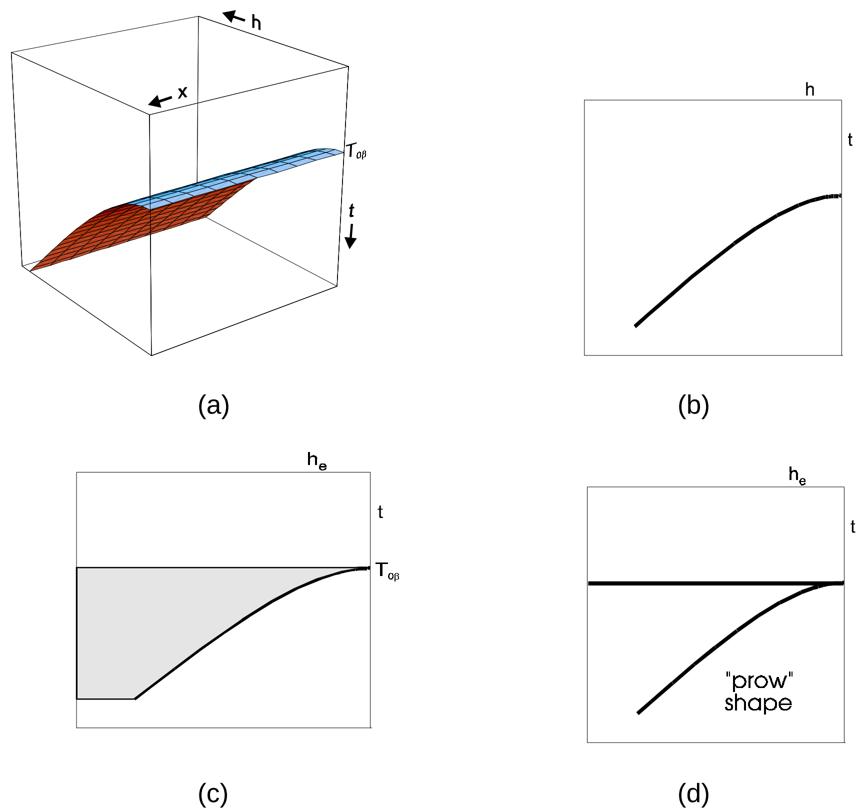

FIG. 2. Two-way travel times to a horizontal reflector at time $T_{0\beta}$ displayed as (a) a surface in a prestack migration volume *(x, h, t)*, (b) a hyperbolic curve in a CMP gather before NMO, (c) a CSP gather with a zone of destructive interference shown in gray, and (d) constructive interference showing hyperbolic and horizontal distributions of energy to form a “prow”.

The horizontal reflector can be considered as a set of closely spaced scatter points along a horizontal line. Energy from the set of Cheop’s pyramids will constructively and destructively interfere to produce the reflection event in the prestack volume as shown in Figure 2a. A Cheop’s pyramid at $T_{0\beta}$ will be tangent with the hyperbolic cylinder at the CMP plane. This is the hyperbolic moveout curve in the CMP plane through the scatter point ($x_{CMP}=x_{CSP}$, h, t) as shown in Figure 2b. Consequently, prestack migration should give identical results to NMO, stack, and post stack migration.

In creating a single CSP gather, all energy from the hyperbolic cylinder that lies within the migration aperture will be stacked without time shifting into the gather at an appropriate equivalent offset. The top of the hyperbolic cylinder is at constant time T_0 in the prestack volume and arises from source-receiver pairs that are already at zero-offset (i.e. collocated). Hence, the equivalent offset for data samples from the top of the cylinder is equal to the distance between the CMP and CSP $(x_{cmp} - x_{csp})$ and the energy appears in the CSP gather as a horizontal event at $T_{0\beta}$ across all offsets (Figure 2c). The remaining energy from the hyperbolic cylinder will lie in the CSP gather between the horizontal event at $T_{0\beta}$ and the hyperbola of maximum constructive interference described above. The 'between' energy will destructively interfere and cancel to zero for typical band-limited seismic wavelets (the gray area in Figure 2c). Thus for a horizontal reflector, all that appears in the CSP gather will be the horizontal event at $T_{0\beta}$ and the hyperbolic event (Figure 2d). These two events meet at time $T_{0\beta}$ and zero equivalent offset to form a prow (bow of yacht) shape.

The prow shape on the CSP gather, produced by horizontal reflectors, illustrates that the formation of the gather combines NMO (the hyperbola) and post stack migration (the horizontal event) into one step. As in post stack migration of horizontal events, the horizontal portion of the prow in the CSP gather will only contribute energy to the zero offset position. The shape of the prow on CSP gathers is illustrated in Figure 3 for real data.

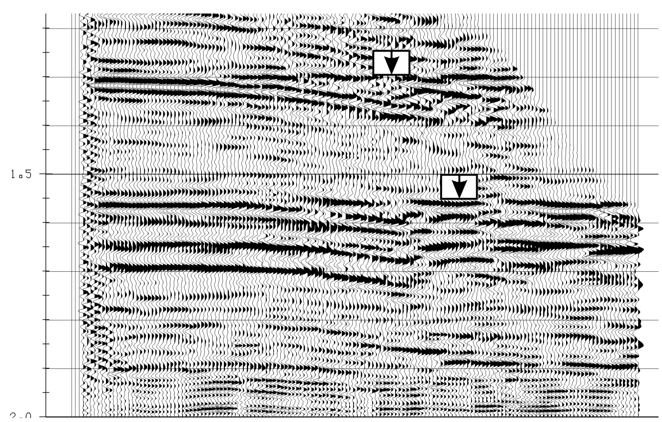

FIG. 3. CSP gather illustrating horizontal events (see arrows) and hyperbolic events forming a prow shape.

ENERGY CONCENTRATION FOR A DIPPING REFLECTOR

The distribution of energy in the prestack volume (x, h, t) for a dipping reflector is much more complex than for a horizontal reflector. For simplicity, we will consider a reflector with geologic dip β formed by a set of scatter points in a media with constant velocity V, where each Cheop's pyramid is asymptotic to a true pyramid with apex at time 0 and surface location x_{CSP}. As we move down-dip, adjacent scatter points are found at progressively later times and give rise to broader topped Cheop's pyramids.

The energy concentrated in a given CMP plane arises from the constructive interference of a number of Cheops's pyramids that lie up-dip from the scatter point of the reflector where it intersects the CMP plane. The concentration of energy can no longer be described as a vertical plane through a single Cheop's pyramid as is the case for a horizontal reflector. However, as is well known (Yilmaz, 1987), the concentration of energy in the CMP plane is hyperbolic, and is given by:

$$T^2 = T_{0\alpha}^2 + \frac{4h^2 \cos^2 \beta}{V^2} \tag{2}$$

where $T_{0\alpha}$ is the zero offset time of the CMP gather on the seismic dip α. This zero-offset time $T_{0\alpha}$ in the CMP gather as defined by in equations (2) is not the same time position as the scatter point $T_{0\beta}$. The two are related by:

$$T_{0\beta} = T_{0\alpha} \cos\beta, \tag{3}$$

where that time at $T_{0\alpha}$ will poststack migrate to the time $T_{0\beta}$ and move up dip on the geologic dip β. The cosine term in equation (2) modifies the velocity to define a stacking velocity V_{stk},

$$V_{stk} = \frac{V}{\cos\beta}, \tag{4}$$

giving a NMO equation for dipping events as,

$$T^2 = T_{0\alpha}^2 + \frac{4h^2}{V_{stk}^2}. \tag{5}$$

These equations are typically derived from the geometry of ray paths on a simple dipping model (Levin, 1971). It should be noted the above equations assume the reflection points in a CMP gather are *not* located at one reflection point on the dipping event, but that the reflection points move progressively updip with increasing offset. They may image to a single point at zero offset time at $T_{0\beta}$ or migration time $T_{0\alpha,}$ but the energy has not been positioned in the correct location.

We now describe the concentration of energy in the prestack volume for a dipping reflector from the perspective of a scatter point. The reflectors at geologic dip β in Figures 4a-c are assumed to be composed of a series of scatter points, each with Cheop's pyramids that constructively interfere to form the hyperbolic surface in (a). Energy in the zero offset plane of the prestack volume lies along the seismic dip α. The energy surface forms hyperbolas in CMP gathers with apex at time $T_{0\alpha}$ as defined by equation (5).

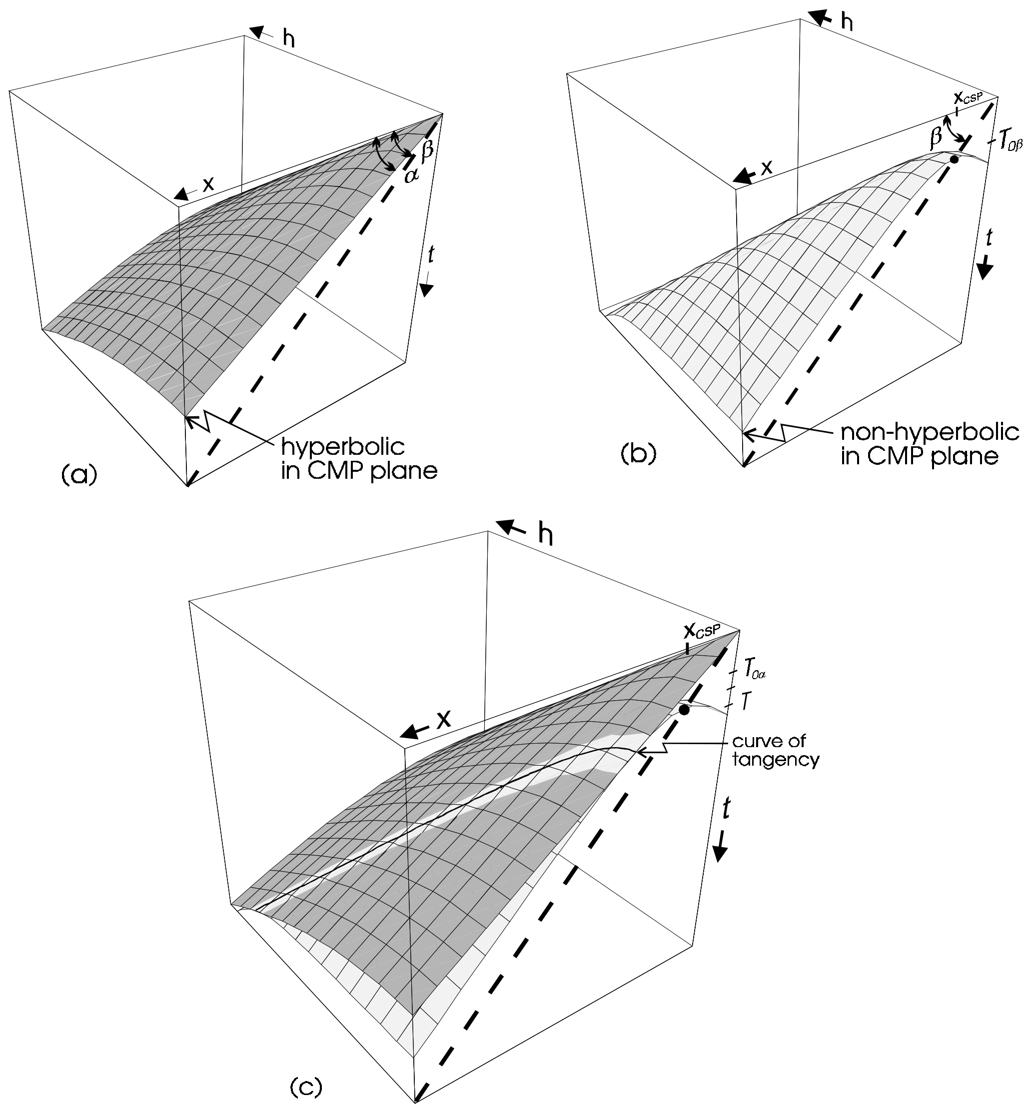

FIG. 4. Prestack volume *(x, h, t)* for 2-D seismic line in constant velocity media showing response of dipping reflector (heavy dashed line in all figures) at geologic angle β. The 'dipping cylinder' shown in (a) as the dark gray surface is the prestack data at time T from the dipping event. In (b), the Cheops's pyramid for a scatter point at location x_{CSP} shown as the light gray surface. A combination of (a) and (b) is shown in (c) where the location of tangency is illustrated by the light gray band.

Figure 4b shows one of the scatter points located at x_{CSP} and time $T_{0\beta}$ with its corresponding Cheops's pyramid shown as the light gray surface. Note that moveout in the CMP plane is non-hyperbolic (except in the plane through x_{CSP}).

The formation of the hyperbolic sheet from a series of Cheop's pyramids is now visualized if Figure 4c where the single Cheops's pyramid in Figure 4b is plotted with the hyperbolic sheet of (a). The light gray band in (c) indicates the area of tangency between the two surfaces where energy will constructively reinforce. This band was produced by slightly reducing the travel times *T* of equation (1) to slightly move the Cheop's pyramid in (b) through the hyperbolic surface of (a). The solid black line is the theoretical location of tangency. Energy from all scatter points will construct the hyperbolic surface and destructively cancel elsewhere.

The image in Figure 4c provides a visual display of the distribution of energy from one scatter point onto the prestack hyperbolic surface for a dipping event. *The inverse concept of the image in Figure 4c is to define the location of energy on the surface of Cheop's pyramid that will prestack migrate dipping energy to one scatter point.*

Energy concentration of a dipping event on Cheop's pyramid

Prestack migration assumes the summation of energy over the surface of Cheop's pyramid as defined by the DSR equation (1) and Figure 4b. The energy concentration on Cheop's pyramid from a dipping event will be found at the area of tangency as illustrated in Figure 4c. An example of energy concentration in the prestack volume obtained from modeling the seismic response of a short dipping event is shown in Figure 5.

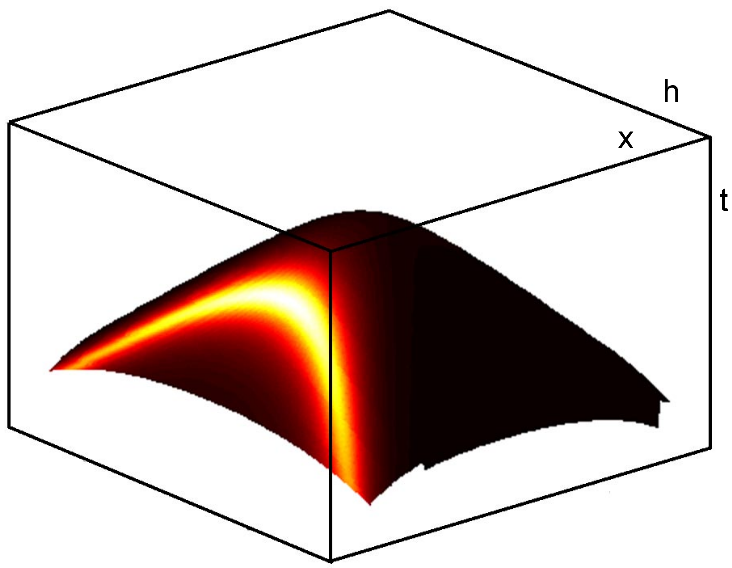

FIG. 5. Energy concentration of a short dipping event.

Energy distribution of a dipping event in CSP gather

All energy within the prestack migration aperture is collected into each CSP gather. Energy that lies on the tangency of Cheop's pyramid with the dipping cylinder will reconstruct on the hyperbola in the CSP gather that passes through the scatter point. The distribution in the CSP gather is found by rotating the energy (in the boomerang) of Figure 4c into the CSP plane.

Energy with the smallest equivalent offset comes from the zero offset point on the curve of tangency of Figure 4c. As the source-receiver offset is increased, the energy concentration on the tangency moves progressively to more distant equivalent offsets. Some dipping energy at zero offset will also reconstruct, however most other data will be attenuated.

Data with dipping structure is often complex and contains more scattered energy than coherent dipping reflectors. Even small structures in horizontally layered media may produce larger distributions of energy along the hyperbolas of a CSP gather. Figure 6 illustrates energy concentration of a dipping event (channel) in real data for an area close to that shown in Figure 3. The location of the energy on the hyperbola gives an indication of the geological dip.

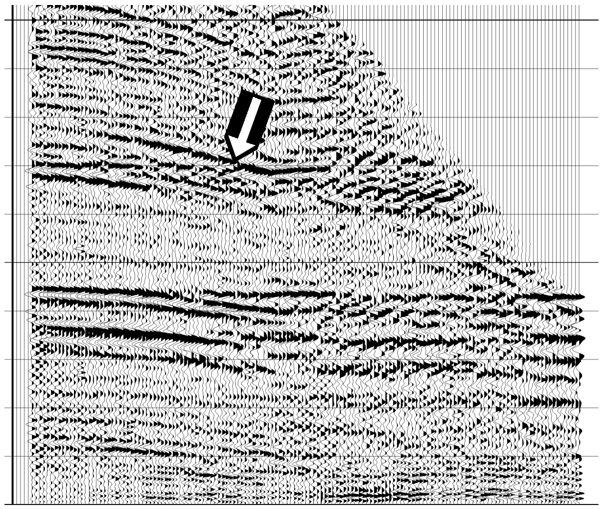

FIG. 6. Illustration of energy concentrated on a hyperbola from a dipping event.

CONCLUSIONS

Scattered energy from linear horizontal and dipping events tends to concentrate in localized areas on Cheop's pyramid. This concentration of energy is also seen on CSP gathers where scattered energy of horizontal data is centered about zero offset and limited to the source receiver offset. For dipping linear reflectors energy is concentrated down the flank of the hyperbolas with little energy at smaller offsets. The lack of smaller offsets may cause some lack of resolution for velocity analysis. Energy from complex structures will tend to have energy distributed across all offsets. The complex formation of energy from horizontal reflectors produces a prow effect which is a combination of scattered and zero offset energy. This energy may be visualized as a combination NMO and post stack migration of horizontal events.

REFERENCES

Bancroft, J. C., and Geiger, H. D., 1994, Equivalent offsets and CRP gathers for prestack migration, Expanded Abstracts 1994 SEG International Convention, pp. 672-675.

Bancroft, J. C., Geiger, H. D., Margrave, G. F., Wang, S., Foltinek, D. S. 1996, Prestack migration by equivalent offset and CSP gathers, CREWES Research Report.

Levin, F. K., 1971, Apparent velocity from dipping interface reflections: Geophysics, **36**, 510-516.

Yilmaz, O., 1987, Seismic Data Processing, SEG, Tulsa.