Linking petrophysical parameters (ρ, σ, μ, λ, κ, φ) with seismic parameters (α, β, R_{PP}, R_{PS}, R_{SS})

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ABSTRACT

Seismic data contains more information than what a conventional seismic section display can offer. Using seismic data as a hydrocarbon indicator lies in the successful extraction of useful petrophysical parameters from simplified Zoeppritz's equation and linking them with lithology or rock properties or even hydrocarbons. The incorporation of other geological well logs and core data with these extracted petrophysical parameters is the key to accurate interpretation. This study investigates the link between the seismic and petrophysical parameters through theory and its application.

INTRODUCTION

Seismic wave propagation in the earth is affected not only by the physical state of the media (solid, liquid or gas) but also by other physical properties such as the density of the rock, the pore size, fluid content, depth of burial and differential pressure, etc. The physical properties of the earth can be measured in situ using acoustic sonic logging system, or by laboratory experiments, or through the reconstruction of "petrophysical image" of subsurface rock property variation through analysis of seismic and other data.

In exploration geophysics, we estimate an earth model from seismic data. Conventional processing yields an earth model in time, which is "smoothly varying" , both in time and space. This smooth characteristic does not require ray bends at layer boundaries to be honored. In contrast, seismic inversion yields an earth model in depth, which carries more stringent accuracy requirements. Strong lateral velocity variations and ray bending at layer boundaries must be accounted for within inversion techniques. Therefore, conventional processing can be largely automated, while an inversion requires interpretation pause at each layer boundary, or multiple interactions. Specifically, the velocity-depth ambiguity inherent with inversion requires independent estimates of layer velocities and reflector geometries - two primary components of an earth model.

The purpose of this study is to look into the link between the seismic parameters (eg. the P-wave velocity - α, S-wave velocity - β, and the reflection coefficients R_{PP}, R_{PS}, R_{SS}) and the petrophysical parameters (e.g. ρ–density, σ–Poisson's ratio, μ–shear modules, λ–Lame's constant, κ–Young's modules, φ–porosity, etc.) and to understand more fully what seismic waves can tell about rock properties and how to extract the desired petrophysical parameters from seismic data.

MATHEMATICAL COMPLEXITY

We shall begin with the various assumptions of the models of the earth and limitations of the methods used in seismic wavefield. Figure 1 shows diagrammatically, the hierarchy of mathematical complexity.

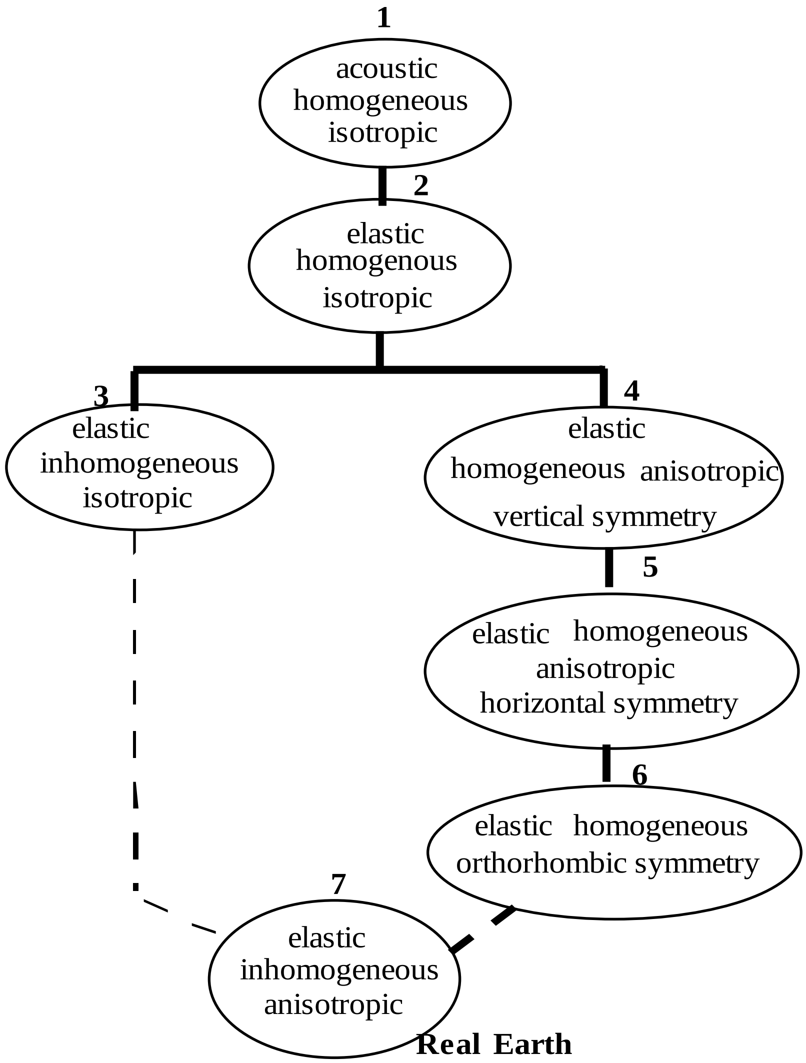

Fig.1. Hierarchy of mathematical complexity of the earth model

Level 1 is the simplest and the most unlike the real earth. No energy conversion, transmission losses, or AVO (amplitude versus offset) effects are admitted and everything can be explained by ray theory. Level 2 admits energy conversion and ray theory is still adequate. At level 3, we use wave equations rather than ray theory. We admit much thinner beds (down to 1/8 wavelength in thickness), AVO effects, velocity gradients, etc. If the thin beds get thinner than 1/8 wavelength, then the seismic wave gets dispersed and its reflection is accompanied by wavelet interference. Seismic waves reacts as if the earth were anisotropic rather than inhomogeneous.

At level 4, anisotropy due to thin layering or shaliness is considered. Level 5 considers anisotropy due to thin vertical cracks where the seismic velocities vary with azimuth. At level 6, P-wave velocities along the three orthogonal symmetry axes differ and different S-wave bifriengence in the three directions. It is at level 4 and above, more factors are taken into account, such as thin bed tuning, reflector curvature, geometric spreading, transmission or anelastic losses, geophone coupling, or source-receiver directivity, etc.

SCALE OF MEASUREMENTS

The commonly used technology to detect lithology, gas and rock properties is well logging. In the laboratory, we simulate the field conditions and measure the petrophysical parameters and attempt to relate these parameters to rock or reservoir properties. In seismic prospecting, velocities are determined by intervals.

Figure 2 illustrates a comparison of the wavelengths from three different measurements: 1) the laboratory measurement, 2) sonic logging, and 3) seismic survey. The laboratory measurement of petrophysical properties are in the range of one-tenth feet. Sonic logging tools measure over a 2 ft interval. Stratigraphers describe rock layers, starting with the thinnest, as lamina, beds, para-sequences, sequences. A lamina is usually a rather homogeneous rock unit, thin (inches) but of significant lateral extent. In terms of seismic data, the thinnest unit we might examine effectively is the sequence (tens of feet).

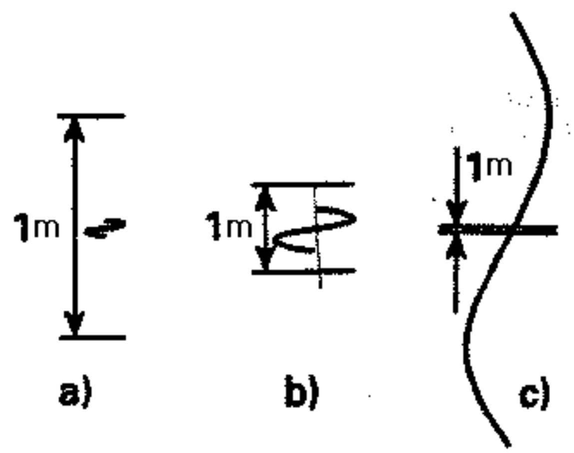

Fig. 2. Schematic wavelet of typical wavelength for a) laboratory measurement, b) sonic logging, and c) seismic survey

The frequencies employed in these different surveys are quite different from each other as shown in figure 3: 50 Hz for seismic survey, 10kHz for logging. The values we obtain from seismic data represent some kind of average. To analyze the changes undergone by an acoustic wave traveling through a bed, it is necessary to be able to distinguish between the reflections at the top and bottom of the bed. Since the resolution of the measurement is proportional to this wavelength, any increase in the maximum usable frequency in the recording enhances the knowledge of the subsurface.

The frequency content of the recorded signal can also be increased by limiting the distance traveled in the subsurface. By recording in a borehole, vertical and multiple offset seismic profiles permit shorter travel paths to recorders inside the borehole. Whatever the frequency is, a wave is reflected from a discontinuities of elastic or anelastic properties, for instance when traveling from a gas-saturated bed to a liquid-saturated bed. This reflection can be detected. This variation in the reflection, as a function of variations in angle of incidence, for example, can provide valuable data about the interface. These problems of scale can be partly limited in certain cases by using the knowledge of the interfaces obtained from different technologies.

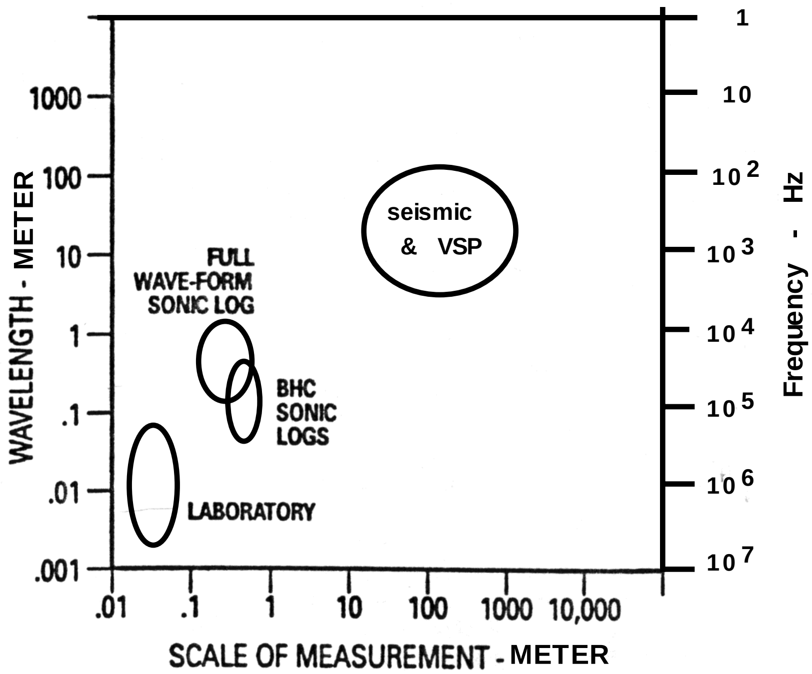

Fig. 3 Relation of wavelengths and frequencies and scale of measurement(modified from Marsden, 1993)

WAVEFIELD AT INTERFACE

The amplitude of a reflected P-wave from an interface is governed by four independent parameters which can be expressed as (α_1/ α_2, ρ_1/ ρ_2, β_1, and β_2). The extraction of any one of these parameters from the variation of reflection amplitudes with offset requires information about the other three; either in the form of borehole logs or else laboratory measurements.

The change from one wave type to the other (i.e. mode conversion) can ensue through reflection & transmission at oblique incidence or through supercritical refraction of diffraction at a solid elastic medium. In the simplest case of a flat horizontal interface (at level 2 in fig. 1) between two isotropic and homogeneous elastic half spaces, both with constant P-wave velocity - α; S-wave velocity - β and density ρ. At the interface, P-wave and PS-wave are in general coupled by the boundary conditions. We can derive the wave equations for P- and SV-waves from the elastic equations of motion and the boundary conditions of both the displacement and the stress are continuos. Four new waves are generated when a P- or SV wave incident upon a plane interface, where ρ and α or β are discontinuous , as shown in Fig. 4. The subscripts 1 refers to the upper medium and 2 refers to the lower medium. Where θ_1 (or θ_2) represent P-wave incident (or P-wave transmitted/refracted) and ϕ_1 (or ϕ_2) represent PS-wave reflected angle (or transmitted/refracted angle). All four waves obey Snell's law. That is

$$p = \frac{\sin\theta_1}{\alpha_1} = \frac{\sin\phi_1}{\beta_1} = \frac{\sin\theta_2}{\alpha_2} = \frac{\sin\phi_2}{\beta_2} \tag{1}$$

and p is the ray parameter.

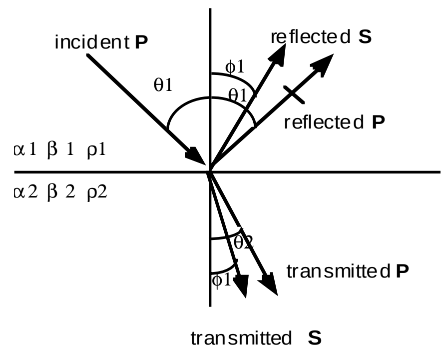

Fig. 4 Energy partitioned at the interface of elastic media.

APPROXIMATION OF ZOEPPRITZ EQUATIONS

The exact P-P and P-S reflection coefficients are expressed by Zoeppritz equations. The complexity of the Zoeppritz equations defies physical insight. A single major problem posed in extracting rock property information from the shape of reflection coefficient curves is that there are more unknowns than there are equations. To arrive at approximate solutions, however, simplifications are made to the equations governing reflections (the Zoeppritz equations), and empirical relationships of dependency are established between some of the parameters. Many authors have developed useful approximations and most of them are derived from Aki and Richard's (1980). We list the Aki and Richard equations of R_{PP}, R_{PS}, and R_{SS} as follows:

$$R_{PP} = \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\alpha} - 4p^2\beta^2\frac{\Delta\beta}{\beta} + \frac{1}{2}(1-4p^2\beta^2)\frac{\Delta\rho}{\rho} \tag{2}$$

$$R_{PS} = -\frac{p\alpha}{2\cos\phi}[(1-2\beta^2p^2 + 2\beta^2\frac{\cos\theta}{\alpha}\frac{\cos\phi}{\beta})\frac{\Delta\rho}{\rho} - (4\beta^2p^2 - 4\beta^2\frac{\cos\theta}{\alpha}\frac{\cos\phi}{\beta})\frac{\Delta\beta}{\beta}] \tag{3}$$

$$R_{SS} = -(\frac{1}{2\cos^2\phi}1-4p^2\beta^2)\frac{\Delta\beta}{\beta} - \frac{1}{2}(1-4p^2\beta^2)\frac{\Delta\rho}{\rho} \tag{4}$$

The impedance contrast across an interface are assumed to be very small, or $\|\frac{\Delta\alpha}{\alpha}\| << 1$, $\|\frac{\Delta\beta}{\beta}\| << 1$, and $\|\frac{\Delta\rho}{\rho}\| << 1$. The P-wave, PS-wave, and SH-SH wave reflection coefficients in the Aki and Richards equations are simplified and expressed in terms of fractional change in P-wave velocity, PS-wave velocity and their density. Where α, β, and ρ are average values of the two media. The fractional changes ($\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$) equal to the actual contrasts from above to below the interface divided by the average value of the two layers.

$$\frac{\Delta\alpha}{2\alpha} = \frac{\alpha_2 - \alpha_1}{\alpha_2 + \alpha_1} \quad (5), \qquad \frac{\Delta\beta}{2\beta} = \frac{\beta_2 - \beta_1}{\beta_2 + \beta_1} \quad (6), \qquad \frac{\Delta\rho}{2\rho} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad (7)$$

Many other forms of reflection coefficients approximation with emphasis on different applications also provide some insight on the reflectivity behavior. We'll look at some of the better known reflectivity approximation:

1. Shuey's (1985) approximation:

$$R_{PP} = R_{P0} + [\frac{1}{2}\frac{\Delta\alpha}{\alpha} - 2(R_{P0} + \frac{\Delta\alpha}{\alpha})\frac{1-2\sigma}{1-\sigma} + \frac{\Delta\sigma}{(1-\sigma)^2}]\sin^2\theta + \frac{1}{2}\frac{\Delta\alpha}{\alpha}(\tan^2\theta - \sin^2\theta) \tag{8}$$

Shuey's equation is arranged in terms of incident angle and eliminates the properties in β and $\Delta\beta$ in favor of Poisson's ratio σ and $\Delta\sigma$ through σ's relationship with α, β :

$$\sigma = (0.5\frac{\alpha^2}{\beta^2} - 1) \Big/ (\frac{\alpha^2}{\beta^2} - 1) \,. \tag{9}$$

Shuey claims that Poisson's ratio is the elastic property most directly related to angular dependence of reflection coefficient. The first term is the zero-offset reflection coefficient, the second term characterizes at intermediate angles, and the third term describes the approach to critical angle. The third term reveals that the reflection amplitude at wide angles relates only to the change in P-wave velocity.

2. Parson's (1986) approximation:

$$R_{PP} = \frac{1}{4}(1+\tan^2\theta)\frac{\Delta(\lambda+2\mu)}{(\lambda+2\mu)} - 2\gamma^2\sin^2\theta\frac{\Delta\mu}{\mu} + \frac{1}{4}(1-\tan^2\theta)\frac{\Delta\rho}{\rho} \tag{10}$$

Parson's equation can be derived from the Aki and Richards equations, where $\gamma = \frac{\beta}{\alpha}$, $\alpha^2 = \frac{\lambda+2\mu}{\rho}$, and $\beta^2 = \frac{\mu}{\rho}$

3. Smith and Gildlow's (1987):

$$R_{PP} = \left(\frac{5}{8} - \frac{1}{2}\gamma^2 \sin^2\theta + \frac{1}{2}\tan^2\theta\right)\frac{\Delta\alpha}{\alpha} - 4\gamma^2 \sin^2\theta \frac{\Delta\beta}{\beta} \qquad (11)$$

Smith and Gildlow use the equation of Aki and Richards, but replace the density with P-wave velocity using Gardner's empirical relation ($\rho \approx k\alpha^{1/4}$ --> $\frac{\Delta\rho}{\rho} \approx \frac{1}{4}\frac{\Delta\alpha}{\alpha}$, k is a constant and $\gamma = \frac{\beta}{\alpha}$) . Smith and Gildow also define the fluid factor as

$$\Delta F = \frac{\Delta\alpha}{\alpha} - 1.16\gamma\frac{\Delta\beta}{\beta} \qquad (12)$$

where the 2nd term is the value of $\Delta\alpha/\alpha$ predicted from $\Delta\beta/\beta$ using Castagnas's mud rock- line.

4. Fatti's (1994) approximation:

$$R_{PP} = \frac{1}{4}(1 + \tan^2\theta)\frac{\Delta I_{PP}}{I_{PP}} - 8\gamma^2 \sin^2\theta \frac{\Delta I_S}{I_S} - \frac{1}{2}(1 - \tan^2\theta)\frac{\Delta\rho}{\rho} \qquad (13)$$

Goodway et. al. (1997) used this linear equation to invert I_P, and I_S. Then they used impedance relationships to extract values of density x Lame`s moduli (i.e. $\lambda\rho$ and $\mu\rho$). They used these two parameters as gas indicator successfully. Note: I_P and I_S are the average acoustic impedances as expressed in eqns 13, 14, and 15.

$$\frac{\Delta I_{PP}}{2I_{PP}} = \frac{I_{PP2} - I_{PP1}}{I_{PP2} + I_{PP1}} = \frac{\rho_2\alpha_2 - \rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1} \qquad (14)$$

$$\frac{\Delta I_{SS}}{2I_{SS}} = \frac{I_{SS2} - I_{SS1}}{I_{SS2} + I_{SS1}} = \frac{\rho_2\beta_2 - \rho_1\beta_1}{\rho_2\beta_2 + \rho_1\beta_1} \qquad (15)$$

5. Hilterman's (1990)

$$R_{PP} = R_{P0}\cos^2\theta + \frac{\Delta\sigma}{(1-\sigma)^2}\sin^2\theta \qquad (16)$$

As noted by Hilterman (1990), the first term is more strongly associated with chronostratigraphy (the macro layer/low frequency) and the 2nd term can be used as a lithostratigraphic (the lithology related information) tool.

where R_{P0} is the zero-offset P-wave reflectivity

$$R_{P0} = \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right) \tag{17}$$

$$\frac{\Delta(\rho\alpha)}{2\rho\alpha} = \frac{(\rho\alpha)_2 - (\rho\alpha)_1}{(\rho\alpha)_2 + (\rho\alpha)_1} = \frac{\rho\Delta\alpha - \alpha\Delta\rho}{2\rho\alpha} = \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right) \tag{18}$$

6. Bortfield's(1961) approximation:

$$R_{PP} = \frac{1}{2}\ln\frac{\alpha_2\beta_2\cos\theta_1}{\alpha_1\beta_1\cos\theta_2} + \frac{\sin^2\theta_1}{\alpha_1^2}(\beta_1{}^2 - \beta_1{}^2)[2 + \ln\frac{\rho_2}{\rho_1}/(\ln\frac{\alpha_2}{\alpha_1} - \ln\frac{\alpha_2\rho_1}{\alpha_1\rho_2})] \tag{19}$$

OTHER RESERVOIR PROPERTIES

Other reservoir properties such as porosity or fluid saturation can also be estimated using seismic data. For isotropic porous media, fluid saturation effects on bulk and shear moduli (κ & μ) at the low frequency limit can be simplified using the Biot-Gassmann equation:

$$\kappa = \frac{\kappa_S - \kappa_d}{\kappa_S\left(1 - \varphi - \frac{\kappa_d}{\kappa_S} + \varphi\frac{\kappa_S}{\kappa_f}\right)} + \kappa_d + \frac{4}{3}\mu_d \tag{20}$$

where κ_S, κ_d, κ_f, κ are the bulk modules of solid material, dry material, porous fluid respectively and μ_d, μ are the shear modules for porous frame and porous frame with fluid saturation.

In AVO analysis, many of the above parameters have to be estimated with uncertainty. Incompatible parameters can generate errors. There is no direct way to judge whether the calculated result is correct or not. We have to understand how the individual, or combination of rock parameters do affect velocity with different fluid saturations.

CONVERTED WAVE VERSUS PURE SHEAR WAVE REFLECTIVITY

From Aki and Richards converted wave reflectivity equation R_{PS} and pure SH reflectivity R_{SS} ,we can derive its relationship as follows:

From equation 3, for small offset, i.e. θ and ϕ are small, then $\cos\theta = \cos\phi = 1$,equation 3 becomes:

$$R_{PS} = \frac{-p\alpha}{2}[(1 - 2\beta^2p^2 + 2\gamma)\frac{\Delta\rho}{\rho} - (4\beta^2p^2 - 4\gamma)\frac{\Delta\beta}{\beta}]$$

$$R_{PS} = \frac{-p\alpha}{2}[-8\sin^2\phi R_{S0} - 8\gamma R_{S0} + (1 - 2\gamma)\frac{\Delta\rho}{\rho}],$$

since θ and ϕ are small , the first term is very small, then

$$R_{PS} = \frac{-\sin\theta}{2}[-8\gamma R_{S0} + (1-2\gamma)\frac{\Delta\rho}{\rho}]\text{, or}$$

$$R_{PS} \cong 4\gamma\sin\phi\left[R_{S0} - \frac{(1-2\gamma)}{8\gamma}\frac{\Delta\rho}{\rho}\right], \tag{21}$$

where

$$R_{S0} \cong -\frac{1}{2}\left[\frac{\Delta\rho}{\rho} + \frac{\Delta\beta}{\beta}\right] \tag{22}$$

in a same manner, R_{SS} can be expressed in terms of R_{S0}

$$R_{SS} = -R_{S0} + \frac{1}{2}\frac{\Delta\beta}{\beta}\sin^2\theta \tag{23}$$

R_{S0} is the zero offset approximate reflectivity for SH wave.

If $\gamma = 1/2$, then the 2nd term in eqn. (21) vanishes leaving

$$R_{PS} = 2 * \sin\phi * R_{S0} \tag{24}$$

We therefore can say that the reflectivity of R_{PS} at a given offset will scale up or down proportionally to the R_{SS} reflectivity and is insensitive to R_{PP} reflectivity at the interface.

In general, P-wave reflection coefficient (R_{PP}) varies with local angle of incidence θ according to :

$$R_{PP} \approx R_{P0} + (R_{P0} - 2R_s)\sin^2\theta \tag{25}$$

This linear equation has a slope (or gradient) equal to (R_{P0} - 2 R_{PS}), it reveals that the reflection amplitude on the near-offset traces (θ = 0) primarily represents the normal incidence P-wave reflectivity. The far-offset controlled by a linear combination of R_{P0} and the normal incidence shear reflectivity R_{S0}. Thus, the new additional piece of information extracted is the shear reflection coefficient which is what we attempt to record by using multicomponent geophones, except that R_{S0} refers to S-waves polarized in the plane of incidence (i.e. SV) while typically field shear records S-wave polarized transverse to that plane (SH wave).

DISCUSSION

The angle of incident can be computed for each sample in a normal moveout corrected CMP gather. Any of the above approximated Zoeppritz equations can then be fitted to the amplitudes of all the traces at each time sample of the gather. The reflectivity information is then transformed into velocity, density or into elastic properties of interpreter’s choice. The extracted parameters from seismic are easily understood deterministically from laboratory and theoretical results. A petrophysical seismic section can be generated and displayed for further interpretation. One common used statistical analysis is the cross-plot or clustering analysis which provides a way to examine multiple seismic parameters to identify interewell regions that statistically resemble the seismic properties at locations known to be favorable. An advantage of clustering analysis is that many attributes are incorporated, even through their physical connections with reservoir properties may not be understood.

If converted seismic data is available, we can map useful petrophysical parameters through the area. Each of these mappings can be incorporated with statistics inferred from well logs and cores, and then used as hydrocarbon or lithology indicators.

The reflection coefficient is related to seismic amplitude through knowledge of the impulse response and wavelet. When the incident angle smaller than 35^o, Zoeppritz's equations or the Aki and Richards equations can be simplified to a linear equation of fractional change of any two petrophysical parameters. These fractional changes of petrophysical parameters are then determined by fitting the curve defined by equation to the reflection amplitudes of an NMO corrected CMP gather. A petrophysical seismic section is then displayed for further interpretation. When incorporate with other geological knowledge derived from well logs, cores, and theory, we can use it as a lithology or even a hydrocarbon indicator.

This seismically derived petrophysical parameter technology does not attempt to recover absolute rock properties but rather involves the construction of an image of subsurface rock property variation and hopefully correlate them to the hydrocarbon or reservoir properties.

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