A new algorithm for converted wave pre-stack migration

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ABSTRACT

A method for converted wave pre-stack time migration is presented. This method splits the Kirchhoff time migration into two steps using the concept of equivalent offset. Migration gathers are first constructed, and these gathers provide accurate migration velocity information. The second step consists of normal moveout (NMO) correction and conventional CDP stacking applied on migration gathers.

The main difficulty in implementing this method is the accurate computing of the equivalent offsets. Different from P-P wave case, we use an approximate solution for the converted wave case. Synthetic and real data experiments prove that this method can provide very good imaging results.

INTRODUCTION

The concept of equivalent offset was first introduced by Bancroft and Geiger(1994). Its applications to seismic data migration have been very successful. Wang, Bancroft and Lawton (1996) applied this concept to converted wave data with very good results.

This paper introduces a new algorithm to perform equivalent offset migration (EOM) for converted wave data. This algorithm is a direct generalization of the algorithm for P-P wave case, so it is easier to understand and implement. Also, this algorithm is faster than the method by Wang, Bancroft and Lawton (1996).

Our algorithm is an approximate solution. Its accuracy analysis is discussed in this paper. Experiments with synthetic and real data prove that our method is practical and accurate.

KINEMATICS OF CONVERTED WAVE MIGRATION

The two-way travel time response of a scatter point in constant velocity case can be expressed as a double square root (DSR) equation. (see Figure 1)

$$t = \frac{\sqrt{(x+h)^2 + z^2}}{V_{down}} + \frac{\sqrt{(x-h)^2 + z^2}}{V_{up}}. \tag{1}$$

For simplicity, we suppose that the source-receiver midpoint is at the origin.

As shown in Figure 2, if the upcoming velocity and the downgoing velocity are all straight line velocities (can be approximated by P-P RMS velocity and the S-S RMS velocity), the travel time response of a scatter point in layered model can also be expresses as equation (1).

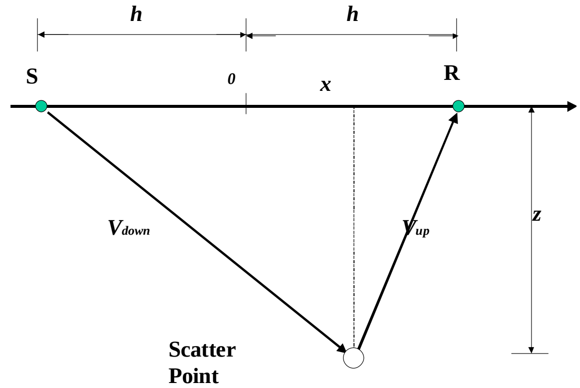

Figure 1: The scatter point model for constant velocity case. This model is the same for seismic experiments with or without mode conversion. The only difference is if the upcoming and the downgoing velocity are the same or not. The velocities have nothing to do with the scattering location at this point.

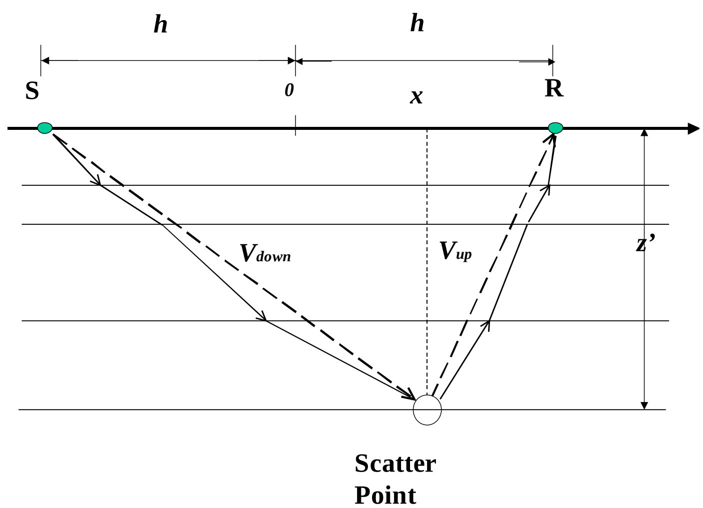

Figure 2: The scatter point model for layered velocity model. The downgoing velocity and the upcoming velocity are used for straight ray paths.

Migration can be considered as the inverse process of seismic experiments. Given a seismic sample, we know its recorded travel time and the offset, provided that the velocity is known, equation (1) is a relation between the depth ***z*** (psuedo-depth) and the migration distance ***x***, i.e.,

$$z^2 = \left(\frac{\gamma}{\beta}\right)^2 \left[\alpha v^2 t^2 + 4\beta x h - 2vt\sqrt{\gamma^2 v^2 t^2 + 4\beta x h}\right] - (x+h)^2 . \tag{2}$$

Where $\gamma = V_{down}/V_{up}$ is the velocity ratio, $v = V_{up}$, $\alpha = \gamma^2 + 1$ and $\beta = \gamma^2 - 1$ are just for shorten the equation. Equation (2) was first obtained by a student at Stanford Exploration Project (SEP) and has been used by Alfaraj and Larner (1992).

We do not really use equation (2) because it is a relation involving the depth which is always unknown in real seismic data. For time migration, we introduce two-way travel time as following,

$$\tau = \frac{z}{V_{down}} + \frac{z}{V_{up}} = \frac{\gamma}{1+\gamma} \cdot \frac{z}{v}. \tag{3}$$

Equation (2) and (3) together provide a relation between zero-offset time and the migration distance. This relation gives us the migration response of an input seismic sample. Figure 3 shows the picture of the migration response of an input sample with arrival time at 1.0 second and source-receiver offset at 1000 meters. The dashed curve is an ellipse which corresponds to migration response of this sample for no-mode-conversion case with the P-wave and S-wave average velocity. The solid curve is shifted and stretched (not elliptic any more) because of the difference of the downgoing and upcoming wave velocities.

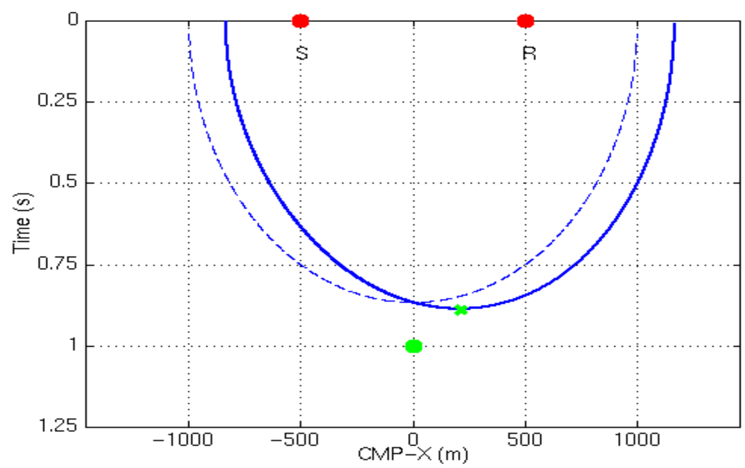

Figure 3: The pre-stack migration response of an input seismic sample

EQUIVALENT OFFSET MIGRATION

Equivalent offset migration splits pre-stack migration into two steps: The first step is a partial migration which completes the energy distribution in space domain, where no time shift is involved. The second step is the process that completes the energy distribution only in time direction. For an input sample, at different migration distance x, we assign a new offset value for the distributed energy instead of moving in time direction. Figure 4 shows some kinematics of EOM process.

Same as in P-P case, the second step of the EOM process is a NMO correction applied on the partially space-direction migrated data,

$$(V_{mig}t)^2 = (V_{mig}\tau)^2 + 4h_e^2 . \tag{4}$$

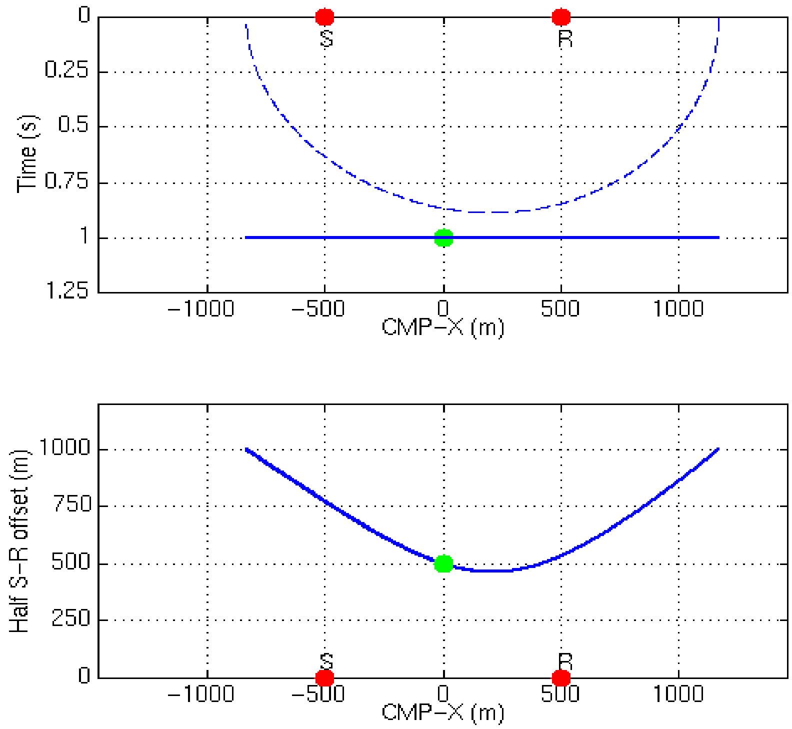

Figure 4: The definition of equivalent offset migration. The **upper part** is shown in space-time coordinates, where the dashed curve is the final migration response of the sample, the solid straight line demonstrates the energy distribution of the first step of EOM process. The limits of the energy distribution are determined by the limits of the migration response. The energy distribution to source direction and to receiver direction is not symmetric. The **lower part** shows how the equivalent offset changes with the migration distance.

It is important to mention that, unlike in the P-P case where equivalent offsets at different migration distances are always greater than the source-receiver offset, the equivalent offsets in converted wave case arrive their minimum value at the location where the mode conversion occurs. This minimum offset value is always smaller than the source-receiver offset. Thus, the equivalent offset can be explicitly expressed as

$$h_e^2 = x^2 + h^2 - 2xh\frac{\gamma - 1}{\gamma + 1} - \frac{16\gamma x^2 h^2}{(\gamma+1)^2 V_{mig}^2 t^2 + (\gamma^2 - 1)xh} + \cdots \tag{5}$$

Where (3) the migration velocity is defined as

$$V_{mig} = \frac{2\gamma}{1+\gamma} v. \tag{6}$$

By equation (4), (6) and (1), equivalent offset has another expression

$$2\frac{\sqrt{h_e^2+z^2}}{V_{mig}}=\frac{\sqrt{(x+h)^2+z^2}}{V_{down}}+\frac{\sqrt{(x-h)^2+z^2}}{V_{up}}. \tag{7}$$

From equation (7), the equivalent offset can be explained as the distance between a pair of co-located source-receiver and the CSP surface location as shown in Figure 5. This also shows that our new approach of the equivalent offset definition is exactly the same as that of Wang, Bancroft and Lawton (1996).

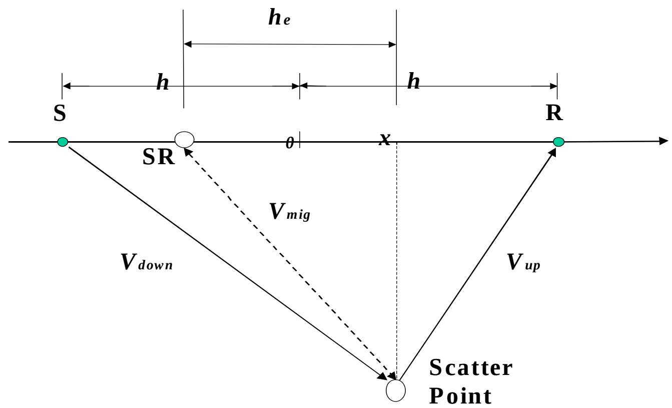

Figure 5: The geometric explanation of equivalent offset definition. The two way travel time from the “co-located” source-receiver SR to the scatter point (the left side of (7)) is equal to the recorded travel time at the receiver R from the source S (the right side of (7)).

MIGRATION VELOCITIES AND THEIR RATIOS

As previously mentioned, the migration velocity can be approximated by

$$V_{mig}=2\frac{V_P\cdot V_S}{V_P+V_S}=2\frac{V_P}{1+\gamma}. \tag{8}$$

This equation is accurate only for constant velocity model case, and it is a second order approximation in layered model case. This equation is simple but is difficult to use in practice as, (1) S-S velocity usually not available, (2) P-P, P-S, and S-S velocities are given in different times, and (3) the preliminary velocity ratio is not accurate.

Motivated by the work of Harrison and Stewart (1991), we tried another way to get migration velocity by using the present converted wave seismic data information,

$$V_{mig}=\frac{2\sqrt{\gamma}}{1+\gamma}V_{TB}, \tag{9}$$

where V_{TB} is the semblance velocity (Tessmer-Behle RMS velocity, Tessmer-Behle (1988)) picked on converted wave CMP gathers. Equation (9) is valid only for constant velocity model, but is robust for not complex structure. The ratio can be a guessed value, as in equation (8).

By using equation (9), we do not need any velocity information from P-P data, which can not really tell us the converted wave velocity information because our data is not re-binned in common conversion point (CCP) gathers. (This binning is another main topic about converted wave data processing.)

As in the P-P case, the first part of the EOM migration, i.e., the construction of common conversion scatter point (CCSP) gathers is not sensitive to velocity information, especially for simple structure cases. Equation (9) provide more than satisfactory preliminary velocity information for our EOM process.

We use the average velocity of P-P RMS velocity and S-S RMS velocity to approximate our migration velocity approximation. By careful consideration, we know this is the same migration velocity defined by Harrison and Stewart (1993).

A synthetic layered velocity model shown in Figure 6 provides an example to discuss the velocities and velocity ratios in more detail. For generality, we suppose that:

(1) the depth of the layer changes randomly between 200m to 1000m;

(2) the P-wave interval velocity changes randomly between 2000m/s to 4500m/s;

(3) the P-wave interval velocity to S-wave velocity ratio changes randomly between 1.5 to 3.8.

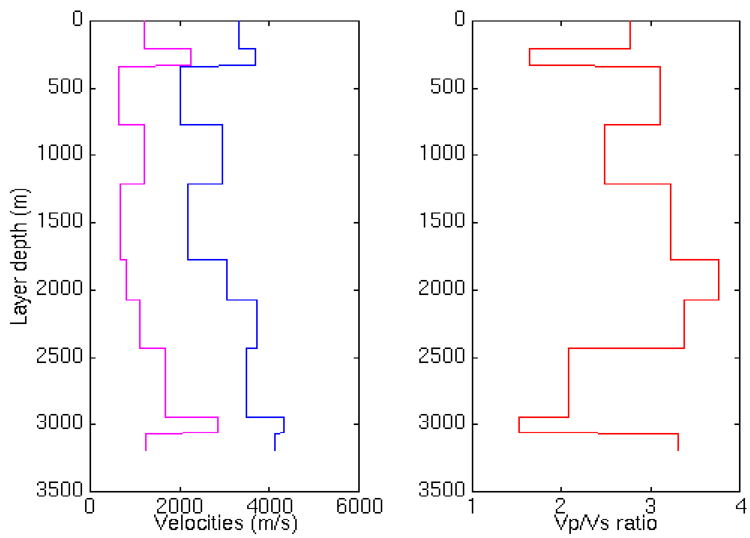

Figure 6: A synthetic layered model with randomly changed layer depth, P-wave layered velocity and P-wave to S-wave layered velocity ratio.

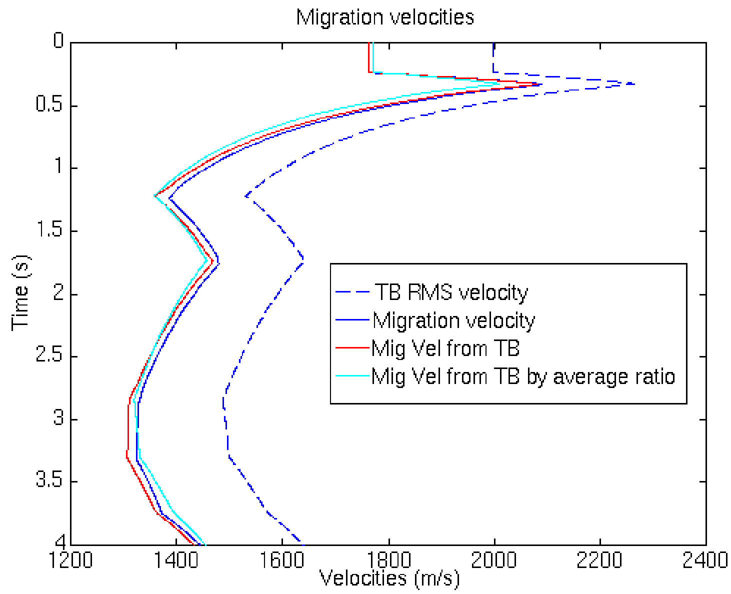

Figure 7: Obtaining migration velocity by equation (9). The “TB RMS velocity” is the RMS velocity defined by Tessmer and Behle (1988); the “Migration velocity” is calculated by using the averaged value of the P-wave RMS velocity and the S-wave RMS velocity, the “Mig Vel from TB” is calculated by equation (9) using the variant ratio of P-wave and S-wave RMS velocities. This ratio function of time is shown in figure 8.

Figure 7 gives some results of migration velocities calculated by different method. As mentioned by Harrison and Stewart (1993), the RMS velocity by Tessmer and Behle (1988) is always higher than migration velocity, the difference between these velocities can be obtained by equation (9). The velocity we used in our algorithm is the one calculated by using the averaged ratio (a guessed value in practice), it is very close to the accurate migration velocity.

A preliminary value of the migration velocity ratio is also important. Although normally we can not get a accurate ratio, a guessed value works well in our method. Figure 8 shows that the migration velocity ratio (solid curve) changes between 2.3 and 3.0, it is much smaller a range than the interval ratio. This also means that an averaged value (around 2.7) has errors within 0.35 instead of 1.2.

EQUIVALENT OFFSET COMPUTATION

The definition of equivalent offset in equation (5) does not have an analytic expression like in P-P case. This involves an approximation similar to Muir second order expansion discussed by Claerbout (1985). Like in the P-P case, we do not use equation (5) to compute equivalent offset for every sample (which has very large computation cost). Instead, we use this equation calculate a time at which an input trace moves to a new bin in the CCSP gather. .

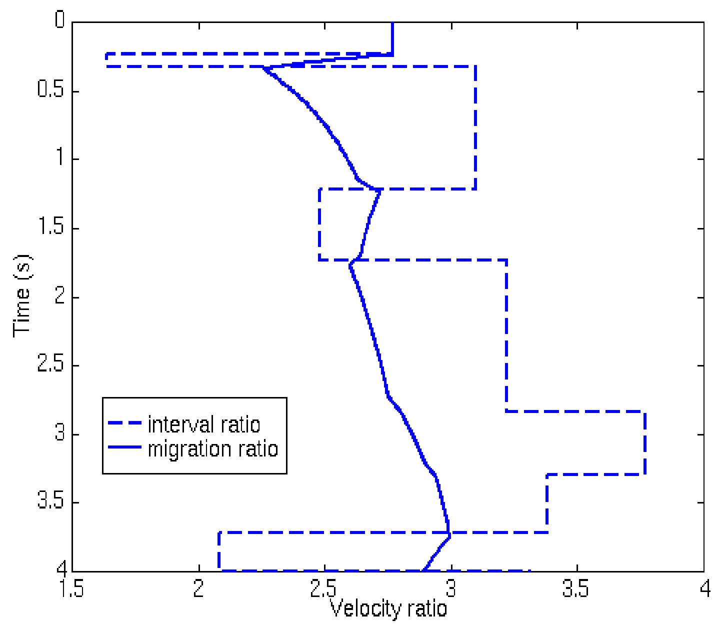

Figure 8: The interval velocity ratio and the velocity migration ratio: The interval Vp/Vs ratio (dashed line) ranges from 1.5 to 3.8 randomly, the ratio (solid curve) for migration has much smaller range: from 2.3 to 3.0

For simplify equation (6), we introduce

$$\theta = 1 - \frac{xh\left(\gamma^2 - 1\right)}{\gamma^2 V_{mig}^2 t^2}, \tag{10}$$

which is called a justifier. This justifier has a very limited range of variation (see Li and Bancroft (1997)). In practice, we simplify this justifier to a simple function only changing with zero-offset time. Equation (6) then can be expressed as

$$\theta\left[\left(V_{mig}\tau\right)^2 + 4h_e^2\right] = \frac{16x^2h^2}{x^2 + h^2 + 2xh\dfrac{\gamma - 1}{\gamma + 1} - h_e^2} \cdot \frac{\gamma}{(\gamma + 1)^2}. \tag{11}$$

Equation (11) is the formula we used to implement our algorithm. Its right hand side term can be easily computed, while the left hand side term function of zero-offset time only, similar as in Li and Bancroft (1996), we can generally say that this function is always increasing with the time. So we can get a zero-offset time value from (11), then it is easy to get the travel time value on the present trace from equation (4).

Figure 9 is a flow for implementing our algorithm. From this flow, there is no difference between our algorithm for converted wave EOM and the algorithm introduced for P-P wave EOM data in Li and Bancroft (1996).

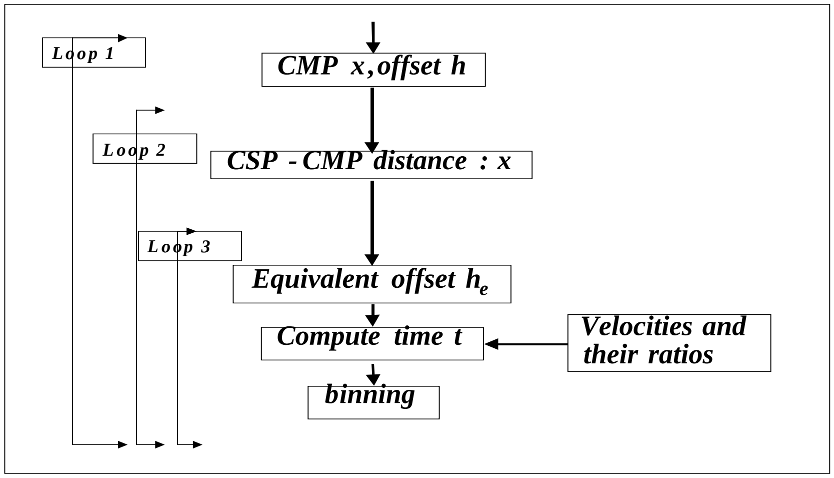

Figure 9: Implementation of our algorithm (exactly same as in P-P case)

ACCURACY ANALYSIS

The approximation approach of our algorithm is complex for analyzing the accuracy problem. By simple kinematics, we found that the accuracy problem has some behavior as following:

- The accuracy becomes better with smaller migration distance and the offset. The behavior of distance and offset is symmetric to our algorithm.

- The accuracy becomes better with higher migration velocity.

- The approximation error becomes smaller at larger zero offset time. In fact, when the zero-offset time is as large as 2 or 3 seconds, our approximation is very accurate. The relative error of the equation (6) and equation (11) is smaller than 1.0%. This is good because usually the target zone for converted wave data is deeper than 2 or 3 seconds.

EXAMPLES

Here we present some examples to demonstrate some important properties of CSP gathers, which is the result of the first step of EOM process.

Insensitive to the preliminary velocity error

Pre-stack migration requires velocity information, but CSP gather construction is a process that is not sensitive to the velocity information. Figure 11 is an example from a simple synthetic model.

CSP gathers provide migration velocity information.

During the construction of CSP gathers, the accuracy of velocity information is not very essential, but for the second step of EOM, which is NMO correction applied on CSP gathers, the velocity accuracy becomes very important. One of the advantages of EOM is that the CSP gathers provide very accurate migration velocity. Figure 12 shows a synthetic example.

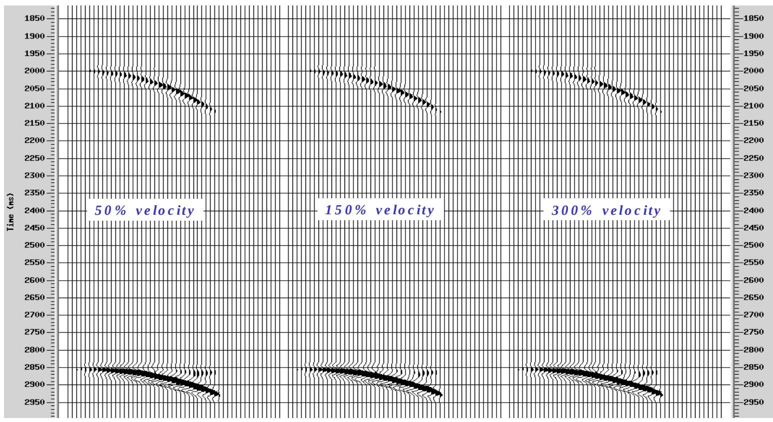

Figure 10: CSP construction process is insensitive to the preliminary velocity information. We show here three CSP gathers at the same location, but with three very different velocity as the input.

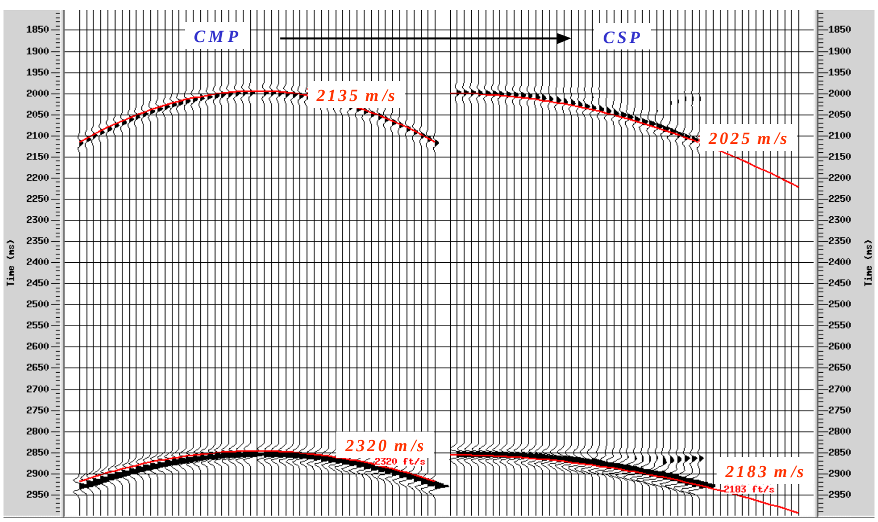

Figure 11: CSP gathers provide migration velocity. The left side is a CMP gather from converted wave data without CCP binning. The events are approximately hyperbolic. The right hand side is a CSP gather at the same location as the CMP gather. The hyperbolic events on CSP gathers provide accurate migration velocity, while the input velocity was the Tessmer-Behle RMS velocity as marked on this CMP gather.

Final image example

We construct a dip reflector model as in Figure 12 to test our algorithm. The result turns out very good. As expected, our result is better than the DMO plus post-stack migration processing result. And, it is important to mention that the computation cost of our method is close to P-S DMO algorithm. The results are shown in Figure 13 and Figure 14.

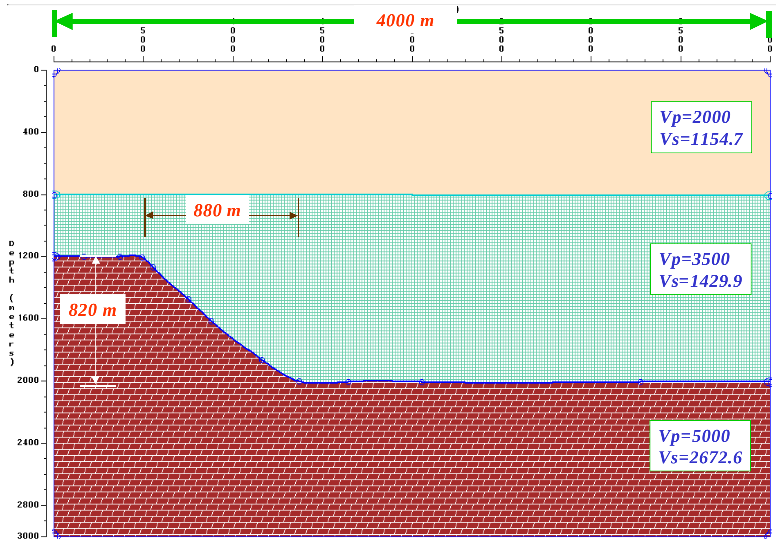

Figure 12: A dip reflector model: the dip is around 45 degree.

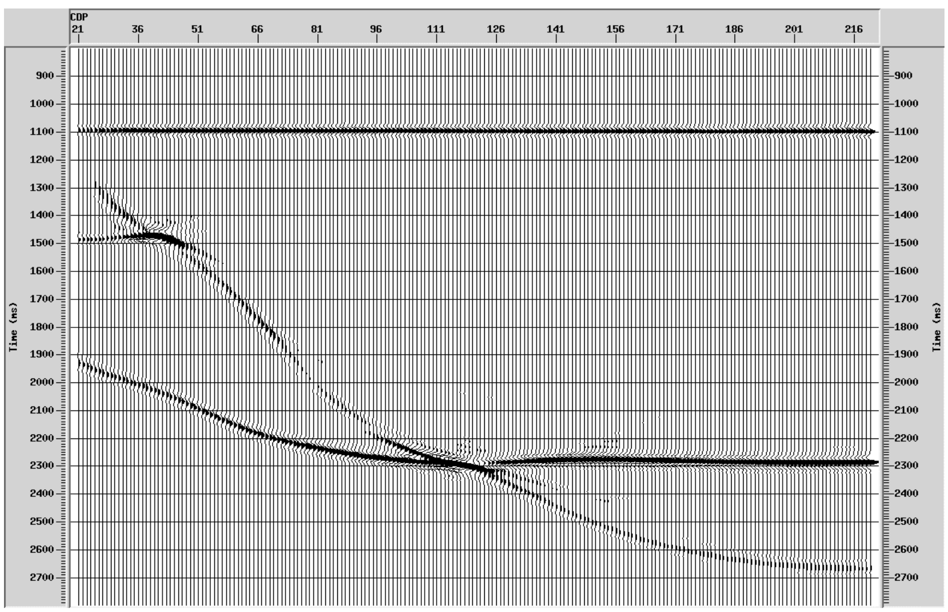

Figure 13: P-SV DMO followed by post stack Kirchhoff migration.

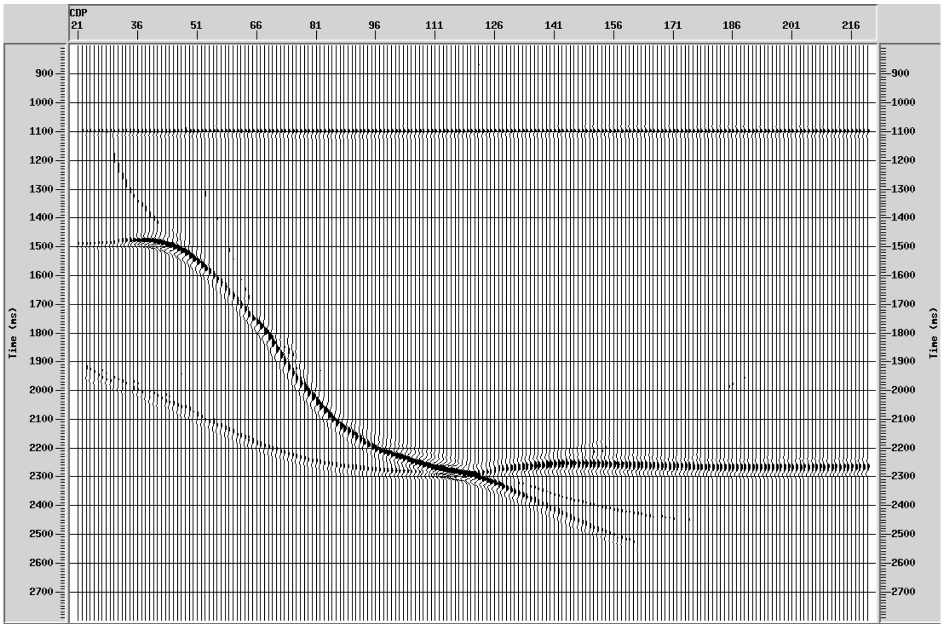

Figure 14: The migration result by our new algorithm. The dip energy is focused much better than the result by DMO plus post-stack migration.

CONCLUSIONS AND FUTURE WORK

The new algorithm we presented here is for converted wave pre-stack time migration. It has some advantages as following:

- Its computation cost is comparable to DMO plus post-stack migration, but the pre-stack image is better as expected.
- The algorithm is mainly insensitive to preliminary velocity. The CSP construction process involves no time shift.
- The CSP gathers provide very accurate migration velocity information for the second step of the EOM process.
- Our method can be an iterative method. More iteration usually results in better imaging. Practically, one iteration is usually enough.
- Like conventional Kirchhoff migration method, our method has no restrictions on the geometry of seismic experiments. CSP locations can be any where, they are not essentially depends on CMP locations or other geometry stations.
- Because our algorithm is a direct generalization of the P-P algorithm, it is very easy to implement.

Some possible future work is related to: (1) more accurate amplitude scaling during the CSP gather construction, (2) residual statics analysis as for the P-P wave case.

ACKNOWLEDGMENTS

The authors would like to thank the sponsors of The CREWES Project for their financial support. We are also grateful to the technical help of Mr. Darren Foltinek and Henry Bland and the work on real data processing using this method by Mr. Carlos Rodriguez, Mr. Christian Strand and Mr. Vladan Simin.

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