Dip limits on pre- and poststack Kirchhoff migrations

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ABSTRACT

Dip limits are considered for poststack and prestack Kirchhoff migrations.

INTRODUCTION (POSTSTACK CASE)

Early poststack Kirchhoff migration programs summed data within a migration aperture that was defined by a fixed distance h_{max}. This fixed aperture resulted in a section that was noisier at shallow times. It was recognized that shallower events had a larger T_0/T ratio at the aperture limit than the deeper events as illustrated in Figure 1. This figure shows a family of zero offset diffractions that result from a vertical array of scatter points in a constant velocity environment.

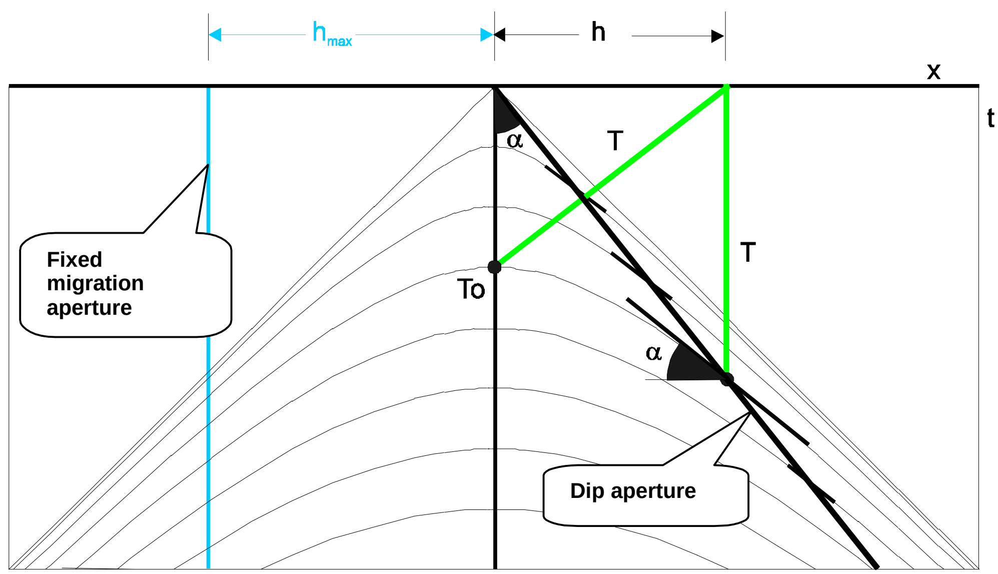

Figure 1 Family of constant velocity diffractions showing a migration aperture defined by h_{max}, and a dip dependent aperture defined by the dip a.

The T_0/T ratio may be used to define the geological dip b that is migrated from time T to the depth defined by T_0. With a fixed distance migration aperture h_{max}, shallower events were being migrated to steeper dips that deeper events. It may appear that a steeper dip migration would produce better results, and that is certainly true in data that contains steeply dipping energy. However, in areas with mild dipping energy, the increased dip range of the shallower summation window will only sum noise into the migrated section. If the noise on the input section is considered to be random, the noise will sum with an amplitude proportional to the square root of the aperture size. This noise would also tend to the square root of T. When the summation crosses other coherent events, the noise (partially due to aliasing) may be proportional to T. I will use the latter definition for noise proportional to T, as it is more convenient.

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In contrast to the noise, reflection energy may span a range that is proportional to T_0. The resulting signal to noise ratio on a Kirchhoff migrated section with a fixed aperture range, SNR_{Kir-f}, is given by

$$SNR_{Kir-f} \approx \frac{T_0}{T} \quad . \tag{1}$$

Consequently, shallower events will have a larger SNR_{Kir-f} than deeper events.

In order to maintain a more even signal to noise ratio, or to reduce the shallower noise, the aperture of each summation diffraction is limited to a range proportional to T_0. For a given T_0, the maximum time T_{max} of the summation diffraction may be defined by a ratio that is also proportion the cosine of the geological dip β, i.e.

$$\frac{T_0}{T_{\max}} = \cos\beta \quad . \tag{2}$$

The dip β is related to the apparent dip α on a time section by the migrator's equation

$$\tan\alpha = \sin\beta \quad . \tag{3}$$

Now the lateral extent of the summation diffractions may be limited to a slope α as illustrated in Figure 1. Note the extent of all diffractions is defined by one line that has an angle α with the vertical line through the scatter points (diffraction apex). The signal to noise ratio for the Kirchhoff migrated section SNR_{Kir} may now be modified to be

$$SNR_{Kir} \approx \frac{T_0}{\left(\dfrac{T_0}{\cos\sin^{-1}\tan\alpha}\right)} \tag{4}$$

or the constant value

$$SNR_{Kir} \approx \cos\sin^{-1}\tan\alpha \quad . \tag{5}$$

When the velocity is constant, the migration of each input time sample produces a semicircle of energy on the migrated section. When a dip limit of β_{max} is imposed on the migration, the semi-circle will only extend to that dip limit as illustrated in Figure 2.

It should be noted that dip limited migrations, whether by choice in a FK or Kirchhoff migration, or as constrained by finite different solutions to the wave equation, the processor must take care to ensure the migration dip limit β_{max} exceed the maximum dip of the migrated section. This limit must also include diffracted energy off dipping events. Failure to include all the diffracted energy may result in a loss of lateral resolution. Choice of a suitable dip limit is based on the dip limits of the stacked section and a reasonable signal-to-noise ratio.

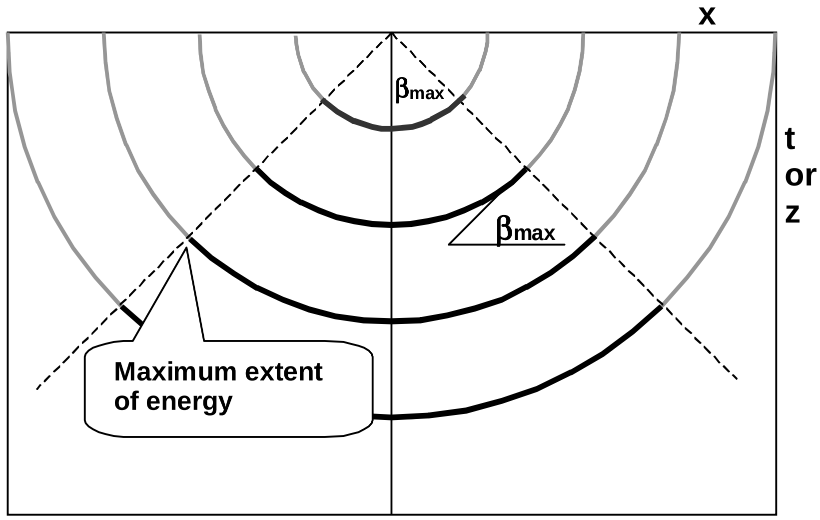

Figure 2 Migration of a number of time sample to produce dip limited semi-circles.

MIGRATED DIPS ON A CONSTANT OFFSET SECTION

The dip limit that is applied to poststack Kirchhoff migration may also be applied to prestack migration. The identification of the constant dip limit, however, becomes more complex than the zero-offset case. In the poststack case, the diffraction was a simple hyperbola (constant velocity or time migration) and in the prestack case the diffraction shape is defined by the double-square-root equation. With a constant velocity environment, the semi-circle of poststack migration becomes an ellipse in the prestack case.

Constant velocity

In the constant velocity case, input time samples migrate to an ellipse. The migration dip limit is equal to the slope on the ellipse as illustrated in Figure 3. This figure shows a family of ellipse and corresponding curves that define constant dips that range from 0 to 90 degrees in 5-degree increments. These constant dip curves (*X*, *Y*) are defined by parametric equations, (6a) for *Y*, and (6b) for *X*, i.e.,

$$Y(\beta, f, b) = \frac{b}{1 + \left(\tan^2 \beta\right)\left(1 + \frac{f^2}{b^2}\right)}, \tag{6a}$$

and

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$$X(\beta, f, b) = \left[\left(b^2 + f^2\right)\left\{1 - \frac{Y^2}{b^2}\right\}\right]^{1/2} = \left[\left(b^2 + f^2\right)\left\{1 - \frac{1}{1 + \tan^2\beta\left(1 + \frac{f^2}{b^2}\right)}\right\}\right]^{1/2}, \quad (6b)$$

where:

- the scatter point is located at *x = 0*,
- *a* is the length of the major axis and is defined by the depth equivalent if the input time *TV/2*,
- *b* is the length of the minor axis and defined by the depth equivalent of NMO corrected time T_nV,
- *f* is the distance to the focus point of the ellipse and defined by $f^2 = a^2 - b^2$, (also half the source-receiver offset *h*), and
- b is the geological dip angle.

In Figure 3, the constant angle curves were plotted with b and *h* constant, while *b* varied. Only one side of the family of ellipse is shown.

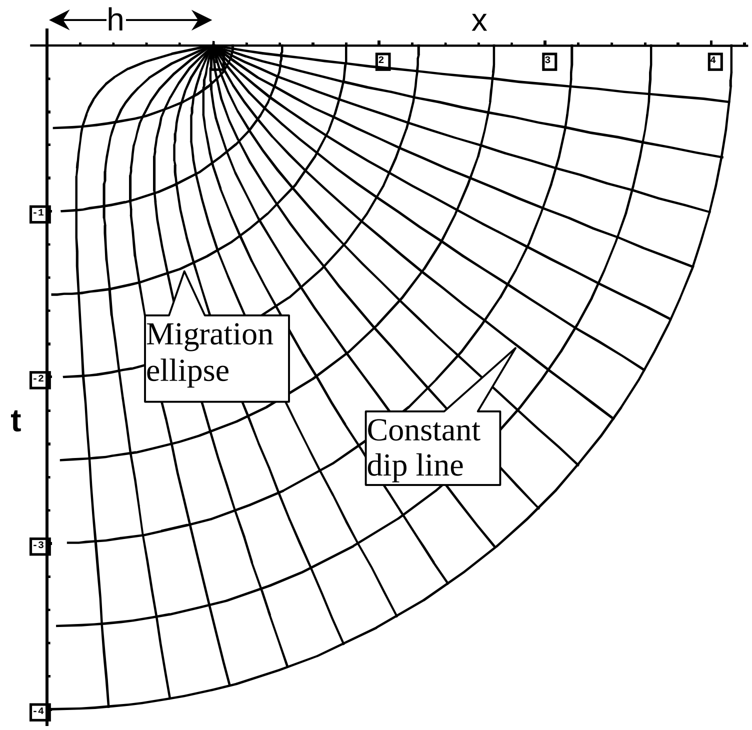

Figure 3 A constant offset section with a family of migrated ellipse. Also shown are lines of constant dip angle in increments of 5 degrees.

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Variable velocities (Vrms)

When the velocity varies, the elliptical case is no longer valid and time migrations become based on the RMS velocity that is defined at the scatter point location. To date, a direct solution for computing a dip limit on a constant offset diffraction has not been found. It is however straight forward to compute a dip limit that may be imposed on the prestack summation surface $T(x, h, T_0)$ that is referred to as Cheops pyramid and defined by the double-square-root equation

$$T = \left[\frac{T_0^2}{4} + \frac{(x+h)^2}{V_{rms}^2}\right]^{1/2} + \left[\frac{T_0^2}{4} + \frac{(x-h)^2}{V_{rms}^2}\right]^{1/2}, \qquad (7)$$

where the scatter point is located at $x = 0$. Figure 4a shows a plot of Cheops pyramid along with a black line that defines a band of energy that will migrate to a defined geological dip b. Note the dip at the zero offset diffraction is a. The constant dip line may be defined from raypaths to a scatter point on a dipping event. Solving for a defined offset is difficult, but if a source location is defined, the image of the receiver (reflected about the reflector) defines the offset h, time T, and midpoint location x.

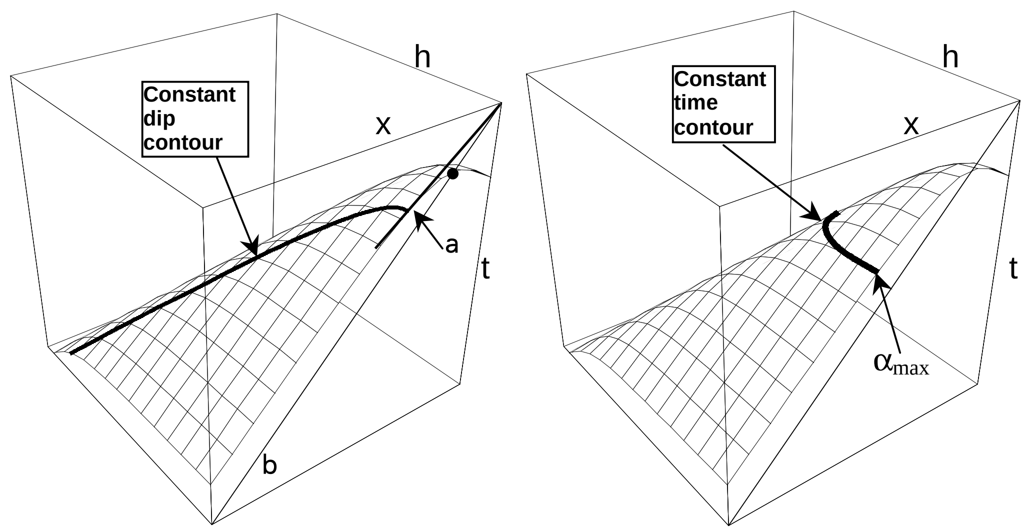

Figure 4 Cheops pyramid with a) showing a line that locates energy that comes from dip b. The corresponding dip a is shown on the zero-offset. Part b) shows a constant time limit contour.

The constant dip line in Figure 4a extends from the dip α at zero offset to larger offsets and larger displacements of x. If a similar constant dipping line represents the maximum dip to be migrated, then the summation aperture becomes the surface of Cheops pyramid that is bound by this line.

If the previous criterion of limiting the aperture to T0/T was used, then the aperture is defined as a constant time slice on Cheops pyramid as illustrated in Figure 4b. Energy from dips smaller than α_{max} will extend beyond this constant time aperture and limit the dipping energy of the migration.

EQUIVALENT OFFSET MIGRATION (EOM)

Equivalent offset migration will sum all points on a constant time slice through Cheops pyramid to one point on the common scatter point (CSP) gather that is located at the scatterpoint (Bancroft et al 1998). Consequently, a dip limit that is imposed on a CSP gather will be similar to the constant time aperture discussed above. Caution is therefore required in defining a dip-limiting migration aperture.

Another approach to limiting the dips on a CSP gather is shown in Figures 5a-d, where one input trace is summed to all neighbouring CSP gathers. The kinematics of the input trace produces a bow (of a ship) shape that is shown from a rear view in (a). Radial planes of constant dip are added to figures (b) through (d) and intersect the bow and may represent a dip limit imposed on the EOM method of prestack migration.

Note that shallower dips (defined from the vertical) tend to leave smaller portions of the input trace, and that these portions tend to a smaller equivalent offset range. This smaller equivalent offset range may permit the input trace to the summed into one bin of the CSP gathers.

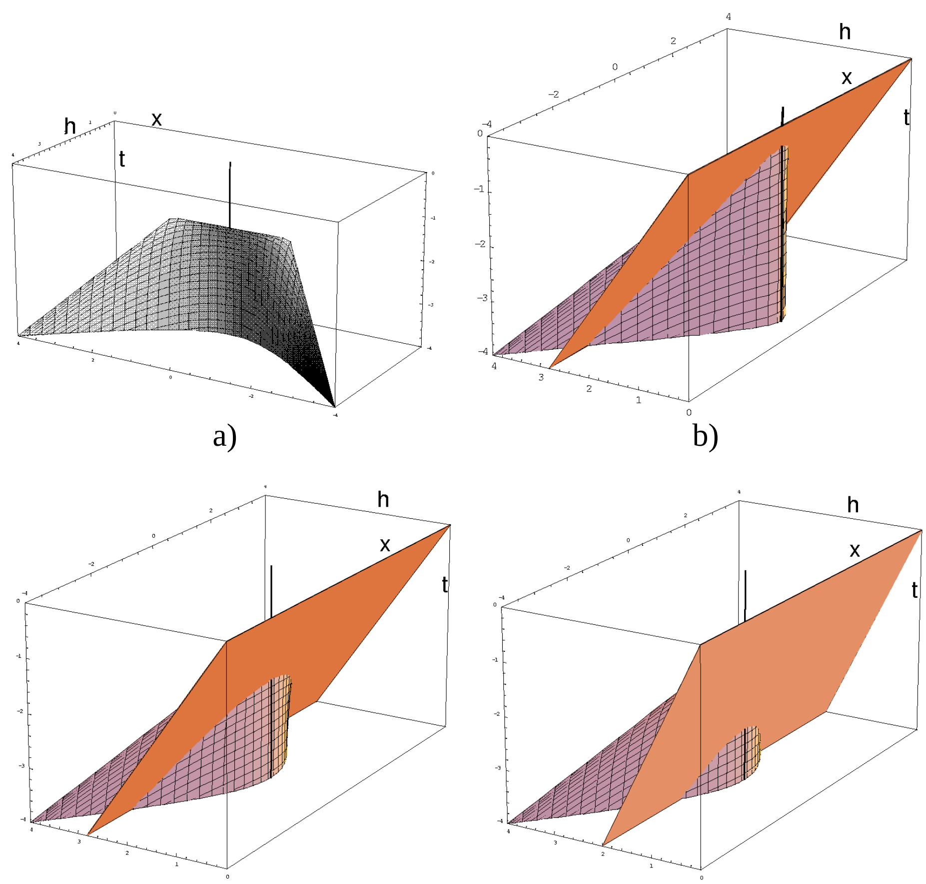

Figure 5 Rear view of bow shape in a) produced by copying one input trace to all neighbouring CSP gathers, b) intersection of large-angle dipping plane, c) mid-angle dipping plane, and d) small-angle dipping plane.

CONCLUSIONS

The aperture for Kirchhoff migrations is best defined by dip limits on the summation diffractions. These dip limits are simple to define for the post stack case but are more complex for the prestack case.

A dip limit imposed on the CSP gathers of EOM migration is comparable to a constant time limit imposed on the summation over Cheops pyramid. Care is required to ensure the energy from dipping events is included in the defined migration dip limit.

REFERENCES

Bancroft, J. C., Geiger, H. D., and Margrave, G. F., 1998, The equivalent offset method of prestack migration, Geophysics, in press for Nov. - Dec. 1998 issue.