3-D converted wave inversion for shear velocity

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ABSTRACT

Weighted stacking of P-S seismic data can provide an estimate of S-wave velocity changes ($\Delta\beta/\beta$). This requires an estimate of the P-wave incident angle, which is derived here. A full processing flow, including modeling, weighted stacking and post stack inversion, is given.

INTRODUCTION

Amplitude-versus-offset (AVO) analysis tries to deduce the subsurface shear velocity from P-wave reflectivity changes in an attempt to provide lithologic discrimination. However, the converted wave reflectivity is directly dependant on the S-wave velocity, without the complication of P-wave velocity changes. Thus if we are interested in S-wave velocity, the P-S reflectivity carries important information. In the current paper, we discuss the theoretical and practical aspects of deriving shear velocity from converted (P-S) seismic data.

METHODS

P-S weighted stack

The Zoeppritz equations allow us to derive the exact plane wave amplitude of a converted S-wave from an incident P-wave as a function of angle, but do not give us an intuitive understanding of how these amplitudes relate to the various physical parameters. Aki and Richards (1980) approximate the equation assuming small changes in elastic properties across an interface (Figure 1):

$$R^{PS} = 4c\frac{\Delta\rho}{\rho} + d\frac{\Delta\beta}{\beta} \tag{1}$$

where:

$$c = -\frac{\alpha\tan\varphi}{8\beta}\left(1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\varphi\right)$$

$$d = \frac{\alpha\tan\varphi}{2\beta}\left(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\varphi\right)$$

$\theta = (\theta_1 + \theta_2)/2, \quad \varphi = (\varphi_1 + \varphi_2)/2$ - average angles across the interface

α, β, ρ - average P-wave velocity, S-wave velocity and density across the interface

$\Delta\beta/\beta,\ \Delta\rho/\rho$ - relative changes in S-wave velocity and density

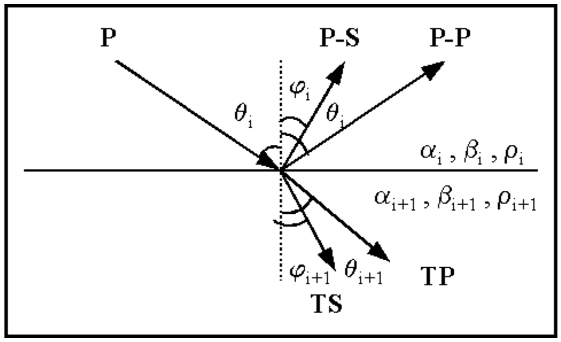

Figure 1: Incident P-wave partitioning at an interface. P-P denotes the reflected P-wave, reflected S-wave (P-S), transmitted P-wave (TP), and transmitted S-wave (TS).

The equation (1) can be cast as a least-squares problem and solved for $\Delta\beta/\beta$ (Stewart, 1990):

$$\frac{\Delta\beta}{\beta} = \frac{\sum_{i=1}^{n} R_i^{PS} d_i - \frac{\Delta\alpha}{\alpha}\sum_{i=1}^{n} c_i d_i}{\sum_{i=1}^{n} d_i^2} \tag{2}$$

where $\Delta\alpha/\alpha$ are the relative changes in P-wave velocity.

To obtain the $\Delta\beta/\beta$ weighted stack P-wave, S-wave and density models in P-S time are required.

Modeling

To create the P-S weighted stack, we need a geological model containing P-wave velocity, S-wave velocity and density. The model, in P-S time, can be build in the following way:

- at the well locations compute the P-S pseudo-velocity logs, defined by:

$$\text{P} - \text{S pseudo - velocity log} = 2\left(\frac{V_P V_S}{V_P + V_S}\right) \tag{3}$$

where V_P and V_S are the measured P-wave and S-wave velocity logs

- using the computed P-S pseudo-velocity log convert the V_P, V_S, and density logs into P-S time

- build a 3-D V_P, V_S, and density models in P-S time by 3-D interpolation

Incident angle approximation

The P-S weighted stack calculation requires knowledge of the incident angle at any particular interface (the reflection and transmission angles than can be found using Snell's law). The incident angle can be found using ray tracing, but in a complex model, as the one discussed above, the required time may be large and thus unattractive. To solve the problem, we derive an approximation for the incident angle as a function of the offset.

P-P case

First we look at the approximation of the incident angle for the P-P case (Figure 2).

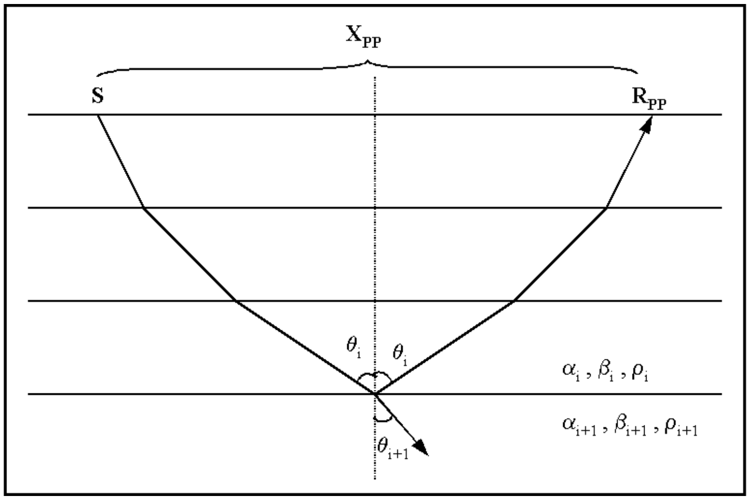

Figure 2: The raypath of a P-P wave in a horizontally layered medium.

The total two-way P-P travel time t_{pp} is:

$$t_{PP}^2 = t_{PP0}^2 + \frac{X_{PP}^2}{\alpha_{RMS}^2} \tag{4}$$

From Snell's law, the incident angle to the interface i can be written as:

$\frac{\sin\theta_i}{\alpha_i} = p, \quad \text{and } p = \frac{dt}{dx}$ at any point including the surface,

$$\sin\theta_i = \alpha_i \frac{dt}{dx} \tag{5}$$

Substitute the two-way travel time in equation (5) and solve:

$$\sin\theta_i = \alpha_i \frac{d}{dx}\left(\sqrt{t_{PP0}^2 + \frac{X_{PP}^2}{\alpha_{RMS}^2}}\right) = \frac{\alpha_i X_{PP}}{t_{PP}\alpha_{RMS}^2} \tag{6}$$

P-S case

From the P-P case, for the same incident angle (Figure 3):

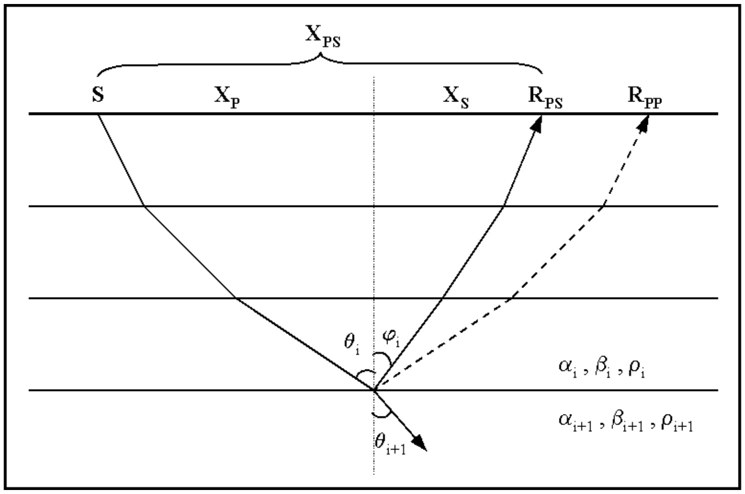

Figure 3: The raypath of a P-S wave in a horizontally layered medium. Note that X_{PP}=2X_P.

$$\sin\theta_i = \frac{X_{PP}\alpha_i}{t_{PP}\alpha_{RMS}^2} = \frac{2X_P\alpha_i}{\alpha_{RMS}^2\sqrt{t_{PP0}^2 + \frac{4X_P^2}{\alpha_{RMS}^2}}} \tag{7}$$

At zero offset:

$$t_{PS0} = t_P + t_S = t_P\left(1+\frac{t_S}{t_p}\right) = \frac{t_{PP0}}{2}\left(1+\frac{\overline{\alpha}}{\overline{\beta}}\right) = \frac{\overline{\alpha}+\overline{\beta}}{2\overline{\beta}}t_{PP0}$$

where: $\overline{\alpha}, \overline{\beta}$ - average P-wave and S-wave velocities

Furthermore, from Tatham and McCormick (1991), we can convert the P-P wave offset 2X_P to P-S offset X_{PS}:

$$X_P = gX_{PS}\text{, where } g = \frac{1}{1+\frac{\overline{\alpha}}{\overline{\beta}}\frac{\beta_{RMS}^2}{\alpha_{RMS}^2}} \tag{8}$$

And then, we write the approximation for the incident angle in the P-S case:

$$\sin\theta_i = \frac{2gX_{PS}\alpha_i}{\alpha_{RMS}^2\sqrt{\left(\frac{2\overline{\beta}}{\overline{\alpha}+\overline{\beta}}t_{PS0}\right)^2 + \frac{4g^2X_{PS}^2}{\alpha_{RMS}^2}}} \tag{9}$$

The angle goes into equation (2) to calculate the weights for the $\Delta\beta/\beta$ stack.

Contents

P-S inversion flow

The P-S inversion flow begins with building the geological model in P-S time, containing P-wave, S-wave and density information for each seismic sample. Then using the model, we calculate the stacking weights for each NMO-corrected CCP gather and perform weighted stacking. The resulted $\Delta\beta/\beta$ volume can be inverted using any available P-P inversion routine to derive the shear velocity.

Figure 4 shows the P-S inversion flow chart:

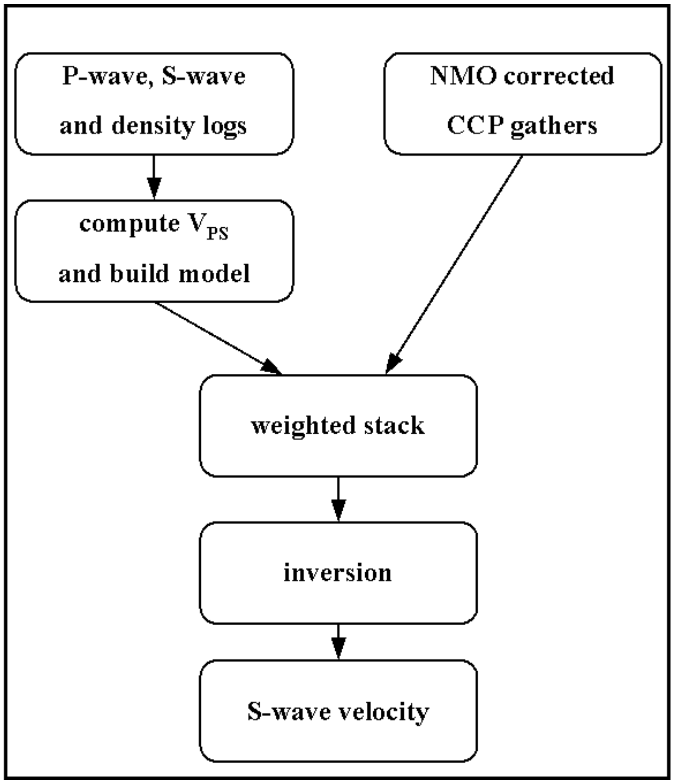

Figure 4: P-S inversion flow chart.

CONCLUSIONS

This paper has outlined a procedure to invert P-S surface seismic data for S- wave velocity. It includes a weighted stacking approach complete with AVO approximation to find $\Delta\beta/\beta$. This normalized velocity change can be used to estimate β itself using conventional inversion routines.

ACKNOWLEDGEMENTS

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