Complex seismic trace analysis and its application to time-lapse seismic surveys

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ABSTRACT

Complex trace analysis effects a natural separation of amplitude from angle (frequency and phase) and then allows definition of instantaneous attributes. The instantaneous amplitude defines single lobes for individual wavelets and, along with the instantaneous phase and frequency, has more power to resolve reflectors. In the case of strongly overlapping wavelets, the instantaneous amplitude and frequency have characteristics that help identify and distinguish wavelet interference. The instantaneous frequency, a measure of most energy-loaded or center frequency, traces frequency change with time. In time-lapse seismic surveys, the power of resolution improves event picks and calculation of time shift and amplitude variation, and the representation of frequency and phase facilitates the study of non-stationary processes.

INTRODUCTION

In reservoir monitoring, the velocities, acoustic impedance and attenuation (anelasticity or Q factor) of reservoir rocks can change in response to changes in fluid saturation, pressure, temperature, porosity, etc., due to production or injection. These changes may be detected in time-lapse seismic surveys, depending on the ability to identify and resolve subtle waveform characteristics, which are measured by seismic attributes.

Conventional seismic attributes in time-lapse seismic surveys are based on a description of the real seismic trace. Event picks on the top and bottom of a reservoir and subsequent calculation of time shift and amplitude variation may be difficult or inaccurate due to interference. This resolution limit could lead to misinterpretation of amplitude change. The Fourier transform in a time window allows us to look at its average properties, but it does not examine local variations, which may change in response to wavelet interference. In addition, Fourier analysis, assuming that all spectral components are stationary in time, is not effective for analysis of non-stationary seismic signals (Bodine, 1984).

Complex trace analysis commonly used in electrical engineering and signal analysis (Gabor, 1946; Bracewell, 1965; Cramer and Leadbetter, 1967; Oppenheim and Schafer, 1975) provides insight into the seismic trace. The seismic trace from surface seismic surveys is treated as the real part of a complex seismic trace, and the imaginary part is minus the Hilbert transform of the real part. This amounts to the phasor representation, which effects a natural separation of amplitude (length of the phasor) from angle (frequency and phase) and enables calculation of instantaneous seismic attributes (Farnbach, 1975; Taner et al., 1979). The envelope of instantaneous amplitude combined with instantaneous phase and frequency improves resolution, and hence seismic events from reservoirs are more clearly defined. Instantaneous attributes can form patterns of change that can be used to identify and distinguish subtle wavelet interference. Response attributes (instantaneous amplitude, phase and frequency) at the maximum of instantaneous amplitude in an event can be used instead of the Fourier transform to trace frequency change with time (Bodine, 1984; Partyka, 2000).

In this paper, we discuss the complex seismic trace, its physical meaning and its application to time-lapse seismic surveys.

DEFINITION AND CALCULATION OF THE COMPLEX SEISMIC TRACE

The use of complex numbers in signal processing can simplify mathematical manipulation (after Robinson and Silvia, 1978). In harmonic oscillation, the real signal we measure or digitize, f(t), is often expressed as $\Psi(t) = A[\cos(\omega t+\phi) + i \sin(\omega t+\phi)]$, of which $f(t) = A\cos(\omega t+\phi)$. In polar representation, $\Psi(t) = Aei(\omega t+\phi)$. Geometrically, $\Psi(t)$ is considered a phasor of length A, which rotates counterclockwise with time at angular frequency ω and phase ϕ. For quasi-harmonic signals, we can construct the expression by modulating either amplitude or frequency and phase. In amplitude modulation, $\Psi(t) = A(t)[\cos(\omega t+\phi) + i \sin(\omega t+\phi)]$, where A(t) is instantaneous amplitude. In frequency and phase modulation, $\Psi(t) = A \{\cos[\omega(t)\ t+\phi(t)] + i \sin[\omega(t)\ t+\phi(t)]\}$. However, the specification of time varying angular frequency is difficult because it is hard to see how a frequency can be established in an instant (Goldman, 1948). In addition, it is difficult to separate frequency modulation from phase modulation. Hence, we define the term $\theta(t)$, which integrates the effect of frequency and phase, i.e., $\Psi(t) = A \{\cos[\theta(t)] + i \sin[\theta(t)]\}$ or $\Psi(t) = A\ ei\theta(t)$. $\theta(t)$ is called instantaneous phase. Then, $1/2\pi\ d\theta(t)/dt$ is defined as instantaneous frequency. If angular frequency and phase are constants, $1/2\pi\ d\theta(t)/dt = 1/2\pi\ d(\omega t+\phi)/dt = f$ and the definition reduces to the usual one for harmonic oscillation. Again $\Psi(t)$ can be viewed as a phasor of time-varying length A(t), which rotates with time-varying angle $\theta(t)$ to the real axis as it translates along the time axis. The advantage of the above representation is that it enables us to separate amplitude from angle (frequency and phase) and allows definition of instantaneous attributes.

The seismic trace, a process much more complicated than quasi-harmonic oscillation, is analogous to hybrid modulation. The corresponding complex trace can be defined as:

$$\Psi(t) = A(t)\ \{\cos[\theta(t)] + i \sin[\theta(t)]\} \text{ or } \Psi(t) = A(t)\ e^{i\theta(t)} \tag{1}$$

where $f(t) = A(t)\cos[\theta(t)]$, the real part of the complex trace, is the seismic trace we measure from surface seismic surveys.

An efficient way to obtain A(t) and $\theta(t)$ from the seismic trace is to calculate the imaginary part of equation (1). Then A(t) and $\theta(t)$ are derived by the following relations:

$$A_{(t)} = \sqrt{[\mathrm{Re}\,\Psi(t)]^2 + [\mathrm{Im}\,\Psi(t)]^2}$$
$$= \sqrt{f(t)^2 + [\mathrm{Im}\,\Psi(t)]^2} \tag{2}$$

$$\theta_{(t)} = \tan^{-1}\{[\mathrm{Im}\,\Psi(t)] / [\mathrm{Re}\,\Psi(t)]\}$$
$$= \tan^{-1}\{[\mathrm{Im}\,\Psi(t)] / f(t)\} \tag{3}$$

From equation (1), the imaginary part lags behind the real part by 900. The imaginary part is found by 900 shifting the frequency spectrum of the real part and performing an inverse Fourier transform. Let F(ω) and H(ω) be the respective frequency spectra of f(t) (the real part) and h(t) (the imaginary part). We have:

$$H_{(\omega)} = A_{(\omega)} \exp[i(\phi_{(\omega)} - \pi/2\ \mathrm{sgn}(\omega)]$$
$$= [A_{(\omega)} \exp(i\phi_{(\omega)})]\ [-i\ \mathrm{sgn}(\omega)]$$
$$= F_{(\omega)}\ [-i\ \mathrm{sgn}(\omega)] \tag{4}$$

$-\pi/2\ \mathrm{sgn}(\omega)$ is used instead of $-\pi/2$ to apply the 90^0 shift in order to keep $\phi(\omega)$ antisymmetric because h(t) is real. From (4), we transform $H_{(\omega)}$ back into $h_{(t)}$ using the inverse Fourier transform:

$$h(t) = 1/(2\pi) \int_{-\infty}^{+\infty} H(\omega) e^{i\omega t} d\omega$$
$$= -1/(2\pi) \int_{-\infty}^{+\infty} F(\omega) i\,\mathrm{sgn}(\omega) e^{i\omega t} d\omega$$
$$= [1/(2\pi) \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega] * [-1/(2\pi) \int_{-\infty}^{+\infty} i\,\mathrm{sgn}(\omega) e^{i\omega t} d\omega]$$
$$= f(t) * \frac{1}{\pi t}$$
$$= -1/\pi \int_{-\infty}^{+\infty} f(\tau) \frac{1}{\tau - t} d\tau \tag{5}$$

Equation (5) is the negative of Hilbert transform. Hence the complex seismic trace is:

$$\Psi(t) = f(t) + i\ h(t)$$
$$= f(t) - i\ f_{hi}(t) \tag{6}$$

where fhi(t) denotes the Hilbert transform of f(t), also termed as the quadrature function of f(t) (Bracewell, 1978).

The Fourier transform of the complex seismic trace, Ψ(t), will have the form:

$$
\begin{aligned}
T(\omega) &= \int_{-\infty}^{+\infty} \Psi(t)\, e^{-i\omega t} dt \\
&= \int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t} dt + i \int_{-\infty}^{+\infty} h(t)\, e^{-i\omega t} dt \\
&= A(\omega)\, e^{i\phi(\omega)} + iA(\omega)\, e^{i[\phi(\omega) - \pi/2 \operatorname{sgn}(\omega)]} \\
&= A(\omega)\, e^{i\phi(\omega)} + iA(\omega)\, e^{i\phi(\omega)} [-i \operatorname{sgn}(\omega)] \\
&= A(\omega)\, e^{i\phi(\omega)} [1 + \operatorname{sgn}(\omega)] \\
&= 0, \quad when \ \ \omega < 0 \\
&= 2A(\omega)\, e^{i\phi(\omega)}, \ when \ \omega > 0
\end{aligned} \tag{7}
$$

As shown in equation (7), the frequency spectrum of the complex seismic trace T(ω) vanishes for ω<0 and has twice the amplitude and the unchanged phase of the real seismic trace for ω>0. Due to being zero for negative frequencies, the complex seismic trace is also called the analytical signal (Ackroyd, 1970; Claerbout, 1992).

Based on equation (7), another way to calculate A(t) and θ(t) is to: 1) Fourier transform the real seismic trace; 2) zero the amplitude for negative frequencies and double the amplitude for positive frequencies; 3) inversely Fourier transform; 4) use equations (2) and (3).

PHYSICAL MEANING OF INSTANTANEOUS ATTRIBUTES

The previous definition of instantaneous attributes is in sharp contrast to the Fourier transform, in which amplitude, frequency and phase are defined over an infinite time interval. This conceptual difficulty lies in our attempt to combine time and frequency domains in some manner. In the real world of signals, frequency does change with time. Listening to a music or speech, one feels the change of frequency and amplitude with time. The effect of sunrise, sun at noon and sunset on eyes varies substantially. Seismic interpreters know that frequency spectrum usually becomes lower with increasing arrival time as the high-frequency components are attenuated faster than low-frequency components. The way to deal with these non-stationary processes is to Fourier transform finite time windows, which are continuously shifted in time. The results tell us how the frequency components evolve with time. However, the time window must have a finite width. The time window should be large enough to make Fourier transform meaningful (Page, 1952). This introduces a resolution problem since there is a tradeoff between the length of time window and the accuracy of frequency estimation. Also, the time window, although very small, may not satisfy the assumption that the process within it is stationary. Hence, it is useful to have a theoretical approach to cope with the continuously varying frequency spectrum in order to define instantaneous attributes.

For a complex signal $\Psi(t)$, we can define the signal power $|\psi(t)|2$ (or energy per unit time), the energy spectrum $|T(f)|2$ (or energy per unit frequency) and the signal energy E (Rihaczek, 1968; Grace, 1981):

$$E = \int_{-\infty}^{+\infty} \left|\psi(t)\right|^2 dt \tag{8}$$

Note from the above equation:

$$\begin{aligned}
E &= \int_{-\infty}^{+\infty} \left|\psi(t)\right|^2 dt \\
&= \int_{-\infty}^{+\infty} \psi(t)\psi(t)^* dt \\
&= \int_{-\infty}^{+\infty} [\int_{-\infty}^{+\infty} T(f)e^{i2\pi ft} df\,]\psi(t)^* dt \\
&= \int_{-\infty}^{+\infty} T(f)[\int_{-\infty}^{+\infty} \psi(t)^*\, e^{i2\pi ft} dt\,]df \\
&= \int_{-\infty}^{+\infty} T(f)T(f)^* df \\
&= \int_{-\infty}^{+\infty} \left|T(f)\right|^2 df
\end{aligned} \tag{9}$$

This is Rayleigh's or Plancherel's or Parseval's theorem (Bracewell, 1978; Grace, 1981; Page, 1952; Sheriff and Geldart, 1995), which shows the relationship between the signal power and the signal energy spectrum. Reorganizing (9), we find that the signal power at time t is distributed through all frequencies:

$$\begin{aligned}
\int_{-\infty}^{+\infty} \left|\psi(t)\right|^2 dt &= \int_{-\infty}^{+\infty} T(f)T(f)^* df \\
&= \int_{-\infty}^{+\infty} [\int_{-\infty}^{+\infty} \psi(t)e^{-i2\pi ft} dt]T(f)^* df \\
&= \int_{-\infty}^{+\infty} [\int_{-\infty}^{+\infty} \psi(t)T(f)^*\, e^{-i2\pi ft} df\,]dt \\
or \qquad &= \int_{-\infty}^{+\infty} [\int_{-\infty}^{+\infty} \psi(t)^*\, T(f)e^{i2\pi ft} df\,]dt
\end{aligned}$$

$$i.e., \quad \left|\psi(t)\right|^2 = \int_{-\infty}^{+\infty} \psi(t)T(f)^*\, e^{-i2\pi ft} df \tag{10}$$

$$or \ \left|\psi(t)\right|^2 = \int_{-\infty}^{+\infty} \psi(t)^*\, T(f)e^{i2\pi ft} df \tag{11}$$

The integrands in (10) and (11) can be defined as the signal power density d(t,f), an amplitude-scaled and phase-rotated and -shifted spectrum. $\psi(t)\ T(f)^*\ e^{-i2\pi ft}$ in (10) is Rihaczek's complex energy density (Rihaczek, 1968) and $\psi(t)^*\ T(f)\ e^{i2\pi ft}$ in (11) is termed as the complex instantaneous power spectrum by Levin (1964). They are the complex conjugates and the choice is immaterial since only the real part of the integral is considered. As a logical extension, in an instant we can consider a frequency where the signal power is most concentrated. Over a time interval δt, the signal energy is:

$$\begin{aligned} E_{\delta t} &= \int_{t-\delta t/2}^{t+\delta t/2} \left|\psi(t)\right|^2 dt \\ &= \int_{t-\delta t/2}^{t+\delta t/2} \int_{-\infty}^{+\infty} d(t,f)\,df\,dt \\ &= \int_{t-\delta t/2}^{t+\delta t/2} [\int_{-\infty}^{+\infty} \psi(t)T(f)^* e^{-i2\pi ft} df]\,dt \end{aligned} \tag{12}$$

Denoting $\psi(t) = |\psi(t)|\, e^{i\theta(t)}$ and $T(f) = |T(f)|\, e^{i\varphi(f)}$, (12) can be rewritten as :

$$E_{\delta t} = \int_{t-\delta t/2}^{t+\delta t/2} \int_{-\infty}^{+\infty} \left|\psi(t)\right|\left|T(f)\right| e^{\theta(t)-\varphi(f)-i2\pi ft} df\,dt \tag{13}$$

In accordance with the principle of stationary phase, the significant contributions to the integral of equation (13) come from the vicinity of the points where the phase is stationary (Rihaczek, 1968). In the case of time integration, the stationary point is found by letting the time derivative of the phase equal zero, i.e.,

$$\begin{aligned} & d[\theta(t) - \varphi(f) - 2\pi ft]/dt = 0 \\ & or \qquad f = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \end{aligned} \tag{14}$$

(14) can be interpreted as the frequency with most energy over the time interval δt, or as the frequency with most signal power at time t. This frequency coincides with the instantaneous frequency we defined previously. So the instantaneous frequency is now meaningful, representing the most energy-loaded frequency in the frequency spectrum. On the other hand, the most energy-loaded frequency at a point in time can also be expressed as the average frequency weighted by signal power density d(t,f):

$$f = \frac{\int_{-\infty}^{+\infty} f d(t,f)\,df}{\int_{-\infty}^{+\infty} d(f,f)\,df} \tag{15}$$

Note that (15) is equivalent to the definition of the instantaneous frequency,

$$\begin{aligned} f &= \frac{1}{2\pi}\frac{d\theta(t)}{dt} \\ &= \frac{1}{2\pi}\frac{f(t)dh(t)/dt - h(t)df(t)/dt}{f(t)^2 + h(t)^2} \end{aligned} \tag{16}$$

where θ(t) is the phase angle and f and h denote the real part and imaginary part of the complex trace respectively (Taner and Sheriff, 1979). Reorganizing (16) yields:

$$\begin{aligned}
f &= \mathrm{Re}\{\frac{i}{2\pi}\frac{[f(t)+ih(t)][df(t)/dt-idh(t)/dt]}{[f(t)+ih(t)][f(t)-ih(t)]}\} \\
&= \mathrm{Re}\{\frac{i}{2\pi}\frac{\psi(t)d\psi(t)^{*}/dt}{\psi(t)\psi(t)^{*}}\} \\
&= \mathrm{Re}\{\frac{1}{-2\pi i}\frac{\psi(t)d(\int_{-\infty}^{+\infty}T(f)^{*}e^{-i2\pi ft}df)/dt}{\psi(t)\int_{-\infty}^{+\infty}T(f)^{*}e^{-i2\pi ft}df}\} \\
&= \mathrm{Re}\{\frac{\psi(t)\int_{-\infty}^{+\infty}f\,T(f)^{*}e^{-i2\pi ft}df}{\psi(t)\int_{-\infty}^{+\infty}T(f)^{*}e^{-i2\pi ft}df}\} \\
&= \mathrm{Re}\{\frac{\int_{-\infty}^{+\infty}f\,d(t,f)df}{\int_{-\infty}^{+\infty}d(t,f)df}\}
\end{aligned} \tag{17}$$

Since only the real part is considered, (15) and (17) are equivalent. Barnes (1993) called (15) the instantaneous center or mean frequency, which is the instantaneous frequency. Another equivalent for (15) is theorem 1 of Ha et al. (1991). Substituting the signal power density defined in (11) into (15),

$$\begin{aligned}
f &= \frac{\int_{-\infty}^{+\infty}f\psi(t)^{*}T(f)e^{i2\pi ft}df}{\int_{-\infty}^{+\infty}\psi(t)^{*}T(f)e^{i2\pi ft}df} \\
&= \frac{\int_{-\infty}^{+\infty}fT(f)e^{i2\pi ft}df}{\int_{-\infty}^{+\infty}T(f)e^{i2\pi ft}df}
\end{aligned} \tag{18}$$

The instantaneous amplitude and phase defined in the previous section may be viewed as the amplitude and phase of this most energy-loaded frequency (instantaneous frequency).

APPLICATIONS TO TIME-LAPSE SEISMIC SURVEYS

Complex trace analysis has been applied to geophysical data processing by Barnes (1990, 1991, 1992 and 1993), Bodine (1984), Farnbach (1975), Ha et al. (1991), Robertson et al. (1984 and 1988), Taner et al. (1977 and 1979) and White (1991). They noted the advantage of separation of amplitude from angle and then calculation of instantaneous attributes, which are independent of each other and contain different information about the seismic trace. The instantaneous attributes characterize waveform. Instantaneous amplitude measures acoustic impedance contrast such as gas accumulation (bright spots) and major lithological variation, and is sensitive to interference such as tuning effect. Instantaneous phase, the angle of a rotating complex vector, is independent of instantaneous amplitude and is sensitive to weak events. Instantaneous frequency traces the change of frequency components and can be used to study low-frequency shadow commonly observed under hydrocarbon reservoirs (Taner, 1977 and 1979). Both instantaneous phase and frequency may change in response to interference. With the aid of instantaneous phase and frequency, the envelope of instantaneous amplitude can give more insight into the compositions of the signal than is apparent in the original signal itself (Farnbach, 1975). Partyka (2000) and Bodine (1984) defined the response energy, phase and frequency as the instantaneous amplitude, phase and frequency at the maximum of an energy lobe and demonstrated that the response frequency and phase are a measure of the dominant frequency and phase of this lobe.

In this section, we deal with the basic properties of wavelets in terms of instantaneous attributes and then propose a model of interference. Finally, two examples are given to illustrate the application to time-lapse seismic surveys.

Basic properties of wavelets

Since the seismic trace is a result of convolving a series of single reflectors with the embedded wavelet (Sheriff and Geldart, 1995), it is important to study the basic properties of wavelets. Figure 1 is the instantaneous attributes of the Ricker wavelet and minimum phase wavelet. The envelope of instantaneous amplitude (red) is a direct response to the magnitude of reflectivity. It is a smooth concave line with a maximum. This single lobe is in contrast to the wavelet, which oscillates for a few cycles. The instantaneous phase is independent of the change of instantaneous amplitude and increases continuously and smoothly with time. The breaks in the figure are not discontinuities, but a period of 2π. Instantaneous frequency, the rate of change of instantaneous phase, is smooth and continuous, although the pattern differs between the two wavelets. The oscillatory portion of instantaneous frequency for the minimum phase wavelet is due to the inaccurate phase angle caused by a very small complex vector. The instantaneous frequency does not change with the magnitude of instantaneous amplitude and phase. Hence, these three instantaneous attributes are independent of each other.

Since most energy is located in the vicinity of the maximum of instantaneous amplitude, the instantaneous phase and frequency at that point correspond to the dominant phase and frequency of the wavelet. The Ricker wavelet is zero-phase and 30Hz dominant frequency, which can be roughly read from instantaneous phase and frequency at $t=0$, where instantaneous amplitude maximizes. For the minimum phase wavelet, the instantaneous phase and frequency at $t=0.01$ approximate the dominant phase and frequency, i.e., $\pi/2$ and 30 Hz.

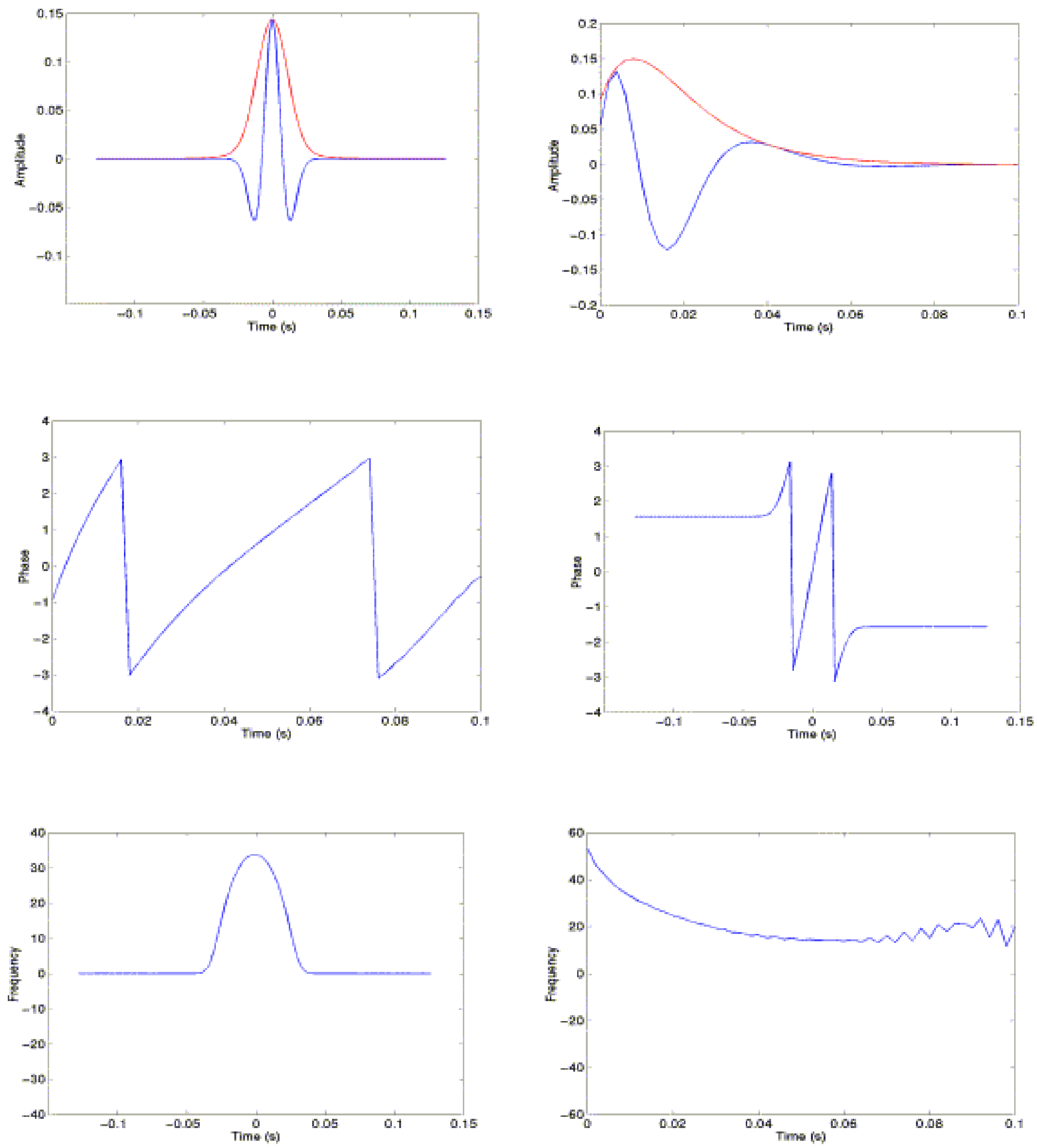


Figure 1. Instantaneous amplitude (red), phase and frequency of the Ricker wavelet (left, 30Hz) and minimum phase wavelet (right, 30Hz).

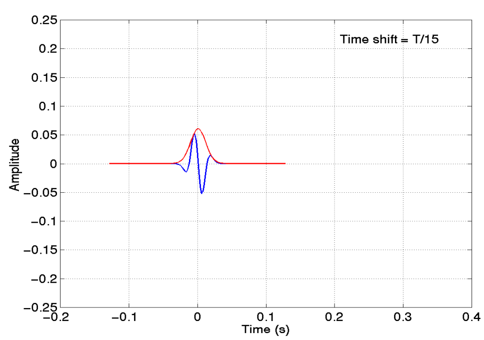
Wavelet interference

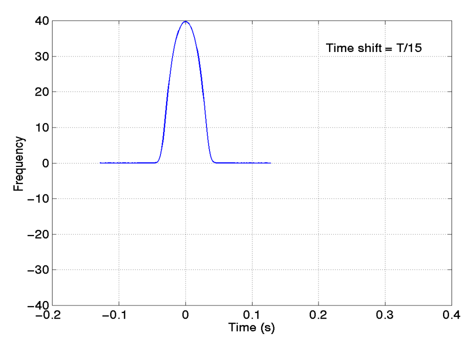
Closely spaced boundaries of acoustic impedance in sedimentary sections cause wavelet interference. Depending on separation, acoustic impedance boundaries may be identified and resolved by using instantaneous attributes. Figure 2 is two opposite-polarity Ricker wavelets separated by T/15, T/4, T/2, 3T/4, T and 2T (T is the period, approximately 0.033 seconds) and the respective instantaneous amplitude and frequency. At a time shift less than T/2, neither the conventional seismic trace nor the instantaneous attributes can separate the two wavelets. However, careful examination indicates an increase in response energy and a decrease in response frequency with time shifting. This pattern is significant in time-lapse seismic surveys. If the event from a reservoir with poorly resolved top and bottom reflectors increases its response energy but decreases its response frequency with recovery, a decrease in velocity can be inferred to cause an increase in time shift.

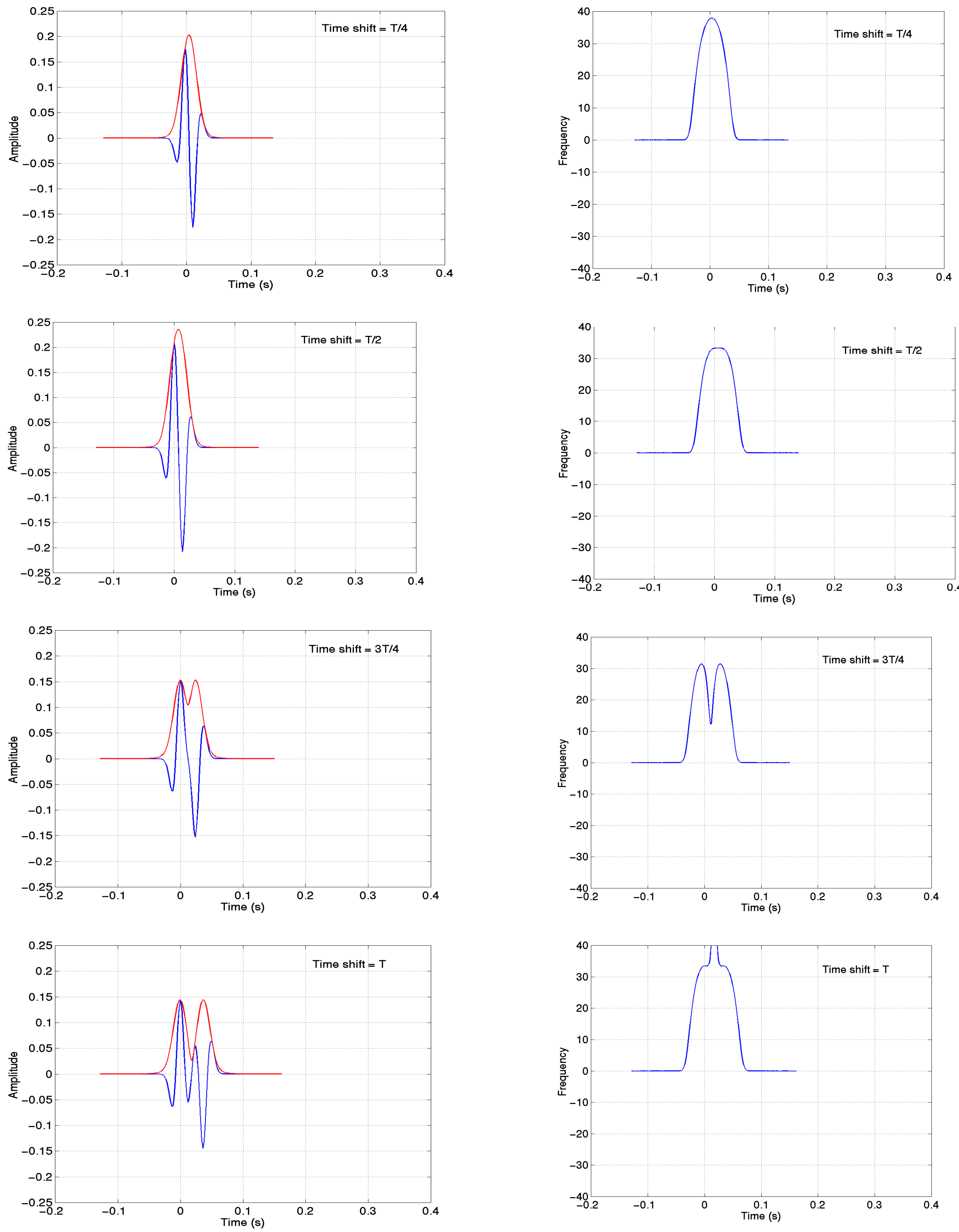
When time shift reaches 3T/4, both instantaneous amplitude and frequency can differentiate two wavelets, whereas the conventional seismic trace can not. The power of resolution is attributed to separate lobes of instantaneous amplitude for individual wavelets and strong frequency interference at the intersection point between the two instantaneous envelopes. Instantaneous frequency appears more sensitive to wavelet interference than instantaneous amplitude, as evidenced by clearer frequency separation at time shift = 3T/4.

When time shift reaches T, the conventional seismic trace is still unable to resolve the number of wavelets in the seismic trace, even though two wavelets are well resolved by the instantaneous amplitude and frequency.

Figure 3 shows two opposite-polarity minimum phase wavelets spaced by T/15, T/4, T/2, 3T/4, T and 2T (T is the period, approximately 0.033 seconds) and the respective instantaneous amplitude and frequency. When time shift is less than T/4, response energy increases but response frequency decreases. This pattern of change is similar to that of the Ricker wavelet. Starting at time shift = T/4, instantaneous frequency identifies two wavelets by abrupt decrease at the intersection point of two envelopes of instantaneous amplitude. The abrupt decrease strengthens until time shift = T, where it changes to an abrupt increase. At time shift = T/2, instantaneous amplitude is able to identify the two wavelets. At time shift = 2T, both instantaneous amplitude and frequency nicely separate two wavelets.






Time shift = T/4
Amplitude
Time (s)
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Frequency
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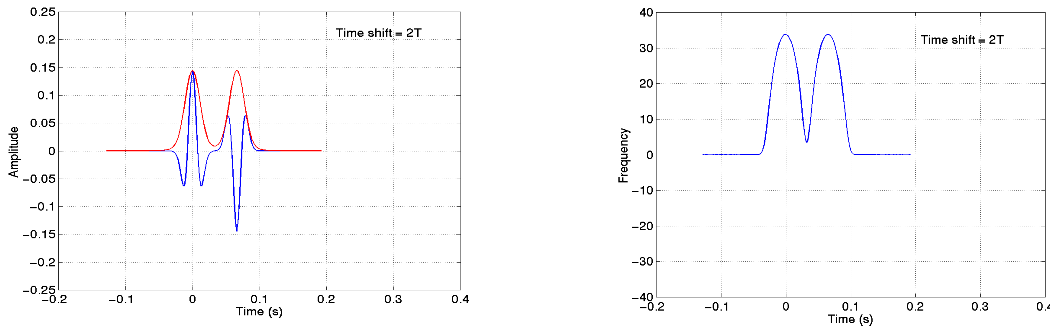
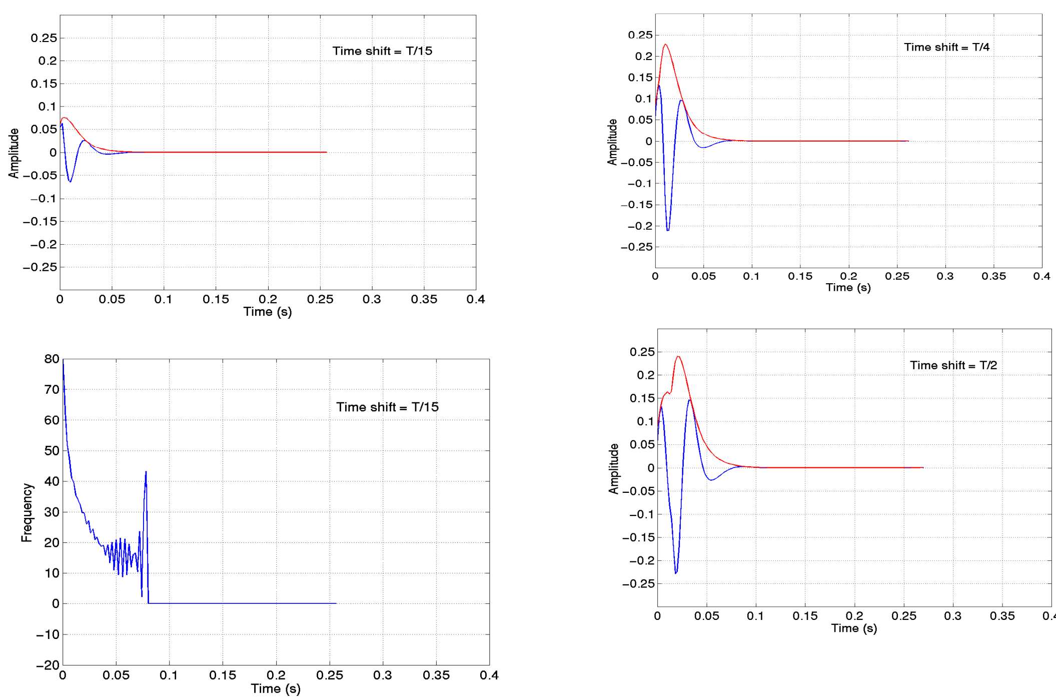


Figure 2. Interfered opposite-polarity Ricker wavelet spaced by T/15, T/4, T/2, 3T/4, T and 2T (1/T is the dominant frequency, 30 Hz) and the respective diagrams of instantaneous amplitude (left, red) and frequency (right, blue).

In summary, at small time shift, when neither instantaneous amplitude nor frequency can separate two wavelets, the pattern of change in instantaneous amplitude and frequency implies the existence of interfering wavelets and predicts the change of time shift so changes in reservoir velocity can be inferred. The conventional seismic trace is more difficult to interpret, and may lead us to interpret the existence of only one wavelet whose amplitude increases due to substantial change in acoustic impedance contrast. With larger time shift, instantaneous amplitude and frequency have power to resolve individual wavelets and instantaneous frequency may be more powerful. The time shift can be calculated as the time difference between two envelopes of instantaneous amplitude.



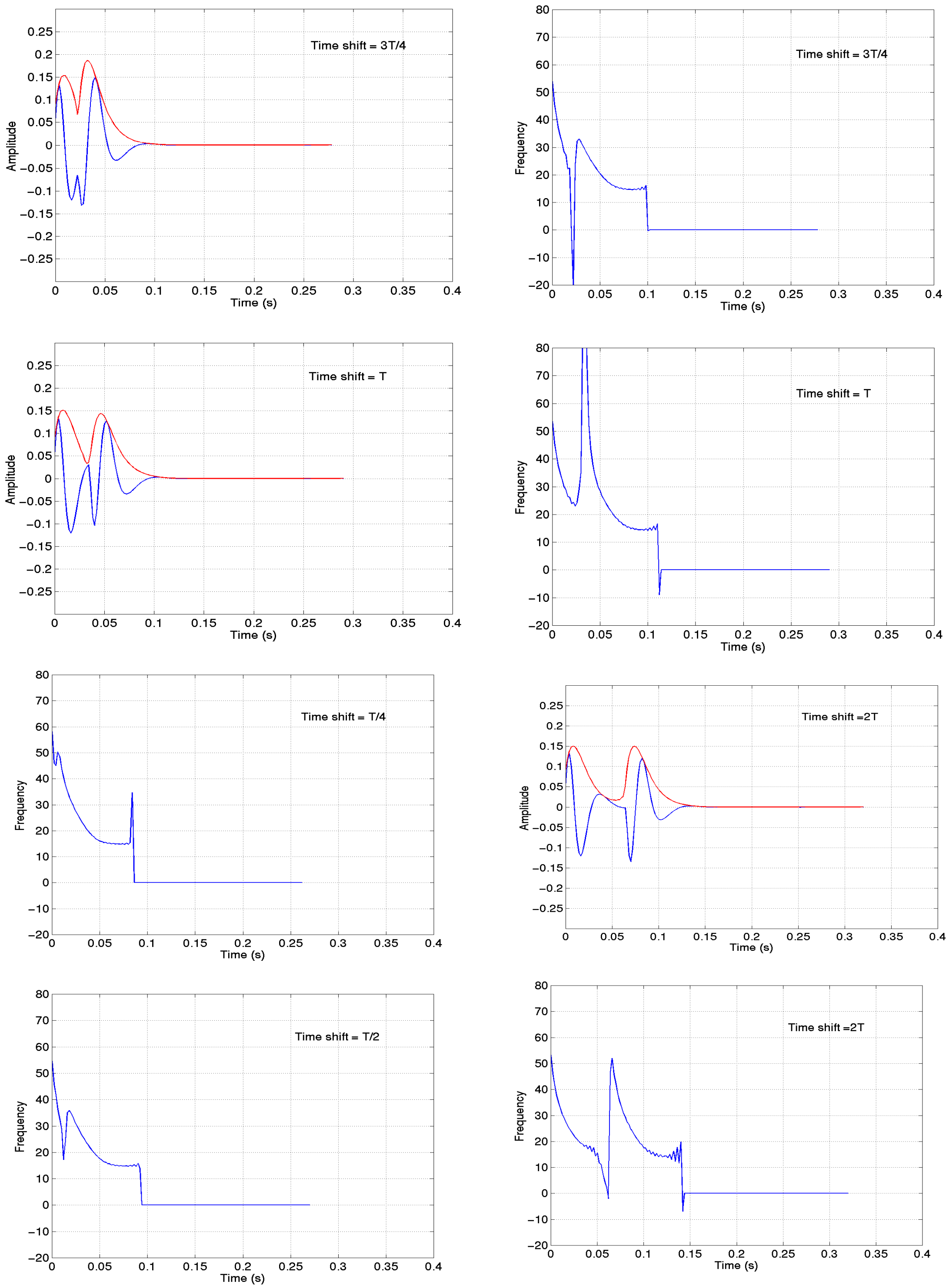


Figure 3. Interfered opposite-polarity minimum phase wavelet spaced by T/15, T/4, T/2, 3T/4, T and 2T (1/T is the dominant frequency, 30 Hz) and the respective diagrams of instantaneous amplitude (left, red) and frequency (right, blue).

Examples

An important method in time-lapse seismic surveys is to pick the events from the top and bottom of a reservoir and to compare time shift and amplitude variation from two or more surveys. In many cases, event picks from the conventional seismic trace may be difficult or inaccurate due to interference. Figure (4) is a random reflectivity series generated with Reflect(1, 0.002) command in Matlab, including approximately 10 major reflectors with a reservoir assumed located between the third and fourth major reflectors from the right side (around 0.7 and 0.74 second).

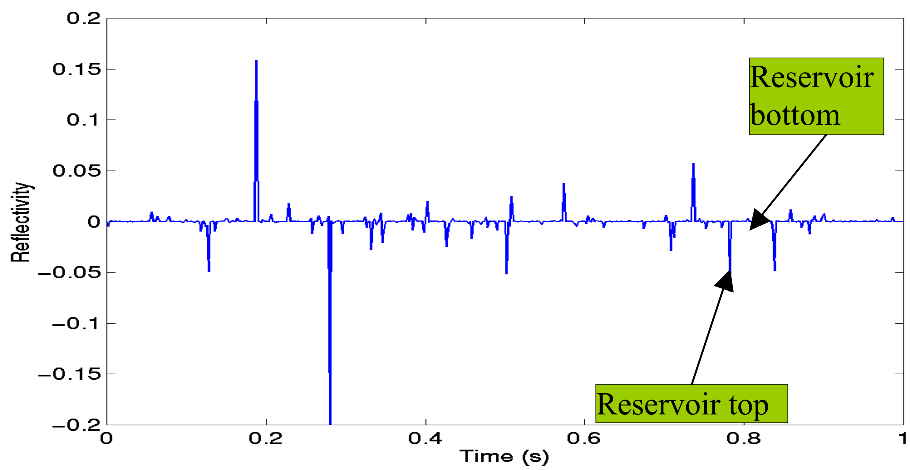


Figure 4. Random reflectivity series generated from Matlab command Reflect(1, 0.002).

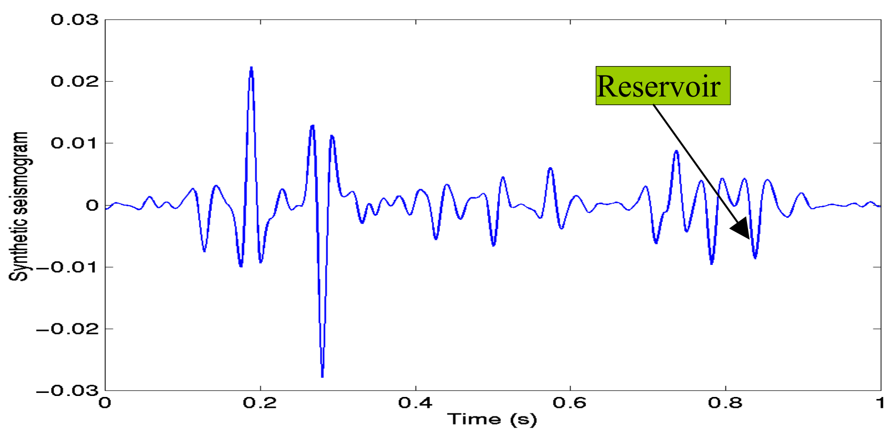


Figure 5. Synthetic seismogram generated from convolution of the random reflectivity series (Figure1) with the Ricker wavelet (30Hz).

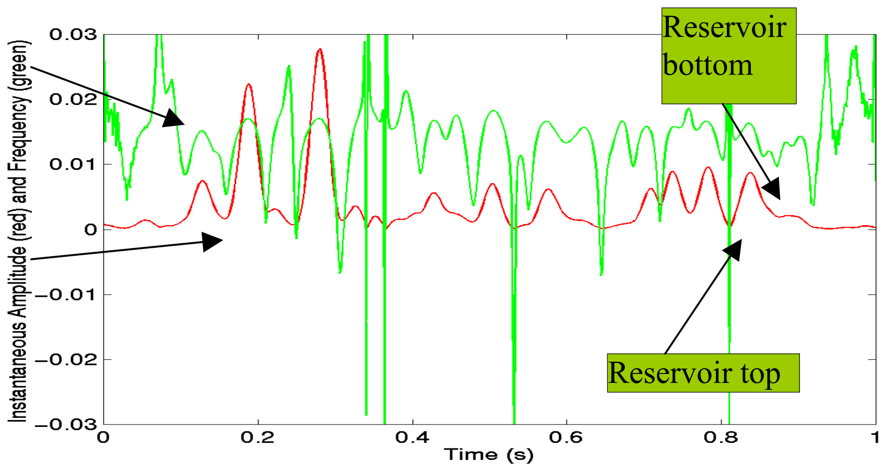


Figure 6. Instantaneous amplitude (red) and instantaneous frequency (green, *2000) calculated based on (5), (2) and (3).

Figure (5) is a conventional seismic trace produced by convolution of the random reflectivity series with the Ricker wavelet (30Hz). As shown in Figure 5, the events overlap and it is difficult to separate them. However, as shown in Figure 6, the envelope of instantaneous amplitude (red) is capable of distinguishing 10 major reflectors, including the two events from reservoir top and bottom. The instantaneous frequency (green) also helps identify event interference by its sudden change. As a result, a more accurate estimate of time shift and amplitude change is possible. Another important factor in Figure 6 is that response frequencies for events not influenced by interference are almost 30 Hz, which is the dominant frequency of the convolved wavelet. In other words, instantaneous frequency captures the wavelet's dominant frequency, and it can be used to study frequency change with time.

Another example is well 0808 from the Blackfoot oilfield in the western Canadian basin. Based on well log data (density and sonic) from 0808, we use 'theosimple' command in Matlab to generate the synthetic seismograms. The density and sonic data are pre-production values from original well log. Those after water flooding are

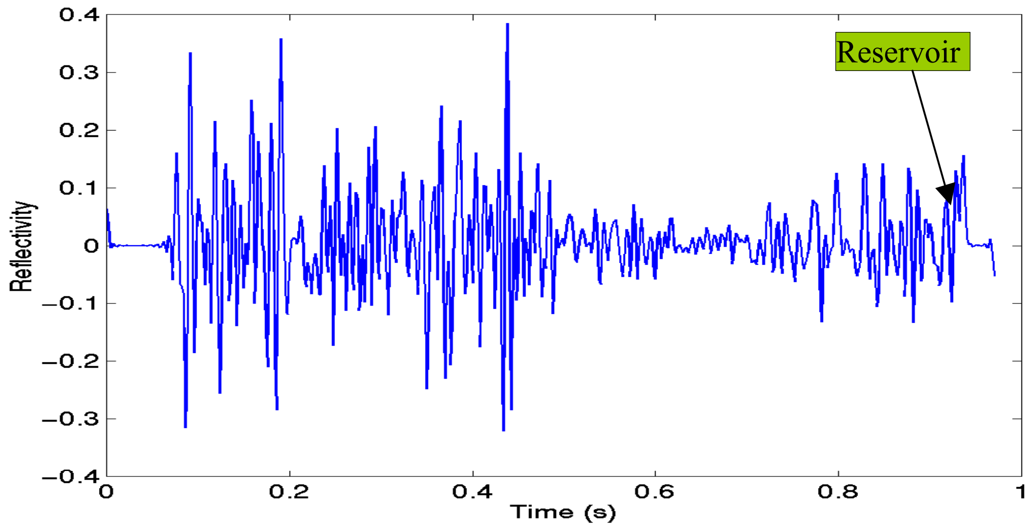


Figure 7. Reflectivity series of well 0808 before production.

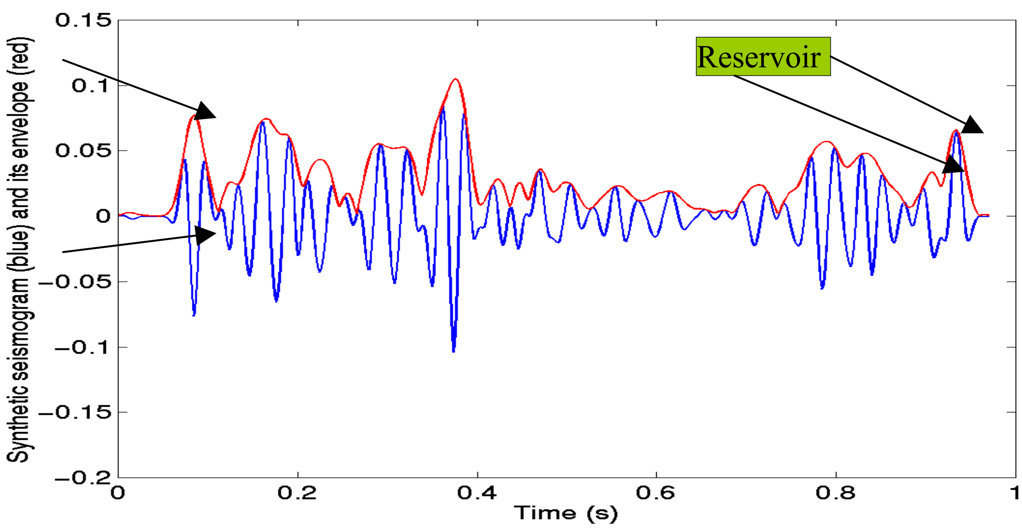


Figure 8. Synthetic seismogram and its envelope of instantaneous amplitude of well 0808 before production.

calculated from fluid substitution using Gassmann equation and Voigt's model (Bentley, Zhang and Lu, 1999). The wavelet is 30 Hz Ricker wavelet. The upper and lower valleys of the reservoir are approximately located between 9.1-9.25 seconds. As shown in Figure 7, reflectors are closely spaced. It is expected that wavelets from individual reflectors interfere with each other strongly.

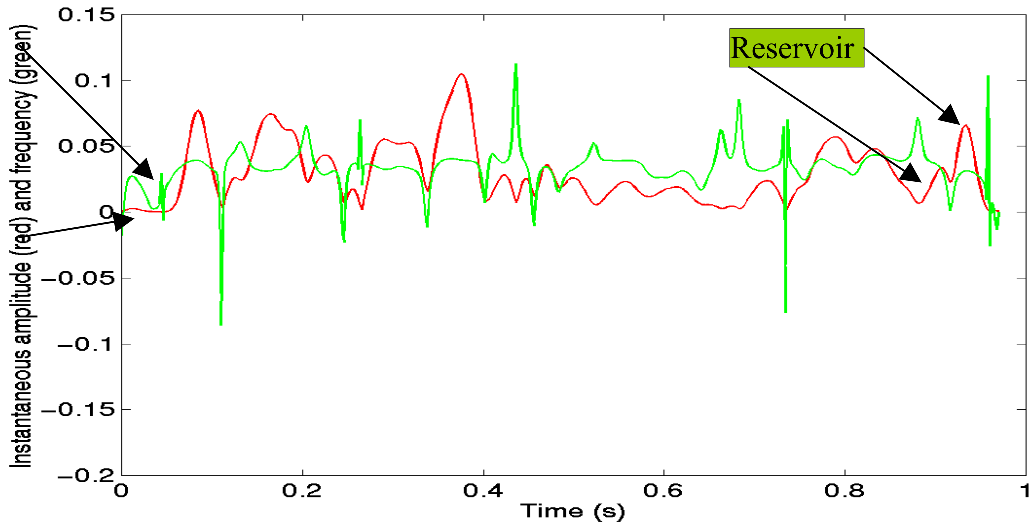


Figure 9. Instantaneous amplitude and frequency of well 0808 before production.

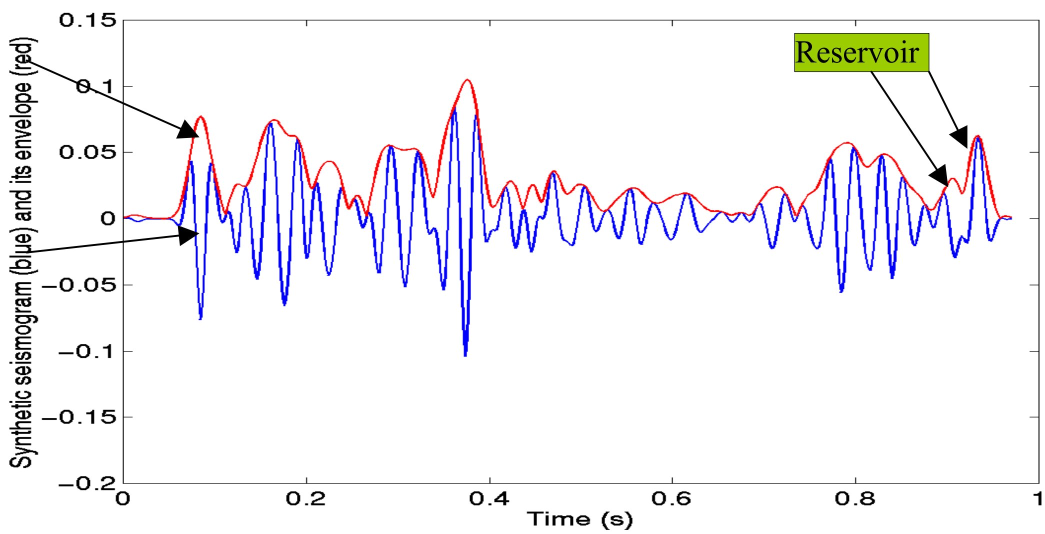


Figure 10. Instantaneous amplitude and frequency of well 0808 after water flood.

In Figure 8, the envelope of instantaneous amplitude shows a series of energy lobes or reflected events, most of which are from a composite of reflectors. The last two energy lobes are from the reservoirs. In Figure 9, the instantaneous frequency indicates that the events are formed by interfered reflectors because the basic properties of the Ricket wavelet are not seen.

Figure 10 is the synthetic seismogram and its envelope of instantaneous amplitude after water flood. It is difficult to see changes in instantaneous amplitude between

Figure 10 and Figure 8. Figure 11 is the envelope of instantaneous amplitude and the instantaneous frequency after water flood. Compared with Figure 9, the frequency minimum between the two reservoir events show some shrink, a possible sign of decreasing time shift (increasing reservoir velocity). Note that the small change may be masked by noise in a true seismic trace.

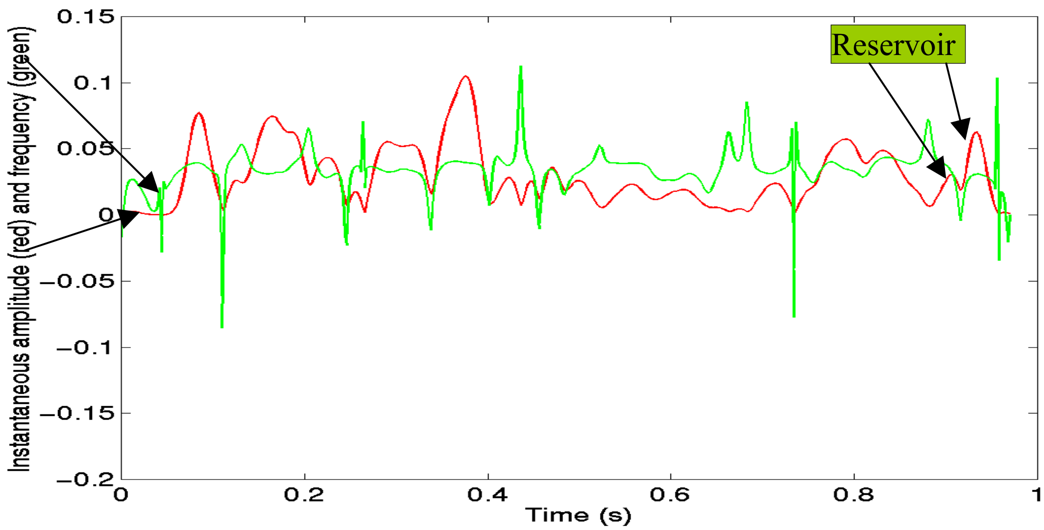


Figure 11. Instantaneous amplitude and frequency of well 0808 after water flood.

CONCLUSIONS

Complex trace analysis enables separation of amplitude from angle (frequency and phase) and definition of instantaneous amplitude, phase and frequency, which are independent of each other. The instantaneous amplitude, a direct response to the magnitude of reflectivity, defines single lobes for individual wavelets and thus has more power to resolve reflectors than the conventional seismic trace, which combines amplitude and angle. The instantaneous frequency is a measure of most energy-loaded frequency or center frequency of the instantaneous power spectrum and, along with the instantaneous phase, traces frequency and phase change with time. It is also sensitive to wavelet interference, manifesting abrupt change at strong interference. In the case of strongly overlapping wavelets, instantaneous amplitude and frequency have characteristics that help identify wave interference. In time-lapse seismic surveys, the power of resolution improves event picks and calculation of time shift and amplitude variation, and the representation of frequency and phase facilitates the study of frequency and phase change with time.

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