# Amplitude scaling for AVO analysis of CSP gathers

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# ABSTRACT

The observation of amplitude variation with offset (AVO) or amplitude variation with angle of incidence (AVA) can give the information of lithology change in the subsurface, which has led to several successful cases of finding oil. AVO or AVA analysis, which is done in common-midpoint (CMP) gathers of un-migrated data often lacks accuracy due to the effects of CMP smearing. It is worth analyzing AVO in common-reflection-point (CRP) gathers after prestack migration.

This paper first introduces the concepts of equivalent offset and common-scatterpoint (CSP) gathers, and derives the relationship between equivalent offset and incident angle for one CRP. It then analyzes four reasonable amplitude scaling factors during CSP gathering, applies them to synthetic data and compares the results. The paper also investigates the possibility of analyzing AVO behavior in CSP gathers for horizontal reflectors. Finally, it applies the most suitable factor to real data.

### **INTRODUCTION**

AVO analysis in CMP gathers gives useful knowledge of lithology changes when the subsurface is bound with horizontal reflectors. When the subsurface has dipping reflectors, one CMP gather includes the reflection information corresponding to a segment of dipping reflector but no longer a reflection point. For AVO analysis in complex media CMP reflection dispersal has to be removed. Since migration is successful in correcting seismic sections for structural effects, it is natural to seek a means of using migration to remove this dispersal, which has led to research work called 'True amplitude migration'. The true amplitude migration process is based on a lossless, isotropic, elastic earth. Several true amplitude migration methods have been developed such as the Delft migration/inversion presented by Berkout and Wapenaar (1993) and others.

The equivalent offset migration (EOM) is a Kirchhoff prestack migration algorithm, which was first introduced by Dr. John Bancroft and his co-workers (Bancroft and Geiger, 1994, Bancroft, et al., 1998). EOM is based on CSP concepts. CSP has more physical meaning than CMP, as each CSP defines a reflection point based on time migration principles. Therefore, it will be valuable to investigate amplitude changes of CSP gathering process based on equivalent offset and see if the amplitude variation with equivalent offset can be used as AVO analysis. This could also be helpful in understanding if the EOM method is an amplitude preserving (true amplitude migration) method.

# METHODOLOGY

#### Scatter point model and prestack migration

In seismic data processing, the earth subsurface is often modeled as a layered medium with each layer having uniform acoustic properties. The reflection energy from the interfaces between the layers can be considered as the sum of the scattered energy from a large number of points 'closely' located on the interfaces. The reflection amplitude at each point is taken as proportional to the reflection coefficient of the interface at this point location. This subsurface model is called the scatter point model, which forms the basis of Kirchhoff migration methods. The traveltime response of one scatter point located at (x, z) with a source-receiver pair at surface location  $x_s$  and  $x_r$  can be expressed as

$$T = \sqrt{\frac{\tau^2}{4} + \frac{(x_s - x)^2}{V^2}} + \sqrt{\frac{\tau^2}{4} + \frac{(x_r - x)^2}{V^2}},$$
(1)

where  $\tau = 2z/V$  is two-way vertical traveltime, V is seismic wave propagation velocity.

Equation (1) can also be expressed in CMP -offset domain as:

$$T = \sqrt{\frac{\tau^2}{4} + \frac{(x_{off} - h)^2}{V^2}} + \sqrt{\frac{\tau^2}{4} + \frac{(x_{off} + h)^2}{V^2}},$$
(2)

where *h* is half source-receiver offset,  $x_{off}$  denotes the horizontal distance (called migration distance) between a CMP location  $x_{cmp} = (x_s + x_r)/2$  and the related scatter point surface location *x*, i.e.,  $x_{off} = |x_{cmp} - x|$ .

Equation (2) describes a surface in  $(x_{off}, h, t)$  space called Cheop's pyramid (Claerbout, 1985). Prestack migration can be thought of as a process to collect from Cheop's pyramid to the scatter point, all the energy scattered from that scatter point.

### Equivalent offset and CSP gather

Equation (2) can be rearranged by splitting the time-related term and the space-offset term as

$$\tau^{2} = T^{2} - \frac{4}{V^{2}} (h^{2} + x_{off}^{2} - \frac{4x_{off}^{2}h^{2}}{(VT)^{2}})$$
(3)

Then the equivalent offset  $h_e$  can be defined as

$$h_e^2 = h^2 + x_{off}^2 - \frac{4x_{off}^2 h^2}{(VT)^2}$$
(4)

The equivalent offset concept introduces a convenient method for gathering reflection and scattered energy in seismic data based on the scatter point model.

A CSP gather at a certain surface location is defined as a two-dimensional rearranging of the seismic energy in equivalent offset and traveltime. Using equivalent offset, CSP gathers can be formed by collapsing all the energy of Cheop's pyramids corresponding to the vertically arranged scatterpoints into hyperbolas.

### Equivalent offset and incident angle

Supposing there is reflection energy reflected by one reflection point on a horizontal reflector, in Cheop's pyramid the energy of reflection forms a hyperbola as shown in Figure 1. Along the hyperbola each offset h corresponds only to one incident angle.



Figure 1. Reflection energy of one reflection point on a horizontal reflector in Cheop's pyramid is a hyperbola.

Forming the CSP gather by equivalent offset, the Cheop's pyramid becomes a hyperbola. The specular energy, i.e. the reflection energy, should be moved to its corresponding position in the equivalent offset section. For the specular energy the incident angle varies with half source-receiver offset h. However equivalent offset  $h_e$  is also a function of h. Therefore, the incident angle of the specular energy can be a function of  $h_e$  as equation (5).

$$h_{e}^{2}(\theta) = h^{2}(\theta) + x_{off}^{2} - \frac{4x_{off}^{2}h^{2}(\theta)}{(VT(\theta))^{2}}$$
(5)

For a horizontal reflector, the migration distance  $x_{off}$  equals zero, in this case, the equivalent offset equals half of the source-receiver offset, so the incident angle versus equivalent offset is shown in the upper side of the curve in Figure 2. The dash circle in Figure 2 shows the zero incident angle position in equivalent offset section. For a dipping reflector, the incident angle of the specular energy versus equivalent offset is shown in the lower side of the curve in Figure 2 and the zero incident angle position is shown by a solid circle.

Figure 2 also verifies that if there is only specular energy then equivalent offset versus the incident angle of the specular energy is one-to-one. This is the bridge between CSP gathers and amplitude analysis in CRP gathers.



Figure 2. a) Contour of Cheop's pyramid relates to equivalent offset, dipping angle  $\beta$  and incident angle  $\theta$ ; b) Relationship between equivalent offset and incident angle.

# Amplitude scaling factors during CSP gathering

In general, Kirchhoff type migration methods can be called diffraction summation methods. For 2D poststack data, the diffractions (the traveltime responses of scatter points) are hyperbolas, and for prestack data, the diffracted energy forms Cheop's pyramid. The amplitude information along the diffractions is not uniformly distributed. According to Yilmaz (1987), the following factors must be considered before diffraction summation:

- 1. The obliquity factor or the directivity factor;
- 2. The spherical spreading factor.

Now consider these two factors during CSP gathering.

# 1. Obliquity factor

In a conventional Kirchhoff migration algorithm, the amplitude obliquity factor tends to zero at the farthest possible migration distance, and tends to maximum when the migration distance is zero. Between these two extreme cases, the scaling factor should change "smoothly" from 0 to 1. By this simple principle, some simplified amplitude scaling factors can be applied to the CSP gathering process.

The definition of equivalent offset implies a simple relationship between migration distance  $x_{off}$  and equivalent offset  $h_e$ , that is, when  $x_{off=0}$ , the  $x_{off}$ -to-  $h_e$  the ratio is zero, and it equals 1, when  $x_{off}=h_e$ , which corresponds to the maximum possible migration distance. Thus the following two scaling functions can be used as approximations of the amplitude obliquity factor.

$$scal_{-1} = 1 - \frac{x_{off}}{h_e}$$
 and  $scal_{-2} = 1 - (\frac{x_{off}}{h_e})^2$ 

The above two factors only consider the migration distance, not source-receiver offset amplitude effects. When  $x_{off}$  tends to he, h tends to zero. In this case using these two factors can deteriorate the amplitude.

The ratio of migration output time  $\tau$  to zero-offset time  $T_n$  approximates the amplitude obliquity factor for zero-offset migration (Bancroft, 1997). This ratio should also be a good approximation for prestack migration. The non-zero-offset obliquity may also be different from the zero-offset cases. The obliquity factor can be approximated by the ratio of  $\tau$  directly to the prestack traveltime T. For the EOM method these two obliquity factors can be expressed as

$$scal_{-3} = \frac{\tau}{T_n} = \sqrt{\frac{T^2 - \frac{4h_e^2}{V^2}}{T^2 - \frac{4h^2}{V^2}}} = \sqrt{\frac{V^2 T^2 - 4h_e^2}{V^2 T^2 - 4h^2}} \text{ and}$$
$$scal_{-4} = \frac{\tau}{T} = \frac{\sqrt{T^2 - \frac{4h_e^2}{V^2}}}{T} = \frac{\sqrt{V^2 T^2 - 4h_e^2}}{VT}.$$

#### 2. Spherical spreading factor

The spherical spreading amplitude factor depends on the distance from a certain source to a certain receiver within a given travel time. The distance used for time migration can be approximated by the value of VT, when V is the migration velocity for the relevant scatter point. This VT value does not change during the CSP gathering process. Thus, similar to the obliquity factor  $scal_4$ , the correction of spherical spreading can be applied efficiently on CSP gathers.

### **EXAMPLES**

In the previous section we have derived four scaling factors, which we now apply to synthetic data and to real data.

#### Synthetic data example

First we create an ideal subsurface model, consisting of only one horizontal reflector located at depth 1500m in a constant velocity medium.

	Density	P-wave velocity	S-wave velocity
	$(kg/m^3)$	(m/s)	(m/s)
Upper layer	2450	4300	2500
Lower layer	2420	4000	2450

Table 1. Parameters for synthetic data.

In this case the AVO effects can be seen clearly in CMP gather as shown in Figure 3. For this CMP source-receiver offset varies from -2500 to 2500m. As the source-receiver offset becomes larger, the amplitude becomes larger.



Figure 3. Amplitude changes with source-receiver offset in CMP gather.

To form one CSP gather there are two loops. The first loops through input traces and the second through equivalent offsets with the amplitude scaling factor being applied during Loop 2. For each equivalent offset, the scaling factor is different for one input trace. Figure 4 shows one CSP gather of synthetic data, with the CSP located in the middle of the survey line.



Figure 4. One CSP gather with the CSP location in the middle of the survey line.

First suppose there is no amplitude attenuation so that the scaling factor equals 1. Next apply each of the four scaling factors. The results are shown in Figure 5.



Figure 5. Amplitude variation with equivalent offset. a) Scaling factor is 1; b) Scaling factor is  $scal_{1}$ ; c) Scaling factor is  $scal_{2}$ ; d) Scaling factor is  $scal_{3}$ ; e) scaling factor is  $scal_{4}$ .

None of the amplitude scaling is obviously wrong compared with the CMP gather within equivalent offset 1000m, which equals source-receiver offset 2000m. For the first two scaling factors *scal\_1* and *scal\_2*, the amplitude of near zero equivalent offset is deteriorated by the reason mentioned before. For the third and fourth scaling factors, which were derived in a reasonable way, they are all good at near offset. The third scaling factor i.e.  $\tau/T_n$ , shows the right trend compared with the CMP gather especially at far offset (source-receiver offset reaches 2800m). It can be concluded that for amplitude scaling this scaling factor is better than the others and is the most suitable to this problem. Next, in Figure 6 consider the result obtained from traditional Kirchhoff prestack time migration using ProMAX.



Figure 6. a) CMP 400 gather is after Kirchhoff prestack time migration. b) Amplitude changes with offset after prestack Kirchhoff time migration in this CMP gather.

The amplitude of CMP gather after prestack Kirchhoff time migration has totally deteriorated. Processing in ProMAX, such as amplitude balancing after migration, may be one of the reasons for the deterioration. Comparing Figure 5 and Figure 6, the amplitude spectrum in the equivalent offset section, no matter what kind of scaling factor is used, is more reliable than that obtained from Kirchhoff prestack time migration. This is another merit of EOM.

### Real data example

The real data is Blackfoot data acquired in 1997. The Blackfoot field is located SE of Calgary, AB, in Canada. Within this data, an amplitude anomaly can be seen clearly around 1.0 second. The pre-processing steps include:

- 1. Elevation statics;
- 2. True amplitude recovery;
- 3. Surface consistant deconvolution;
- 4. TV spectral whitening;
- 5. Refraction statics;
- 6. Residual statics;
- 7. Trim statics;
- 8. Trace muting.



After all these processes the CMP gather is as shown in Figure 7, in which an amplitude anomaly is indicated by the rectangle.

Figure 7. Amplitude anomaly of real data in CMP gather is shown in rectangle.

Input data after pre-processing to form the CSP gather is shown in Figure 8, with the amplitude anomaly again shown in the rectangle.



Figure 8. Amplitude anomaly in CSP gather of real data is shown in rectangle.

Does the amplitude anomaly exist after Kirchhoff prestack time migration? In Figure 9, the amplitude anomaly is no longer visible.



Figure 9. Amplitude in CMP gather after prestack Kirchhoff time migration has no anomaly.

#### PRELIMINARY CONCLUSION

The relationship between equivalent offset and incident angle for one commonreflection-point is unique. It is the bridge of AVO analysis in CSP gathers.

Using amplitude scaling factor of  $\tau/T_n$  gives the amplitude change tendency especially at far offset.

The amplitude in CSP gathers can be used for AVO analysis of horizontal reflectors.

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