The spectral ratio technique for determining sea floor reflectivity

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ABSTRACT

The convolutional model of seismic signals has inspired a number of computational techniques over the years, many of them aimed at extracting one or more convolution factors, such as the wavelet or the reflectivity sequence. One such technique for marine streamer data uses the ratio of a simple multiple event to its primary to extract the reflectivity function of the sea floor interface, or the ratio of autoconvolved primary to multiple to extract an estimate of the source wavelet. This technique has been implemented in ProMAX. The method is described here and demonstrated on a model source gather as well as on a field record from the modelled region. Unique to this study is the extension of the method to the radial trace (R-T) domain, which provides a more proper geometry for the computation than the X-T domain.

INTRODUCTION

The convolutional model of seismic signals is one of the more successful and intuitive devices in seismic processing theory. Its representation of a seismic trace as the convolution of an underlying earth function with various response functions due to the seismic source, the seismic detector, and other systematic effects intuitively suggests that the various convolutional factors may, under some conditions, be mathematically separable.

One situation that especially lends itself to a simple separation technique is that of marine seismic data recorded by a conventional hydrophone streamer cable. Because the sea is a simple medium, and the sea floor usually provides a strong impedance contrast, clean sea floor reflections and often several orders of simple multiples are prominent features of many seismic records gathered in this environment. When the sea floor is flat, the primary reflection and its common raypath multiple, illustrated in Figure 1, have a particularly simple mathematical representation. Since the sea surface is a single discrete interface, and its impedance contrast with the air is large, its upgoing reflectivity function may, with negligible error, be assumed to be a spike of negative unity amplitude. Also, the equivalence of the path segment lengths of the primary reflection and multiple events imposes a simple amplitude relationship between them, based on spherical divergence. This means that the mathematical expression of seismic energy that follows a single raypath consists of terms for primary and multiples that are particularly simple and which contain common factors. Transformed to the Fourier domain, the expression simplifies further, in that the expressions for reflection and multiple events become multiplications rather than convolutions. In this domain, terms representing primary and multiple reflections can be combined by multiplication and/or division to cancel common factors; it is this procedure that can be used to isolate the factor representing the sea floor reflectivity or the source wavelet, as shown in the following section.

THE SPECTRAL RATIO TECHNIQUE

For seismic energy travelling the path shown in Figure 1, the amplitude function of the primary sea floor reflection $P(t)$, measured at the surface, can be written

$$P(t) = (1/2R) * W(t) * r(t) \quad , \tag{1a}$$

and the amplitude function of the first order multiple $M(t)$, can be expressed as

$$M(t) = (-1) * (1/4R) * W(t) * r(t) * r(t) \quad , \tag{2a}$$

where R is the path length of one segment of the travel path, $W(t)$ is the unknown source wavelet, $r(t)$ is the unknown reflectivity function of the sea floor, -1 is the reflectivity function of the sea surface, t is the two-way travel time, and $*$ represents the convolution operation. The spherical divergence factor $(1/2R)$ in equation (1a) is not a function of t, the convolutional variable, so the convolution symbol following it may be replaced by multiplication. Likewise, the first two convolutions in equation (2a) may also be replaced with multiplication, resulting in the equations

$$P(t) = (1/2R) \cdot W(t) * r(t) \quad , \tag{1b}$$

and

$$M(t) = (-1) \cdot (1/4R) \cdot W(t) * r(t) * r(t) \quad . \tag{2b}$$

Further simplification of these expressions may be made by transforming them to the Fourier domain, where convolution becomes multiplication:

$$\overline{P}(f) = (1/2R) \cdot \overline{W}(f) \cdot \overline{r}(f) \quad , \tag{1c}$$

and

$$\overline{M}(f) = (-1) \cdot (1/4R) \cdot \overline{W}(f) \cdot \overline{r}(f) \cdot \overline{r}(f) \quad , \tag{2c}$$

where $\overline{P}(f), \overline{M}(f), \overline{W}(f)$, and $\overline{r}(f)$ are the Fourier transforms of the respective time functions in equations (1b) and (2b), and f is the frequency.

Forming the ratio of equation (2c) to equation (1c) yields:

$$\frac{\overline{M}(f)}{\overline{P}(f)} = \frac{(-1)\cdot(1/4R)\cdot\overline{W}(f)\cdot\overline{r}(f)\cdot\overline{r}(f)}{(1/2R)\cdot\overline{W}(f)\cdot\overline{r}(f)} \quad . \tag{3a}$$

Cancelling common factors in numerator and denominator on the right, and rewriting the equation results in:

$$\overline{r}(f) = \frac{(-2)\cdot\overline{M}(f)}{\overline{P}(f)} \quad . \tag{3b}$$

This equation can be made more computationally robust by multiplying top and bottom by the complex conjugate of the primary spectrum, $\overline{P}^*(f)$, and adding a small stability factor, λ, to the denominator;

$$\overline{r}(f) \cong \frac{(-2)\cdot\overline{M}(f)\cdot\overline{P}^*(f)}{\overline{P}(f)\cdot\overline{P}^*(f)+\lambda} \quad . \tag{3c}$$

Equation (3c) states that the Fourier spectrum of the sea floor reflectivity function may be estimated from the cross-spectrum of the multiple and primary and the power spectrum of the primary. Since there are no unknown factors in this equation, the inverse Fourier transform of the function $\overline{r}(f)$ is the properly calibrated reflectivity sequence $r(t)$, whose sample values are actual reflection coefficients.

An equation for estimating the source wavelet can be constructed by squaring (autoconvolving) equation (1c) and dividing it by equation (2c):

$$\frac{\overline{P}(f)\cdot\overline{P}(f)}{\overline{M}(f)} = \frac{(1/2R)\cdot(1/2R)\cdot\overline{W}(f)\cdot\overline{W}(f)\cdot\overline{r}(f)\cdot\overline{r}(f)}{(-1)\cdot(1/4R)\cdot\overline{W}(f)\cdot\overline{r}(f)\cdot\overline{r}(f)} \quad . \tag{4a}$$

Cancelling common factors in numerator and denominator on the right, and rearranging the equation yields:

$$\overline{W}(f) = \frac{(-R)\cdot\overline{P}(f)\cdot\overline{P}(f)}{\overline{M}(f)} \quad . \tag{4b}$$

Once again, the equation may be stabilised by multiplying numerator and denominator by the complex conjugate of the multiple spectrum, $\overline{M}^*(f)$, and adding a stability factor, λ, to the denominator;

$$\overline{W}(f) \cong \frac{(-R)\cdot\overline{P}(f)\cdot\overline{P}(f)\cdot\overline{M}^*(f)}{\overline{M}(f)\cdot\overline{M}^*(f)+\lambda} \quad . \tag{4c}$$

Equation (4c) contains an unknown scale factor, the path length R, so the wavelet amplitudes $W(t)$, are only relative to each other and have no absolute significance. Equations (3c) and (4c) have been implemented in a ProMAX operation to be described later.

DETAILS

One important detail of the technique not yet addressed is that the above mathematics and the following ProMAX module are strictly applicable only to a seismic trace that follows a single raypath as shown in Figure 1. Actual field traces, however, correspond to the situation shown in Figure 2, with primary reflection energy and multiple energy actually coming from segments of different raypaths, with different lengths and angles of incidence. While the spectral ratio technique can be applied to conventional fixed offset traces, at least for deep water (large R) at the smallest offsets (large α), the results can be expected to suffer from the inexact geometry. There is no reason to use conventional traces, however, since the situation depicted in Figure 1 corresponds to constant takeoff angle traces, which is just another name for radial traces (Henley, 1999). Application of the radial trace transform to a raw marine shot gather produces the traces needed for the spectral ratio technique, and the transformation is exact in this case because the raypath segments in the water layer are straight and of constant angle for all orders of the sea floor multiples (as long as the sea floor is level).

It should be evident that gain correction should not be applied to raw traces prior to computing the radial trace transform or the reflectivity ratio, since any gain function destroys the scaling between primary and multiple.

For actual shot records, the process of transforming to the radial trace domain may be complicated by the fact that the sea floor reflection and its multiples are badly spatially aliased on shot gathers for many marine surveys. For aliased records, an additional processing step required is the spatial interpolation of the gather, for which there are a number of methods available, including one using the radial trace transform (Henley, 2000).

THE PROMAX MODULE

The spectral ratio module in ProMAX is a simple one which reads ensembles of seismic traces and applies the spectral ratio technique to each trace to yield in seismic trace format an estimate of either the sea floor reflectivity function or the source wavelet (scaled by the path segment length, R). For each trace, the algorithm uses input parameters to extract one trace segment containing the primary sea floor reflection event and another trace segment of the same length containing the first order multiple of the sea floor reflection. These segments are padded and placed in power-of-two arrays which are subsequently used for the Fourier transforms. After the segments are transformed, the spectral arithmetic described by either equation (3c) or equation (4c) is performed. The result, either a reflectivity or wavelet spectrum, is inverse transformed and output as a seismic trace with the same headers as the input trace.

Following is a description of the parameters presented in the menu of the ProMAX spectral ratio module:

- *Switch for reflectivity or wavelet* – This parameter is used to select the desired output of the module, either the calibrated reflectivity sequence of the sea floor, or a scaled estimate of the source wavelet.
- *Length of extraction gate in ms* – The length of the trace segment used for both primary and multiple events is set by this parameter. The main criterion is to make the gate long enough to capture the entire multiple waveform without including significant energy from background reflections. If the gate is long enough to include the multiple, it will always be long enough to capture the primary, if the start of the gate is properly positioned.
- *Begin time of primary gate in ms* – This parameter positions the first gate at the beginning of the primary sea floor reflection.
- *Begin time of multiple gate in ms* – The second gate is positioned at the beginning of the multiple event by this parameter.
- *Stability factor for spectral division* – This is a decimal fraction that is multiplied by the peak power of the divisor spectrum to determine the actual factor added to the divisor spectrum for either reflectivity computations or wavelet computations. It prevents division by zero or very small numbers and controls the degree of whitening in the spectral division.
- *Apparent velocity or offset for the first trace to use* – The algorithm selects input traces by a range of offset values (X-T traces) or velocity values (R-T traces). This parameter is the lower limit for the desired range.
- *Apparent velocity or offset for the last trace to use* – The upper limit for the desired trace range.

Since both X-T gathers and R-T gathers have moveout, the spectral ratio module can only handle small groups of input traces with one pair of gate begin times. In order to handle larger groups of traces in one computation, linear moveout may be applied to an input gather to approximately flatten the sea floor primary reflection and allow both primary and multiple to remain in the gates for a larger range of offsets (velocities). Linear moveout must be used, since NMO stretches waveforms, destroying the phase information needed for the reflectivity/wavelet estimates.

EXAMPLES

To demonstrate the spectral ratio technique, the algorithm was used on a synthetic shot gather generated by a finite difference modelling program to simulate a survey off the east coast of Canada. As well, the program was applied to a real shot gather acquired in the same region used as the basis for the model. In both instances, the primary and multiple sea floor reflections were badly aliased on the raw shot records, so the radial transform de-aliasing technique described in Henley (2000) was applied to interpolate the gathers. Next, the gathers were transformed to the radial trace domain, and sufficient linear moveout was applied to roughly flatten the primary sea floor reflection. Finally, the spectral ratio module was applied to groups of traces from each R-T gather.

Model test

Figure 3 shows several traces containing the sea floor primary from the model shot gather, while Figure 4 shows the corresponding multiple events. Figure 5 shows the sea floor reflectivity functions estimated from the gated radial trace domain primary/multiple pairs, while Figure 6 contains the wavelet estimates. The amplitude values of the reflectivity sequence samples indicate that the sea floor has a large positive reflectivity. The individual sample values in the peak of the reflectivity spike are about 0.15 to 0.2; so integrating the samples in the vicinity of the peak gives a total sea floor reflectivity of ~ 0.5 to 0.7. Also, because each reflectivity function estimate is associated with a specific radial trace (constant angle), the reflectivity functions capture the angular dependence of the sea floor reflectivity over some range of angles corresponding to the velocity range of the selected radial traces.

Field data example

The gated sea floor primaries from the radial transform of the real shot gather are shown in Figure 7, while the multiples appear in Figure 8. Figures 9 and 10 contain the estimated reflectivity functions and wavelets, respectively. As with the model data, the sample values of the reflectivity functions can be used to estimate the overall reflectivity of the sea floor. One notable feature of these reflectivity functions, in comparison to those extracted from the model, is that there appears to be a negative postcursor for each function, indicating that the sea floor here may consist of a relatively thin hard layer overlying somewhat softer sediments. Unfortunately, the limited bandwidth precludes a more detailed analysis.

EXTENSIONS

Although not shown or discussed, the spectral ratio technique can also be used on reflections from other horizons than the sea floor. This application, however, requires a very strong primary reflection as well as a strong simple multiple on a background of very weak primary reflections. An estimate for a reflection from a deeper interface will not be as robust as that for the sea floor, however, because the primary reflection and multiple are both deeper and weaker than those for the sea floor—the strongest and shallowest reflector in the seismic record.

Also, although not discussed earlier, higher order multiples can be used in spectral ratio computations, with certain adjustments. It can easily be shown that, except for the scale factor, a ratio of the second order multiple to the first order multiple will also yield a reflectivity function estimate for the sea floor. The scale factor, which is due just to differences in spherical divergence, can be shown to be –3/2 instead of –2. Likewise, third and second order multiples can be used in a ratio, but with a scale factor of –4/3. If N is the order of the multiple, then $-(N+1)/N$ can be shown to be the appropriate scale factor to calibrate the reflectivity function computed from the ratio of multiple of order N to multiple of order $N-1$. Similarly, wavelet estimates can be made from ratios of higher order multiples. In general, estimates of either reflectivity functions or wavelets are increasingly narrow in band with increasing multiple order, however.

CONCLUSIONS

The spectral ratio technique can be used to give reasonable and useful estimates of sea floor reflectivity functions and source wavelets using gated seismic events corresponding to the primary and first order multiple reflections from the sea floor on radial trace transforms of marine source gathers (hydrophone streamer data only, *not* OBC or OBS data).

ACKNOWLEDGEMENTS

The author wishes to acknowledge the support of the sponsors of CREWES and various staff members for discussion.

REFERENCES

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Henley, D.C., 2000, More radial trace domain applications. CREWES Research Report, Vol. 12.

FIGURES

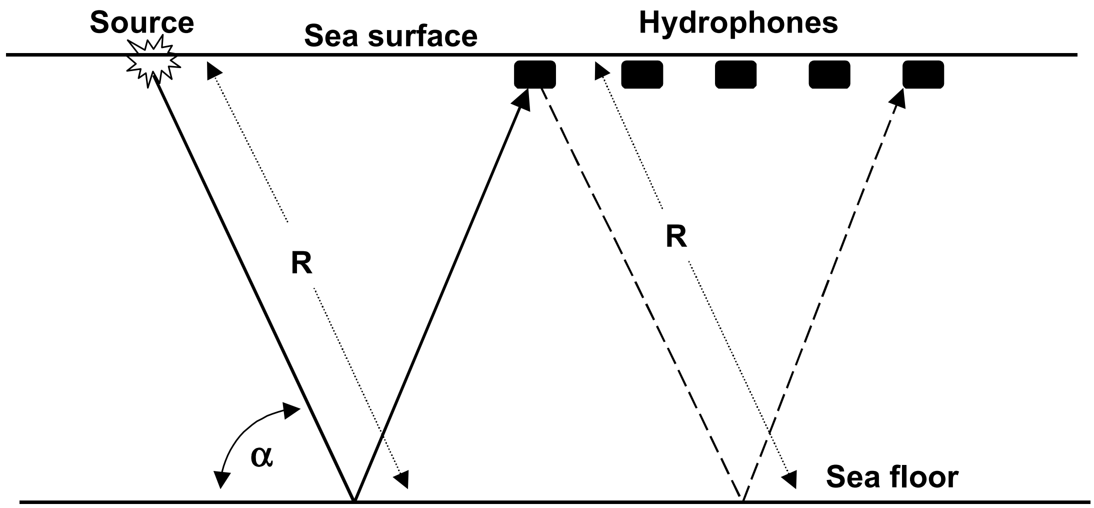

FIG. 1. Schematic showing the relationship between a sea floor reflection and its single bounce multiple as recorded along a single raypath. Here the multiple path length, 4R, is twice the primary path length, 2R, the reflection angle, α, for primary and multiple is the same, and primary and multiple share the first reflection point. Since the signal along a single raypath cannot be physically recorded, it must be constructed from the signals recorded by many hydrophones using the radial trace transform.

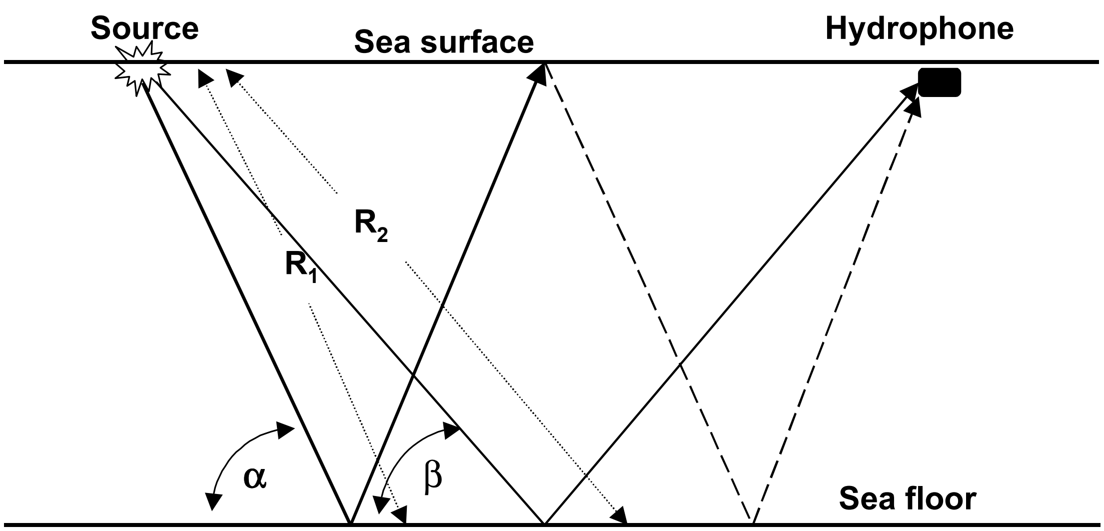

FIG. 2. Schematic showing the relationship between a sea floor reflection and its single bounce multiple as recorded by a single hydrophone in a streamer cable. The path length of the multiple, $4R_1$, is less than twice the primary path length, $2R_2$, the angle of incidence of the multiple, α, is not the same as that of the primary, β, and none of the reflection points are in common.

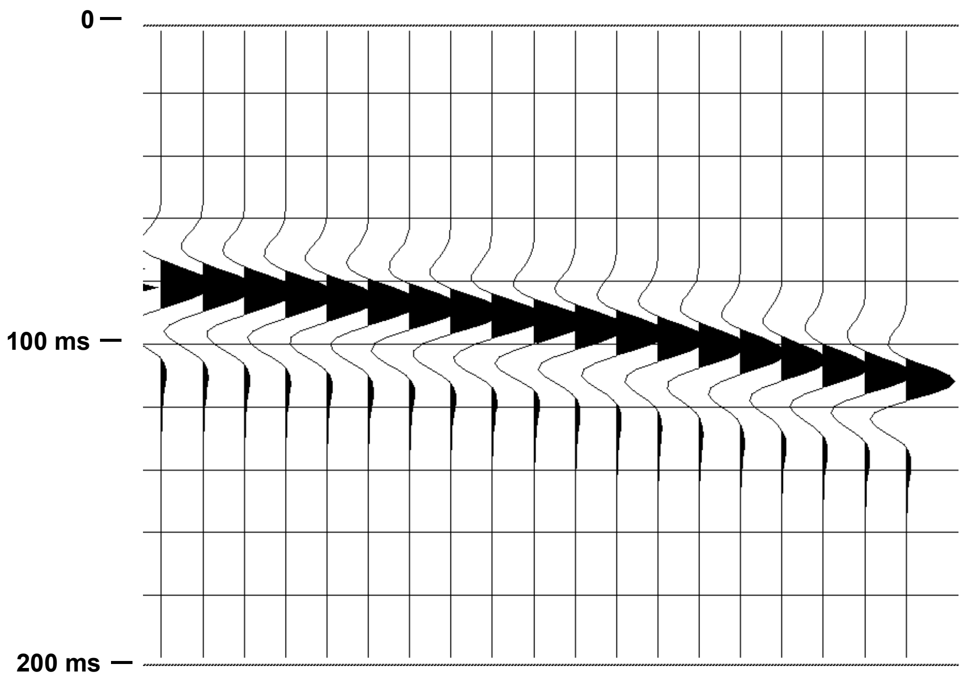

FIG. 3. Primary sea floor reflection from a shot gather associated with a synthetic finite difference model.

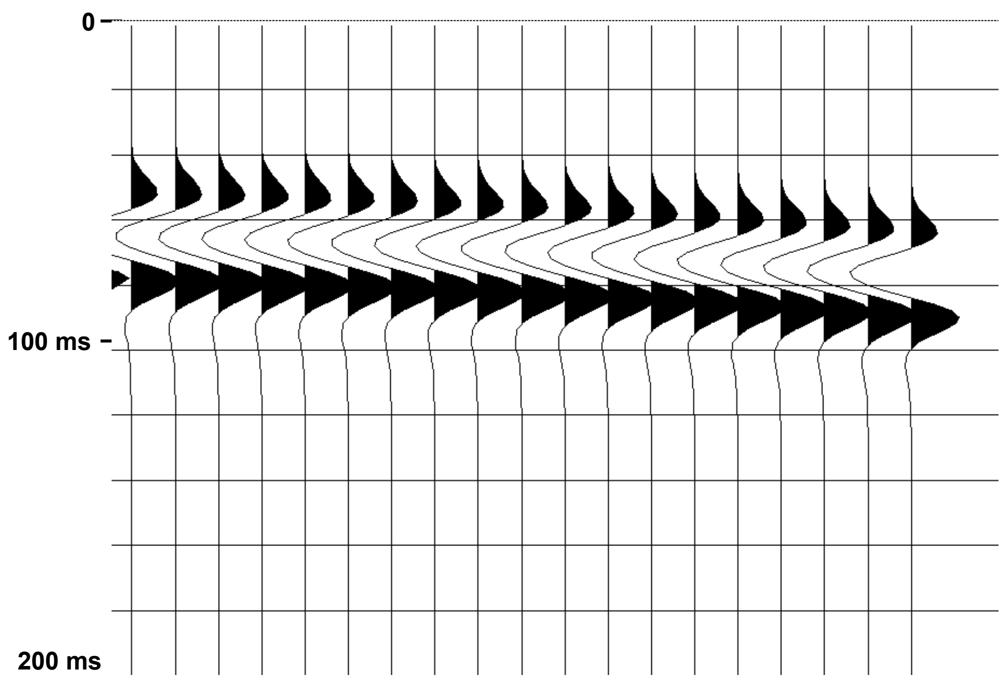

FIG. 4. Single bounce multiple from synthetic shot gather.

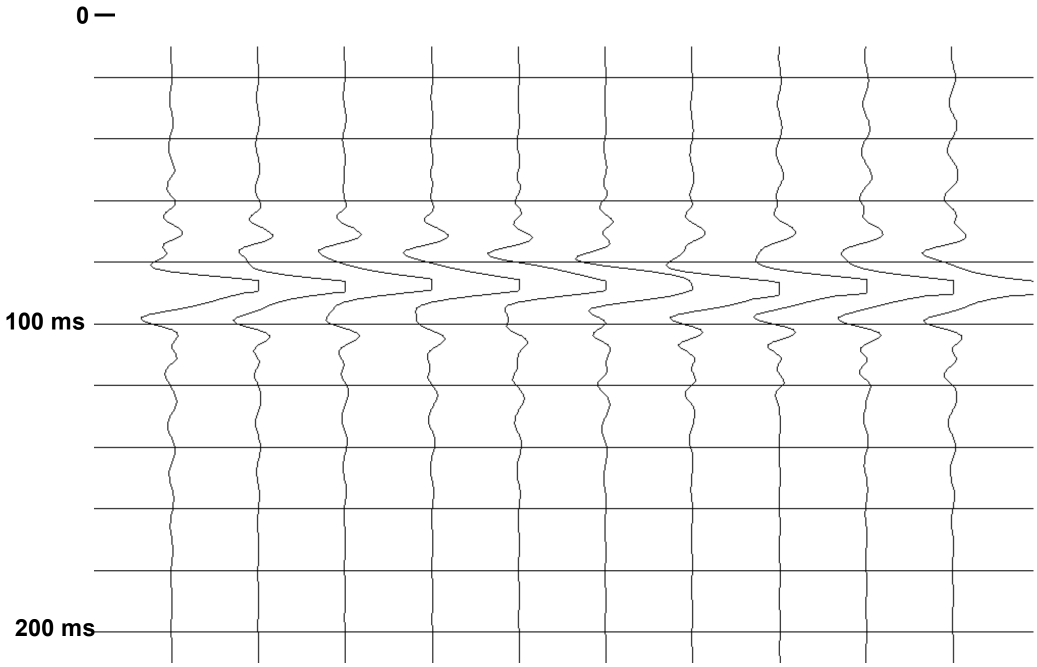

FIG. 5. Reflectivity series computed from the synthetic shot gather, using the spectral ratio technique on radial traces formed from the gather. The amplitudes of these waveforms are calibrated reflectivity values. The offset between traces corresponds to amplitude of 0.2.

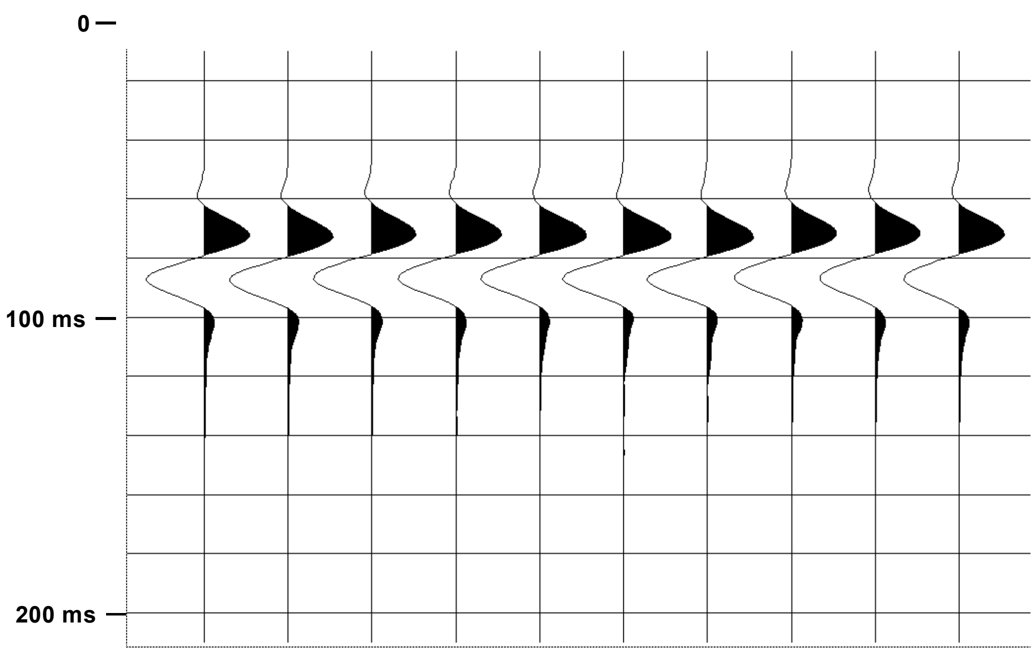

FIG. 6. Wavelets estimated from the synthetic shot gather using the spectral ratio technique on radial traces formed from the gather. The amplitudes of these waveforms are uncalibrated.

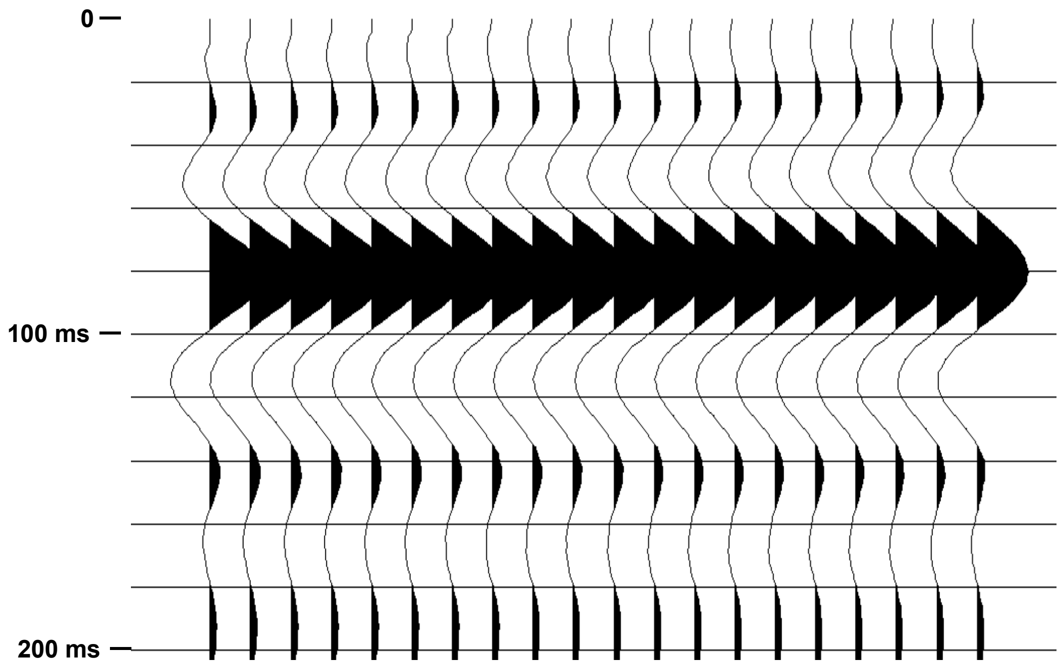

FIG. 7. Primary sea floor reflection gated from the radial trace transform of marine field record.

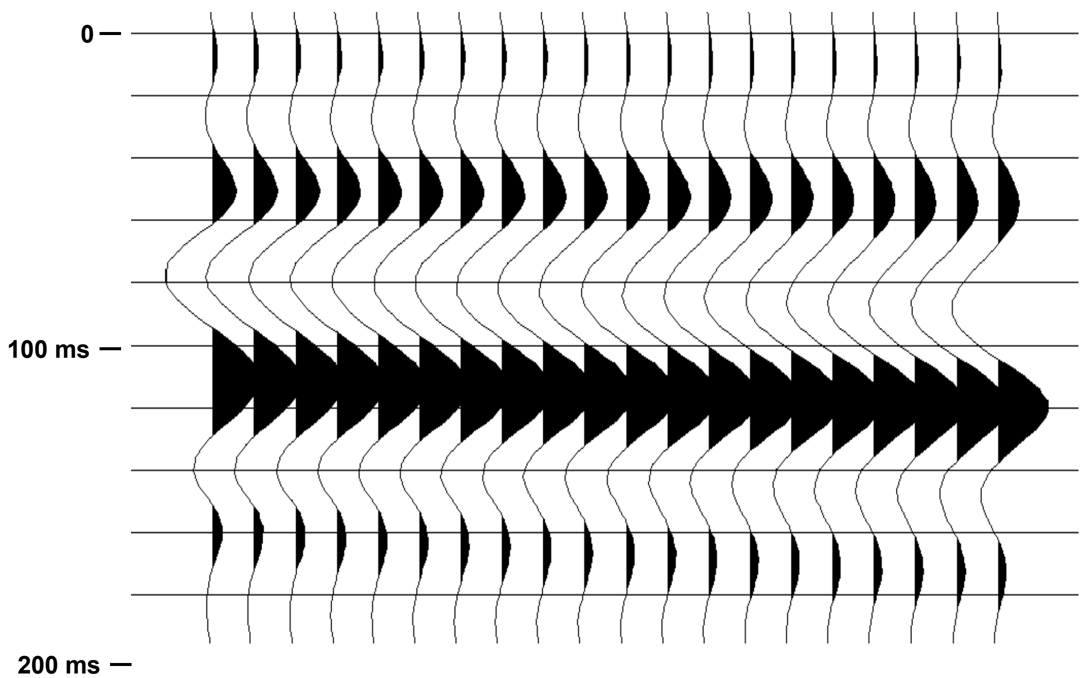

FIG. 8. Simple multiple of sea floor gated from the radial trace transform of marine field record.

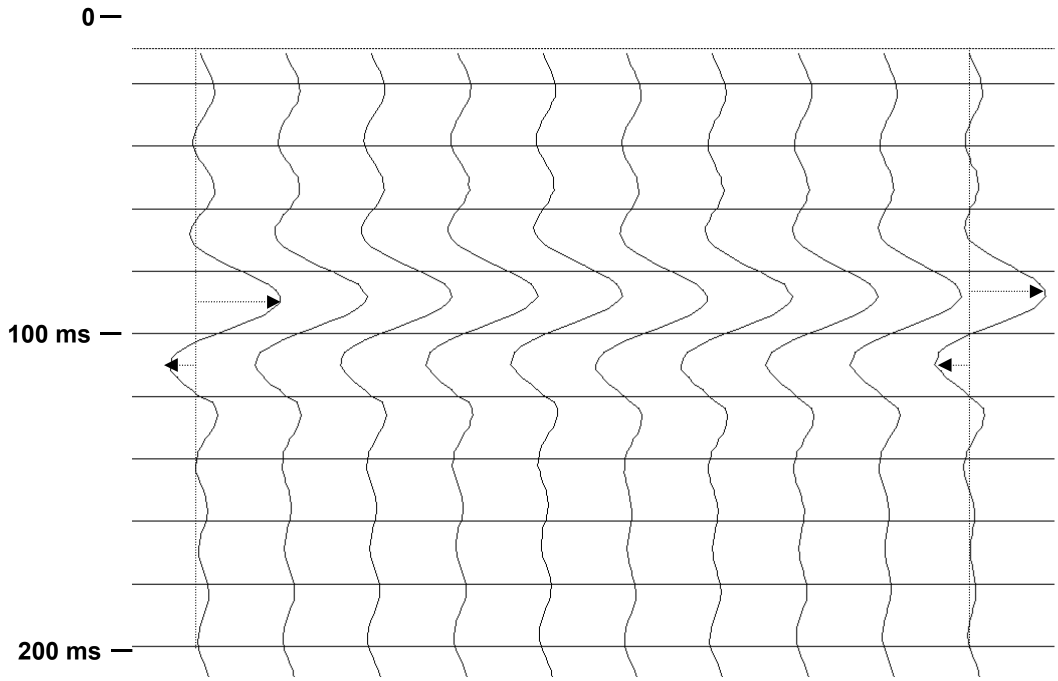

FIG. 9. Reflectivity series derived from marine field record using the spectral ratio technique on the gated primary and multiple from the radial trace transform of the gather. Trace offset corresponds to an amplitude of 0.2. These reflectivity functions appear to have significant negative postcursors, possibly signifying a relatively thin hard layer overlying somewhat softer sediments.

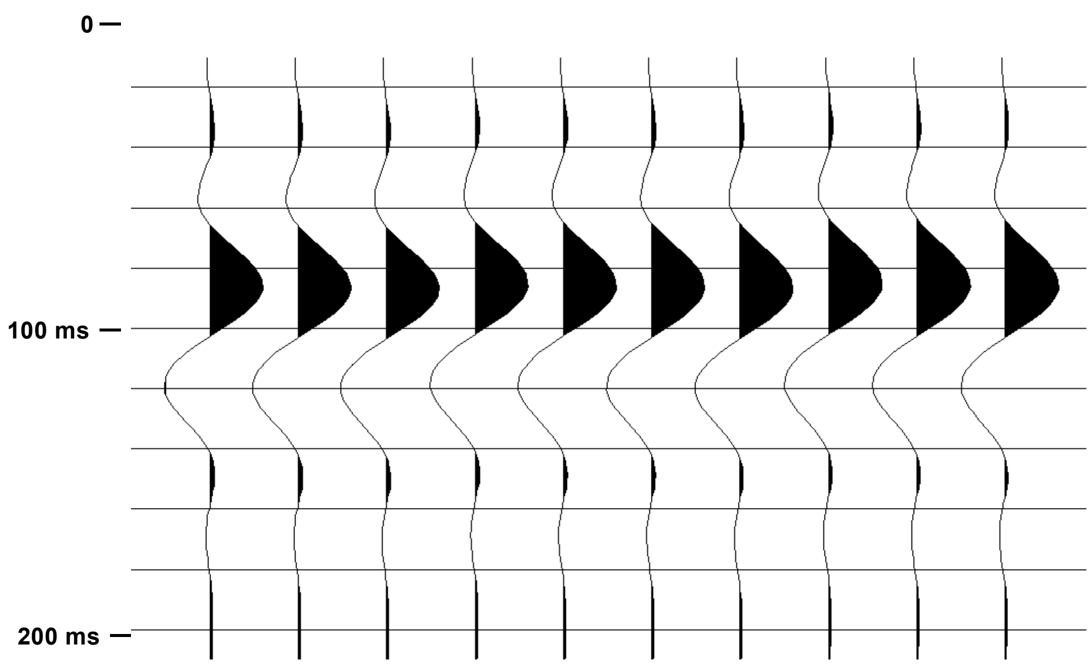

FIG. 10. Wavelets estimated from marine field record using the spectral ratio technique on the gated primary and multiple from the radial trace transform of the gather.