Generalized Gardner relation for gas-saturated rocks

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ABSTRACT

Gardner's relation has previously been extended to include dependence on both shear and compressional velocity simultaneously. The former work, carried out on water-saturated lithologies, is here extended to gas-saturated rocks and also to lithologies that may be either gas- or water-saturated.

INTRODUCTION

Gardner's relation ($\rho = C\alpha^{1/4}$, ρ = density, α = compressional velocity) has been widely used as a means of providing approximate densities or compressional velocities, when one was available and the other not (Gardner et al., 1974). It has also been applied to developing formal AVO expressions (Smith and Gidlow, 1987).

The previously proposed empirical relation (Ursenbach, 2001, hereafter referred to as Paper I) involves the variables ρ, α, and β (β = shear velocity):

$$\rho = C\alpha^{A}\beta^{B}. \tag{1}$$

A, *B*, and *C* are empirically determined parameters. This expression allows one to predict shear-velocity logs from the more-commonly obtained density and sonic logs. When lithology is known, α-β relations are generally used to predict β from α. Wang (2000b) has also given a different β prediction technique for sandstones. However if lithology is not known, a single lithology-independent $\beta(\rho,\alpha)$ relation is useful. Another potential application analogous to the Smith-Gidlow AVO method employs the relation

$$\frac{d\rho}{\rho} = A\frac{d\alpha}{\alpha} + B\frac{d\beta}{\beta}, \tag{2}$$

derived from Equation (1).

The original Gardner relation was developed for water-saturated lithologies, but there are certainly cases of interest involving gas-saturated lithologies, and one is often interested in cases where the saturating fluid is unknown. In these cases, one would desire an expression not designed specifically for water-saturation. In this paper we derive generalized Gardner relations for gas-saturated lithologies and for both gas- and water-saturation (Ursenbach, 2002).

REVIEW OF METHOD

To generate empirical relations one requires appropriate data sets. As described in detail in Paper I, pseudo-data was generated from empirical relations due to Wang (2000a). One key set of parameters specifies the density ranges in which data is generated. Table 1 contains the information relevant for this study:

Lithology	Range of ρ (liquid-saturated)	Range of ρ (gas-saturated)
Dolostone	2.45-2.85	2.45-2.85
Limestone	1.85-2.75	2.30-2.70
Sandstone (sst)	2.10-2.60	2.10-2.60
Shaley sst	2.05-2.55	2.20-2.55
Unconsolidated sst	2.05-2.30	2.05-2.30
Shale	2.30-2.70	Not used

Table 1: Ranges of densities values employed for generating input data for each lithology.

From these density values we employ Wang's equations [Equations (6) and (7) from Wang, 2000a] to generate values of α and β. These are of the form $\alpha = a\,\rho^b$, which is simply a rearrangement of Gardner's relation, and similarly $\beta = c\,\rho^d$. We then use a least-squares procedure to fit the resulting (ρ,α,β) triples from all lithologies to the expression

$$\ln \rho = \ln C + A \ln \alpha + B \ln \beta \qquad (3)$$

to obtain A, B, and $\ln C$. Associated with the least-squares minimization is a correlation coefficient describing the degree to which the data is well-described by the linear function, Equation (3). The expression for the correlation coefficient is well known for systems with one dependent variable. By analogy this was generalized to two dependent variables, and results were given in Paper I. For completeness we include the relevant expression here. The correlation coefficient is given formally by

$$R = \frac{\Sigma_i (\ln \rho_i - <\ln \rho>) \times (A[\ln \alpha_i - <\ln \alpha>] + B[\ln \beta_i - <\ln \beta>])}{\sqrt{\Sigma_i (\ln \rho_i - <\ln \rho>)^2} \times \sqrt{\Sigma_i (A[\ln \alpha_i - <\ln \alpha>] + B[\ln \beta_i - <\ln \beta>])^2}} \qquad (4)$$

$$= \frac{A(<\ln \rho \ln \alpha> - <\ln \rho><\ln \alpha>) + B(<\ln \rho \ln \beta> - <\ln \rho><\ln \beta>)}{\sqrt{<(\ln \rho)^2> - <\ln \rho>^2} \times \sqrt{A^2(<(\ln \alpha)^2> - <\ln \alpha>^2) + B^2(<(\ln \beta)^2> - <\ln \beta>^2)}}$$

where <…> denotes an average.

In Paper I we obtained results of A = 0.0799, B = 0.164, and C = 1.87 with a correlation coefficient value of R = 0.83.

METHOD & RESULTS FOR GAS-SATURATION AND COMBINED CASE

The method for obtaining an expression for gas-saturated rocks was similar to that of the liquid, but with a differing treatment of the α/β ratio. Paper I describes how additional effort was required to make the generalized Gardner relation consistent with established α/β relations. However, gas-saturated rocks typically have an α/β similar to that of their mineral constituents. Ranges of α/β predicted by Wang's relations were compared to values for mineral α/β ratios (Mavko, 1998; Christensen, 1982) and an empirical relation for dry sandstone (Wang, 2000b). It was concluded that the extra measures required for the water-saturation case were not necessary for the gas-saturated case. The other difference is that the shale lithology has not been included (Wang, 2000a).

From the fitting procedure for the gas-saturation case we obtained values of $A = 0.192$, $B = 0.106$, $C = 1.612$, and $R = 0.911$ using 100 generated data points. The sum of A and B, 0.298, is relatively close to the Gardner value of ¼, reminiscent of the results from water-saturation.

A visual representation of the fit is presented in Figures 1 and 2 below. Density is plotted against shear and compressional velocity in the following way. The horizontal and vertical axes represent α and β respectively. For each data triple, the density is plotted at the appropriate location as a short line, whose angle with the horizontal represents the magnitude of the density. A horizontal line represents the minimum density in Table I, and a vertical line represents the maximum density. The solid lines represent input densities from Wang's lithology-specific relations, and the dotted lines over top represent the density predicted by Equation (1) after fitting to all lithologies simultaneously. For each (α,β) pair, the input and output ρ values cross over at their midpoints. For a very good fit the dashed line lies over the black line and is not visible. For clarity, the displayed results present only 7 density points for each lithology instead of the full number used in the fitting procedure. One observes a few results that are noticeably less satisfactory than others. These correspond to certain density ranges of sandstones. In general though, this appears to be a reasonable method for fitting data from several lithologies.

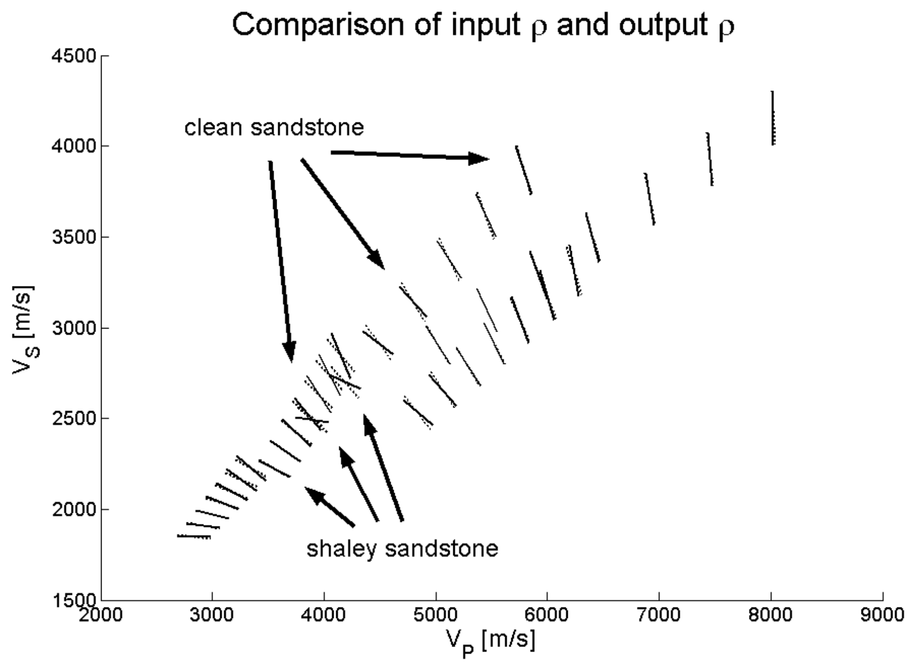

FIG. 1: Comparison of fitted density to input density. The density for a given (V_P=α, V_S=β) pair is given by the angle of the line with respect to the horizontal (horizontal = minimum density in range, vertical = maximum density). The solid lines represent the original input densities due to Wang (2000a). This data was then fitted to Equation (1), and the values of ρ predicted by the fit of Equation (1) for the same (α, β) pairs are given by the dashed line intersecting the solid line. For a good fit the dashed line is not visible but can be seen, for instance, with some sandstones.

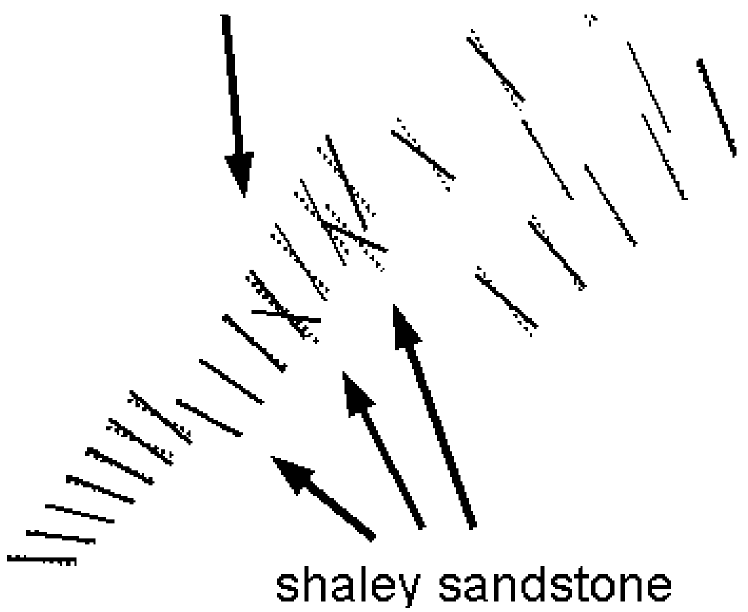

FIG. 2: An expansion of the lower left corner of Figure 1. Note that the densities of clean and shaley sandstones (equal to the slope of the solid lines) are unequal in the overlapping region. The densities from the fitted function (the slopes of dashed lines) are continuous, fitting as closely as possible to the solid lines.

For completeness, we have obtained a fitting using both water-saturated and gas-saturated values (without generating any extra points). The result is $A = 0.205$, $B = 0.053$, $C = 1.688$, and $R = 0.847$. We note that the sum of A and B, 0.258, is intermediate between the values from only gas- or water-saturated data. The results of this fitting are displayed in Figure 3 below:

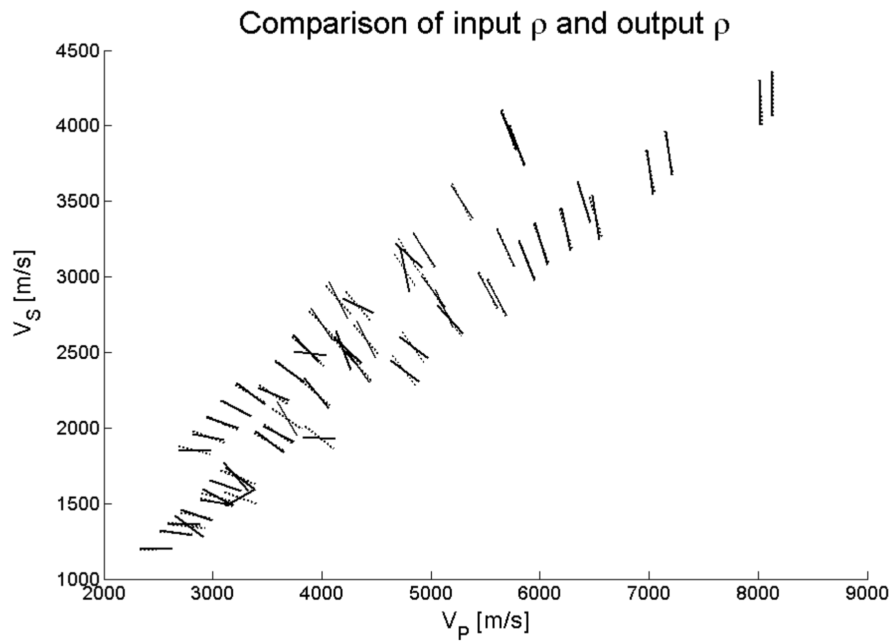

FIG. 3: Similar to Figure 1 but for both water- and gas-saturated rocks.

CONCLUSIONS

Gardner's relation, $\rho = 1.741\ \alpha^{1/4}$ (for ρ in g/cm^3 and α in km/s), is generalized herein to $\rho = C\ \alpha^A\ \beta^B$, where C is similar in size to 1.741 and $A + B \approx 1/4$. Specifically for water-saturated rocks we have previously obtained values of $A = 0.0799$, $B = 0.164$, and $C = 1.87$, while for gas-saturated we have here obtained $A = 0.192$, $B = 0.106$, and $C = 1.612$. Drawing on both types of saturations simultaneously we obtain $A = 0.205$, $B = 0.053$, and $C = 1.688$. These results are expected to be reasonably accurate for several lithologies. As with the original Gardner relation, these expressions are not expected to possess any physical meaning, but simply serve as empirical correlators of experimental data. As such, these results are expected to be useful both for predicting shear velocities and for substituting velocity contrasts for density contrasts in AVO inversion approximations.

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