Kinematic determination of Vp/Vs in multicomponent seismic data

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ABSTRACT

This paper discusses a method of determining the zero-offset velocity ratio V_p / S_s (γ_0) via velocity analysis, using Thomsen's (1999) non-hyperbolic traveltime equation. Processing of synthetic shot gathers from a four-layer geologic model provides the Pwave velocities (V_p) and PS velocities (V_{ps}) used to stack the data sets. A semblance program written in Matlab code to compute semblance in 3-dimention is employed to scan for depth varying γ_0 . It was observed that post-critical angle reflected events adversely affect the V_p/V_s values determined by this method. Thus, it is necessary to mute the data prior to velocity analysis. Accuracy increases as the depths to reflecting horizons increase. This is probably because the conversion point moves closer towards the midpoint with increasing depth. Furthermore, accuracy increases by decreasing the sampling interval in each variable used in the computation of semblance. That is increasing the number of samples in each variable. This however, linearly increases the computational cost. By converting the γ_0 and the PS zero-offset times to the corresponding P-wave times, the correlation of P-wave and PS stacked sections can be implemented. Correlation of the transformed PS data and the P-wave stacked data sets is found to be good (the difference being less than 5%).

INTRODUCTION

A primary goal of exploration geophysicists is to be able to translate acquired seismic data and other related information into drilling sites and hydrocarbon volumes. One of the critical values/parameters that governs the translating process is the stacking velocity. It constitutes a vital characteristic in exploration seismology; and its determination forms a key step in seismic data processing. The task is more challenging in PS propagation because the raypath is not symmetric about the conversion point. Tessmer and Behle (1988) derived expressions for the conversion point. They provided approximate and exact equations for data gathering. Thomsen (1999) extended their work to multi-layer, anisotropic and inhomogeneous media. Thomsen's traveltime equation is complicated but is applicable to both long and short offsets in isotropic, and also with slight modification, anisotropic media. The equation contains several variables such as the γ_0 value (defined as the ratio of P-wave velocity to S-wave velocity measured at zero-offset time), P-wave velocity V_{P} , a PS velocity V_{PS} , and the PS zero-offset traveltime t_{ps0} . To use this equation to perform velocity analysis, it is useful to make some simplifying assumptions to reduce the number of variables to three. In the non-hyperbolic NMO equation, $V_{ns}^{2}\gamma_{0}$ is substituted for V_p^2 (Tessmer and Behle, 1988). Thus, instead of computing semblance as SC ($\gamma_0, V_{ps}, V_p, t_{ps0}$), it will be computed as SC ($\gamma_0, V_{ps}, t_{ps0}$). The semblance values can be considered as being contained in a volume whose dimensions are defined by the

variables. From a semblance peak at given time and velocity, using V_{ps} and t_{ps0} , we expect to find a PS event that will be mappable to a P-wave event to correlate the PS event whose time value was used to compute the P-wave time. Computing γ_0 and t_{ps0} values via velocity analysis, will facilitate the transformation of PS stacked data to P-wave times. Transforming PS data to P-wave times offers a quicker and better method of correlation (Garotta, 2000). The objectives of this work are to:

- 1. Compute the PS stacking velocities,
- 2. Derive associated S-wave velocities, and
- 3. Automatically register and correlate P-wave and PS sections

METHODOLOGY

A four-layer geologic model, with the associated physical attributes as shown in Figure1 was constructed using the GX2 raytracing modelling package. The dimensions of the model are 4000 meters long and 4000 meters deep.

Simulated field acquisition parameters are shown in Table 1.

 $V_p = 3000m/s, V_s = 1395.35m/s, V_p/V_s = 2.15$, Density =2.5, Thickness =1000 metres

 $V_p = 3500m/s, V_s = 1635.51m/s, V_p/V_s = 2.14$, Density = 2.52, Thickness = 900 metres

 $V_p = 4000m/s, V_s = 1877.9m/s, V_p/V_s = 2.13$, Density = 2.54, Thickness = 1700 metres

 $V_p = 4500m/s, V_s = 2195.12m/s, V_pV_s = 2.05$, Density = 2.55, Thickness = 400 metres

FIG 1. Showing geologic model with associated physical attributes

Table 1. Acquisition parameters

Geophone spread length	4000 meters
Acquisition geometry	Split spread
Geophone spacing	20 meters
Shot interval	100 meters
CDP interval	10 meters
Total number of shots	41
Total CDP locations	400
Source wavelet	30 Hz Ricker

The resultant shot gathers were exported to the Promax environment for processing. To process the PS data, the asymptotic common conversion-point binning was used. The velocity ratio (γ_0) used was 2.12, the arithmetic average of the four input velocity ratios. Subsequent CCP gathers were stacked after routine velocity analysis. The P-wave and PS stacked sections are shown in Figures 2 and 3.

Having obtained the stacked sections, the next step is to transform the PS section to Pwave times for the purposes of correlation and interpretation. This is a crucial stage in PS and P-wave interpretation. Transformation of PS data to P-wave times entails using the appropriate velocity ratios for the various geologic formations. This implies using depthvarying γ_0 values. To derive this function we turn to the non-hyperbolic equation (Thomsen, 1999).



Traveltime equation and velocity analysis

Tessmer and Behle (1988) first derived the expressions for the PS conversion point and the corresponding traveltimes. Thomsen (1999) provided a Taylor's series expansion and extended it to cover anisotropic and inhomogeneous media. Yilmaz (2001) gave another derivation for the PS traveltimes.



After stacking, we need to be able to correlate the PS stack section to the P-wave stack section. To do this, we need an expression, which ties the sections at their zero-offset time. Tessmer and Behle (1988) derived this expression as:

$$t_{ps0} = \frac{1}{2} (1 + \gamma_0) t_{p0}, \qquad (1)$$

where, t_{p0} and t_{ps0} are respectively the P-wave PS zero-offset traveltime and $\gamma_0 = V_p/V_s$. For an isotropic medium γ_0 is assumed constant; but in transversely isotropic media, this parameter is depth-variant. Equation (1) is the expression we need to transform the PS stack section to P-wave times. Now, if it is possible to perform velocity analysis such that we scan for the velocity ratio, the PS velocity together with the corresponding PS times, then we can compute the equivalent P-wave times for correlation.

Thomsen's (1999) traveltime equation:

Thomsen's non-hyperbolic traveltime equation is given below as:

$$t_{ps}^{2}(X) = t_{ps0}^{2} + \frac{X^{2}}{V_{ps}^{2}} - \left(\frac{(\gamma_{0} - 1)^{2}(V_{p}^{2} - V_{ps}^{2})}{4(V_{p}^{2} - V_{ps}^{2})(\gamma_{0} + 1)t_{ps0}^{2}V_{ps}^{4} + (\gamma_{0} - 1)^{2}V_{p}^{2}V_{ps}^{2}X^{2}}\right)X^{4}, \quad (2)$$

where V_{ps} is the P-S-wave shot-spread moveout velocity, defined by Tessmer and Behle (1988) as:

$$V_{ps}^{2} = \frac{V_{p}^{2}}{\gamma_{0}},$$
(3)

and t_{ps0} is as defined in equations (1) above (see Thomsen, 1999 for further details). The V_{ps} in both equations (2) and (3) are the same, and represent the PS moveout velocity. Substituting equation (3) in (2), we obtain:

$$t_{ps}^{2}(X) = t_{ps0}^{2} + \frac{X^{2}}{V_{ps}^{2}} - \left(\frac{(\gamma_{0}-1)^{2}}{4(\gamma_{0}+1)t_{ps0}^{2}V_{ps}^{4} + \gamma_{0}(\gamma_{0}-1)V_{ps}^{2}X^{2}}\right)X^{4}.$$
 (4)

In this paper, equation (4) was used to compute of semblance for various velocity ratios. The equation applies to both short and long offsets (Thomsen, 1999). Variables contained are:

 t_{ps0} = PS zero-offset times, X = offset distance V_{ps} = PS stacking velocities

 γ_0 = velocity ratio (values at zero-offsets).

Equation (4) can be written in simpler form thus:

$$t_{ps}^{2}(X) = a + bX^{2} + cX^{4}$$
(5)

where
$$a = t_{ps0}^{2}$$
, $b = (V_{ps})^{-2}$ and $c = -\left(\frac{(\gamma_{0} - 1)^{2}}{4(\gamma_{0} + 1)t_{ps0}^{2}V_{ps}^{4} + \gamma_{0}(\gamma_{0} - 1)V_{ps}^{2}X^{2}}\right)$.

SEMBLANCE COMPUTATION AND γ_0 DETERMINATION

The semblance coefficient is a statistical measure introduced into velocity analysis by Tanner and Koehler (1969). Simply stated, it is defined as the normalized output/input energy ratio, where the output trace is a simple compositing or sum of the input traces (Neidell and Taner, 1971). Mathematically, the semblance coefficient SC can be stated as:

$$SC = \frac{\sum_{j=k-(N/2)}^{k+(N/2)} \left\{ \sum_{i=1}^{M} f_{i,j(i)} \right\}^2}{\sum_{j=k-(N/2)}^{k+(N/2)} \sum_{i=1}^{M} f_{i,j(i)}^2} , \qquad (6)$$

where, k is the time of the event calculated using the traveltime equation [in this case, equation (1)], N is the window length within which semblance is calculated, M is the number of traces, i is the channel (in this case the offset), and j is the time sample, and $f_{i,i(i)}$ is the seismic amplitude at offset i, and at time sample, j.

Matlab code

Using equations (4) and (6), a Matlab code was developed to compute semblance as a function of PS velocity, velocity ratio (V_p/V_s) , and zero-offset, two-way time; i.e. semblance = $SC(V_{ps}, \gamma_0, t_{ps0})$. To execute the code, a range of values of PS velocities, velocity ratios, and zero-offset traveltimes are provided. And in a manner akin to routine velocity analysis, values corresponding to maximum semblance are extracted. The objectives of the semblance computation are to:

- 1. determine the velocity ratios (V_p / V_s) values.
- 2. determine PS stacking velocities and corresponding P-wave velocities.

RESULTS AND DISCUSSIONS

The results from semblance analysis are shown in Figures 4 to 16. In these Figures, time is increasing upward. Figure 4 shows the 3D (volume) display of semblance at the three horizons. Figure 5 shows the time slice at horizon 3. From this figure, the location of maximum semblance can easily be seen. Values corresponding to zero-offset time t_{ps0} , velocity ratio γ_0 , and PS velocity V_{ps} can be extracted by taking vertical slices parallel to the time and velocity axes (see Figures 6 and 7). In a similar manner, Figures 8 to 13, demonstrate how the various variable values can be obtained. At horizon 1, however, some inconsistencies are observed. This is because at shallow depths, (Tessmer and Behle, 1988), the asymptotic conversion point (ACP), deviates from the actual common conversion point (CCP). Results obtained from these displays are tabulated in Table 2. Computed P-wave times agree very closely with the observed data. Accuracy can be increased by, using a finer grid interval; but this would increase the computational cost. Run-time on a dedicated machine is about 10-15 minutes.

t_{ps0}	Maximum	V_{ps}	$t_{p0} = (2 * t_{ps0}) / (1 + \gamma_0)$	t_{p0} from PP data
	γ_0			
3.1	2.12	2430	2.05	1.974
1.95	2.057	2170	1.28	1.243
1.18	2.147	2030	0.75	0.715

Table 2. Showing computed variable values; beginning with horizon 3

Transformation of P-S-wave stack data to P-wave times

To transform the PS stack data to P-wave times, three steps are involved:

1. Interpolate the γ_0 values obtained from semblance analysis, to generate a γ_0 function. I.E. $\gamma_0(t_{ns0})$.

2. Next, using this function in equation (1), we compute P-wave times (t_{p0}) .

3. Finally, we plot the PS amplitudes from a given PS-time at their new P-wave times. The results of these steps are shown in Figures 14 to 16. The interpolated γ_0 function is shown in Figure 14. Figure 15 shows the transformed converted wave stack data while Figure 16 shows the comparison of the transformed PS stack data and the P-wave stack section. From these figures, it can be seen that the transformed PS data agrees very closely with the P-wave data.

Error analysis

The differences between the computed P-wave times and the actual are shown in Table 3A. The difference between the actual and computed P-wave times varies from 3.2% to 4.9%. This shows that the computed results very closely agree with the actual values. The differences occur because gathers used in the velocity analysis were formed using the ACP instead of CCP gathers. For better accuracy, gathers should be formed using CCP or a better approximation than ACP.

Horizon	Computed P-wave times	Actual P-wave time	Difference	% Difference
1	0.75	0.715	0.035	4.9
2	1.28	1.24	0.04	3.2
3	2.05	1.974	0.076	3.8
-				

Table 3A showing the difference between actual and computed P-wave velocities

Table 3B showing the difference between input and the scanned-for γ_0

Horizon	Computed γ_0	Input γ_0	Difference	% Difference
1	2.147	2.15	-0.003	-0.14
2	2.057	2.14	-0.083	-3.9
3	2.12	2.13	-0.01	-0.47
		2.05		

From table 3B, the differences between the estimated and the input V_p/V_s values vary from -3.9 to -0.14 %. These differences are large errors; however by, using the derived γ_0 function to re-bin, and performing semblance analysis again, these errors would be narrowed down.

CONCLUSION

It is possible to determine γ_0 and V_{ps} , values kinematically via velocity analysis, using a non-hyperbolic traveltime equation from PS seismic data. Post-critical angle reflected events adversely affect the V_p/V_s values determined by this method. Thus, it is necessary to mute the data prior to velocity analysis. Accuracy increases as the depths to reflecting horizons increase. This is probably because the conversion point moves closer towards the midpoint with increasing depth. Furthermore, accuracy increases by decreasing the sampling interval in each variable used in the computation of semblance. That is increasing the number of samples in each variable. This however, linearly increases the computational cost. By converting the γ_0 and the PS zero-offset times to the corresponding P-wave times, the correlation of P-wave and PS stacked sections can be implemented. Correlation of the transformed PS data and the P-wave stacked data sets is found to be good (the difference being less than 5%).

FUTURE WORK

Having used synthetic data to test the practicability of this method, the next stage is to implement it on real data sets. In addition to this, the scope of work would be extended to anisotropic data to determine anisotropic parameters. For this method to be commercially viable, another programming language such as C++ would be utilized to develop the algorithm for faster and better results.

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FIG 4. A 3D display of semblance showing: Time, V_p/V_s and Vps axes. Note that the time axis is inverted in all the 3-D displays in this paper.



FIG 5. A 3D display of maximum semblance on horizon 3 time-slice



FIG 6. A 3D display of semblance showing a vertical plane drawn parallel to the Time-Vps plane and intersecting horizon-3 time slice. The line of intersection is along line of maximum semblance occurrence; and the corresponding V_p/V_s value along the V_p/V_s axis is 2.12.



FIG 7. A 3D display of semblance showing a vertical plane drawn parallel to the Time- V_p/V_s plane and intersecting horizon-3 time slice. The line of intersection is along line of maximum semblance occurrence; and the corresponding V_{ps} value along the V_{ps} axis is 2430 m/s.



FIG 8. A 3D display of maximum semblance on horizon 2 time-slice



FIG 9. A 3D display of semblance showing a vertical plane drawn parallel to the Time-Vps plane and intersecting horizon-2 time slice. The line of intersection is along line of maximum semblance occurrence; and the corresponding V_p/V_s value along the V_p/V_s axis is 2.057.



FIG 10. A 3D display of semblance showing a vertical plane drawn parallel to the Time- V_p/V_s plane and intersecting horizon-2 time slice. The line of intersection is along line of maximum semblance occurrence; and the corresponding V_{ps} value along the V_{ps} axis is 2170 m/s.



FIG 11. A 3D display of maximum semblance on horizon 1 time-slice



FIG 12. A 3D display of semblance showing a vertical plane drawn parallel to the Time- V_{ps} plane and intersecting horizon-1 time slice. The line of intersection is along line of maximum semblance occurrence; and the corresponding V_p/V_s value along the V_p/V_s axis is 2.147.



FIG 13. A 3D display of semblance showing a vertical plane drawn parallel to the Time- V_p/V_s plane and intersecting horizon-1 time slice. The line of intersection is along line of maximum semblance occurrence; and the corresponding V_{ps} value along the V_{ps} axis is 2030 m/s.



FIG 14: V_p/V_s function generated by, interpolation using the derived V_p/V_s values and corresponding P-S-wave times obtained from semblance analysis.



FIG 15. PS stacked data transformed to P-wave times



FIG 16: A comparison of the transformed P-S-wave and P-wave stack data. Both sections have been enlarged.