The Foster-Mosher hyperbolic Radon summation curve and the shifted-hyperbola formulation

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ABSTRACT

The Foster-Mosher hyperbolic summation curve, or integration path, was chosen somewhat arbitrarily. This note shows that the Foster-Mosher form can be derived as an approximation to, or special case of, the shifted-hyperbolic summation curve. This relationship would account for the enhanced focusing power of the more general shifted-hyperbola summation curve.

INTRODUCTION

Ten years ago, Foster and Mosher (1992) introduced a hyperbolic adaptation of the Radon transform (Radon, 1917) for the reduction of multiple reflections. Their technique involves application of a hyperbolic convolutional filter to stacks of normal-moveout-corrected (NMOC) seismic gathers, followed by reproduction of the undesirable energy (partial inverse transformation) and subtraction of the multiples from the original data. On NMOC gathers, they chose hyperbolic stacking surfaces over parabolic ones because, as they claim, the residual moveout of multiples is closer to hyperbolic than parabolic.

Foster and Mosher (1992) also mentioned two conditions that are important for keeping the costs of these computations – which are comparable to those of prestack migration – within reason. The first is that the stacking surface should be time-invariant so that computations may be performed in the frequency-space (ω-x) domain, and the second that the matrix operators should have Toeplitz form so that fast solvers may be used. They went on to state that it is easy to specify hyperbolic surfaces satisfying both of these conditions.

THE FOSTER-MOSHER HYPERBOLIC SUMMATION CURVE

The Radon transform, generalized to arbitrarily curved integration paths, may be defined as

$$m(\tau, p) = \int_{x_{\min}}^{x_{\max}} d\left[\tau + q\theta(x), x\right] \mathrm{d}x \tag{1}$$

where d is the original seismic *data*, m is the transform in *model* space, and τ is intercept time. We use q instead of p for the parameter that, along with $\theta(x)$, specifies the moveout curve or integration path. We refer to q as *a* ray parameter, as distinct from p, which is *the* ray parameter (or horizontal slowness). For the classical slant stack (straight-line integration paths), $\theta(x) = x$ and $q \equiv p$.

Transforming from the time to the frequency domain, we can write:

$$\tilde{m}(\omega, p) = \int_{x_{\min}}^{x_{\max}} \tilde{d}(\omega, x) \exp[i\omega q \theta(x)] \, \mathrm{d}x \tag{2}$$

and curved-path integration in the time domain has become an integration of phase shifts in the frequency domain. Foster and Mosher (1992) discuss further manipulations of this transform and conditions on the data that allow a formulation with optimal computational efficiency. In so doing, they invoke the two conditions mentioned above and adapt certain results of Thorson and Claerbout (1985) and Hampson (1986).

Foster and Mosher then motivate their choice of integration path or summation curve, specified by $\theta(x)$, by stating that, since "multiples have moveout curves that are hyperbolic with respect to traveltime and offset,... the factor of the time delay function (phase shift) is given as":

$$\theta(x_k) = \sqrt{x_k^2 + z_{\mathrm{ref}}^2} - z_{\mathrm{ref}} \tag{3}$$

and (1) would become:

$$m(\tau, p) = \int_{x_{\min}}^{x_{\max}} d\left[\tau + q\left(\sqrt{x_k^2 + z_{\mathrm{ref}}^2} - z_{\mathrm{ref}}\right), x\right] \mathrm{d}x \tag{4}$$

where x_k are the discrete receiver offsets "and z_{ref} is a constant parameter defined as the reference depth". In fact, they had to choose a constant in order for the earlier stated condition of time-invariance to be met. Although this form was clearly chosen because it is hyperbolic, the choice of z_{ref} is somewhat, though not entirely, arbitrary. The better the match between these hyperbolae (their summation curves) and those in the NMOC seismic data, the more compact the events will be in the transform domain. Foster and Mosher (1992) state that "events reflected from this depth [z_{ref}] are optimally resolved". We note, however, that in making an equivalence between (3) or (4) and the standard hyperbolic expression for single-layer traveltime of a reflection recorded at offset x, the reflecting horizon would lie at a depth of $z_{\mathrm{ref}}/2$ rather than z_{ref}.

THE SHIFTED-HYPERBOLA SUMMATION CURVE

Castle (1994) describes the shifted-hyperbolic NMO equation for a horizontally layered earth model as

$$t = \tau_s + \sqrt{\tau_0^2 + \frac{x^2}{v^2}} \tag{5}$$

where

$$\tau_s = \tau_0 (S - 1) \tag{6}$$

$$\tau_0 = t_0 / S \tag{7}$$

$$v^2 = SV_{\text{rms}}^2 \tag{8}$$

$$S = \mu_4 / \mu_2^2 \tag{9}$$

and

$$\mu_n = \sum_{i=1}^{N} \Delta t_i V_i^n \Bigg/ \sum_{i=1}^{N} \Delta t_i \ . \tag{10}$$

The shifted-hyperbola curve represents a Dix NMO equation shifted by the time τ_s and is exact through fourth order in offset. Equation (5) can now be written as

$$t = t_0 + \sqrt{\frac{t_0^2 \mu_2^4}{\mu_4^2} + \frac{x^2 \mu_2^2}{\mu_4 V_{\text{rms}}^2}} - \frac{t_0 \mu_2^2}{\mu_4} \ . \tag{11}$$

Given that $\mu_2 = V_{\text{rms}}^2$, we make the approximation $\mu_4 \approx V_{\text{rms}}^4$, given reasonable offsets and laterally homogeneous velocities. Direct substitution of these approximations into equation (11) leads to the Dix equation for reflection traveltime. Then, by approximating $q = \frac{1}{V_{\text{rms}}}$, equation (11) simplifies to:

$$t = t_0 + \frac{1}{V_{\text{rms}}} \left(\sqrt{{t_0}^2 V_{\text{rms}}^2 + x^2} - t_0 V_{\text{rms}} \right) \tag{12}$$

or, since $t_0 = 2z / V_{\text{rms}}$

$$t = t_0 + \frac{1}{V_{\text{rms}}} \left(\sqrt{4z^2 + x^2} - 2z \right). \tag{13}$$

Generalizing $1/V_{\text{rms}}$ to q, and discretizing x for the seismic recording situation, we arrive at a summation curve where

$$t = t_0 + q\left(\sqrt{x_k^2 + 4z^2} - 2z \right). \tag{14}$$

DISCUSSION

Equation (14) is strikingly similar to the hyperbolic curve in equation (4) given by Foster and Mosher (1992). They differ in two significant respects, one being the focusing-depth parameter ($z_{\text{ref}} \Rightarrow 2z$). So we might expect the focusing depth to be closer to $z_{\text{ref}}/2$ than z_{ref}. The two expressions also differ in that, whereas z varies in proportion to t_0 in (14), z_{ref} is a constant in (3) and (4) with respect to variation of τ [which corresponds to the t_0 of (14)]. Thus, the summation curve of Foster and Mosher focuses on a particular depth and yields accurate focusing only over some depth range around that focusing depth.

The approximations made by Foster and Mosher (1992) to formulate equations (3) and (4) cause some smearing of events in the Radon domain and somewhat inaccurate estimations of curved events. The shifted-hyperbolic equations used for summation in the Radon domain [i.e. equations (3.28) and (4.1) of Oppert (2002)], from which (14) was approximated, avoid the approximation of μ_4, creating an equation with enhanced focusing power over a greater depth range.

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