Influence of a thin layered viscoelastic surface zone on seismic traces recorded at the earth's surface

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ABSTRACT

The influence of a thin layered viscoelastic surface zone on the reflected *SH* wavefield in a simple model consisting of a single layer over a halfspace is investigated. This thin weathering zone is generally assumed to be composed of any number of layers as long as the "thin" assumption is retained. The free surface vacuum – solid interface is replaced by this zone of finite thickness, which is assumed small when compared with the predominant wavelength which is defined in terms of the predominant frequency of the band limited wavelet employed and the near surface velocity. A ray-reflectivity analogue of the surface conversion coefficient is derived and comparisons of synthetic traces computed for an elastic and viscoelastic thin surface zone are made. For surface receiver seismology the effect of the viscoelastic versus elastic response detected by the receivers should be examined in that all deeper reflections within the structure must pass through this layer before being recorded at the geophones and all are consequently affected in some manner.

The seismic velocity of the surface layer is very often the lowest when compared with the velocities in the underlying geological structure and it is not uncommon for the impedance contrast of the weathering layer and the subsequent layer to be relatively high. As matrix methods are employed to obtain an analogous expression for the traditional surface coefficient, the full seismic response of the thin layered zone is inherent in the expression. Combining this with a high impedance contrast and the frequency, incident angle and zone thickness dependence of the surface conversion coefficient analogue could possibly lead to a ringing event being recorded at the surface. There could also be gaps in the frequency amplitude spectrum due to so-called "tuning" effects.

SH wave propagation is used to explore this problem due to its simpler nature when compared with the coupled $P-SV$ problem. As might be expected there is a trade off in that the effects of introducing this concept into the $P-SV$ case can be much more pronounced than those for the *SH* case. However, it is necessary to introduce displacement potentials in the $P-SV$ case to determine the thin layer propagator matrices, introducing added complication into an already complex problem. In the *SH* derivation this step is bypassed, after a fashion, as the results obtained using potentials are the same as those using displacements.

INTRODUCTION

A practical consideration that should be addressed in realistic geological models is the presence of a thin layered surface zone or weathering layer. In seismic modeling programs this zone is often either omitted or assigned the status of a thick layer. This is done even though it may be classified as thin when compared with the predominant wavelength, which is related to the predominant frequency of the source wavelet being used.

It is also usually assumed that the total layered structure is elastic, which is fairly idealistic in general, but even to a greater extent for a thin layered surface zone where it has generally been accepted to be viscoelastic to some degree. For receivers located at the surface all arrivals reflected from interfaces deeper within the medium are affected by the passage of seismic energy associated with each of these individual events through the thin layered structure.

The propagation of SH seismic waves for this type of geological structure will be examined in what follows. In particular, analogues of the reflection coefficient from the thin layered zone (reflectivity) and the corresponding surface conversion coefficient will be derived with the solid – vacuum boundary at the surface replaced with a thin layered, viscoelastic, finite thickness structure. Viscoelasticity will be introduced in this thin layered zone by using complex velocities, which may be frequency dependent.

The free surface zone is developed using the ray-reflectivity method (Daley and Hron, 1982), which although based on the reflectivity method, differs in that what is sought is the continuity of the particle displacement – stress vector through a thin layered structure between what can be generally be termed as two halfspaces. This is similar to what is done at a free surface interface to determine reflection and surface conversion coefficients.

The mathematical mechanism to introduce viscoelasticity into a medium based on the earlier works of Futterman (1962) and Azimi et al. (1968) and presented more recently in the literature by Müller (1983) and Zharadnik et al. (2002) was used here as the basis of introducing a frequency dependent quality factor, $Q(\omega)$, into the numerical computations. The mathematical analysis is somewhat biased by this formulation but can, in general, be taken as valid for any frequency dependent quality factor specification.

BASIC ASYMPTOTIC RAY THEORY

Consider a plane layered half space $(z \geq 0)$ with a thin viscoelastic layered surface zone of total thickness H overlying an elastic thick layer of thickness h and an elastic halfspace. The thick layer and halfspace are assumed to be in welded contact and are both isotropic and homogeneous. The shear wave velocities in the thick layer and halfspace will be denoted β_I and β_B, respectively with ρ_I and ρ_B being the densities. The Lame′ coefficient is defined as $\mu = \rho\beta^2$, with appropriate subscripts. The velocities $(\beta_\ell, \ell = 1,\ldots,n)$ of the thin layers comprising the surface layer are in general complex and frequency dependent with the corresponding densities, ρ_ℓ, being real quantities.

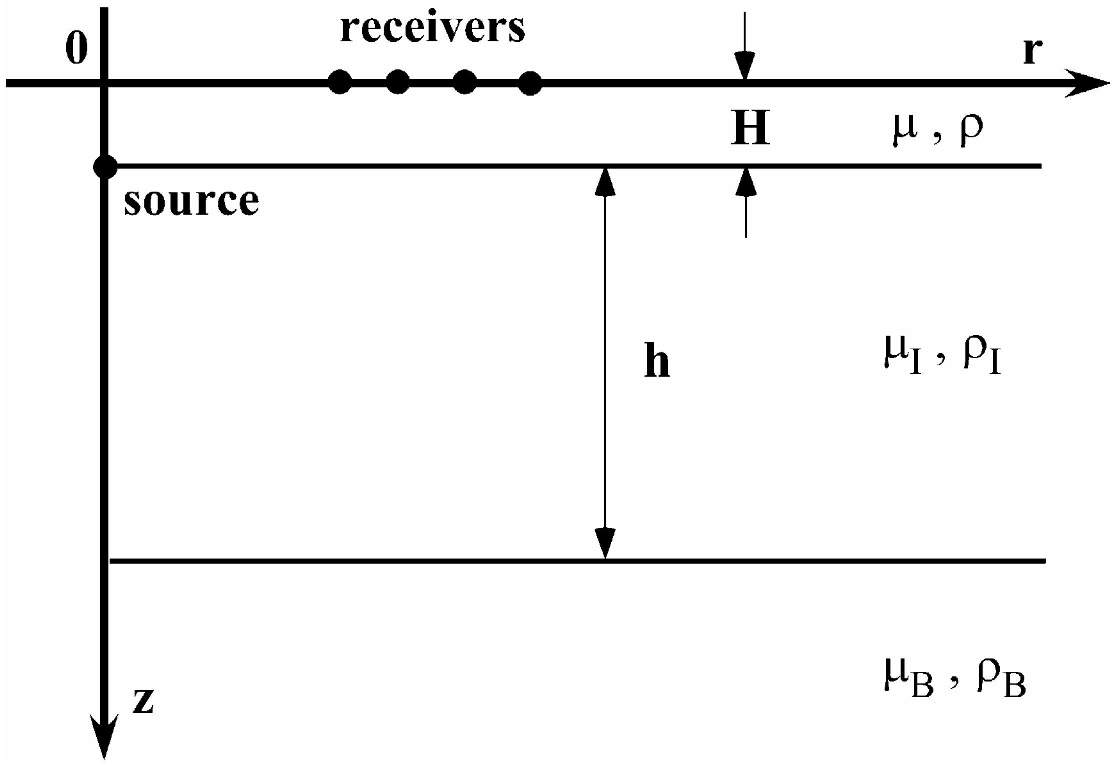


FIG. 1. Schematic of the model used for the method presented in this paper.

A point torque source is located at the origin just below the surface zone $\left(r=0, z=H\right)$ and the receivers are positioned on the surface $\left(z=0\right)$, (Figure (1)). The Fourier time transform of the particle displacement, $u\left(r,z,\omega\right)$, of a multiply reflected *SH* ray with *2k* ray segments in the thick layer for this geometry is given, at the surface, in cylindrical coordinates with radial symmetry (Aki and Richards, 1980, Daley and Hron, 1982 and 1990) by

$$\mathbf{u}\left(r,0,\omega\right)=u_{\phi}\left(r,0,\omega\right)\mathbf{e}_{\phi} \tag{1}$$

with

$$u_{\phi}\left(r,0,\omega\right)=\frac{i\omega^{2}L\left(\omega\right)}{2}\int_{\Omega}R\left(p,\omega\right)H_{1}^{(1)}\left(\omega pr\right)\exp\left[i\omega 2kh\eta_{I}\right]\frac{p^{2}dp}{\eta_{I}} \tag{2}$$

where

- p – horizontal slowness and integration parameter which must initially be assumed to be complex,

- ω - circular frequency $\left(2\pi f\right)$, f being frequency,

- $L(\omega) = \frac{A_0}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{i\omega t}\, dt$ - Fourier transform of the time dependence of the source wavelet, $g(t)$,

- A_0 - a proportionality constant dependent on the elastic parameters of the medium surrounding the point torque soure,

- $\eta_j = \left(\beta_j^{-2} - p^2\right)^{1/2}$, $\operatorname{Im}\left(\eta_j\right) \geq 0$, $\left[j = I, B\right]$,

- $R(p,\omega)$ - the product of the k (frequency independent) reflection coefficients from the layer – halfspace boundary, the $k-1$ reflection coefficients from the thin layered surface zone and the surface conversion coefficient, which are both frequency dependent. The surface layer reflection coefficient and surface conversion coefficient are analogues of the similar free surface quantities and are discussed in a subsequent section.

- $H_1^{(1)}(\omega pr)$ - the Hankel function of order 1 and type 1, whose asymptotic expansion for large argument is $\left(\frac{2}{\pi\omega pr}\right)^{1/2} \exp\left[i\omega pr - 3i\pi/4\right]$,

- r – source receiver offset,

- Ω - a suitable steepest descents contour for the zero order saddle point approximation of the integral in equation (2).

- $\mathbf{e}_\phi$ - a unit vector perpendicular to the plane of incidence (*SH* polarization vector).

A change of the integration variable, from the complex p to a real variable y is introduced to transform the contour of integration from the real p axis to one more suitable for a steepest descent approximation to the reflected wave. This parameterization is given as (Cerveny and Ravindra, 1970)

$$\left(\beta_I^{-2} - p^2\right)^{1/2} = \left(\beta_I^{-2} - p_0^2\right)^{1/2} - y e^{-i\pi/4} \quad , \quad -\infty < y < \infty \tag{3}$$

The quantity p_0 is the saddle point associated with the reflected arrival from the layer – halfspace interface and located on the real p axis. Only this set of reflected arrivals will be considered here.

Introducing the variable change given by equation (3) and employing standard steepest descent methods, the zero order saddle point approximation to the integral in equation (2), which is the expression for the reflected arrival is found to be (Daley and Hron, 1990)

$$u_\phi\left(r,0,\omega\right)=\frac{i\omega p_0 L(\omega)}{\Re}R\left(p_0,\omega\right)e^{i\omega\tau_R} \tag{4}$$

where $\Re$ is the geometrical spreading (spherical divergence) of the ray in the thick layer and $R\left(p_0,\omega\right)$ is the product of reflection coefficients and the frequency dependent surface conversion coefficient. In the reflectivity method of producing synthetic traces the total seismic wavefield, consisting of all possible rays in the *n* layer surface layer, is implicitly contained in the surface conversion coefficient. For multiple reflections within the thick layer, the same is true of the free surface frequency dependent reflectivity coefficient. The saddle point, p_0, is the solution of

$$\frac{d}{dp}\left[rp+2mh\eta_I\right]_{p_0}=0\,, \tag{5}$$

with $2m$ being the number of *SH* ray segments in the thick layer.

The reflected travel time for the path which the ray traverses in the thick layer is obtained from equation (5) and given by

$$\tau_R=\frac{2mh}{\beta_I\cos\theta_0}\quad,\quad \cos\theta_0=\beta_I\hat{\eta}_I \tag{6}$$

with $\hat{\eta}_I$ being the value of η_I at the saddle point and θ_0 is the acute angle that the ray makes with the vertical (z) axis. Minor travel time corrections for the surface layer zone are automatically contained in the free surface reflection coefficient and surface conversion coefficient analogues.

In the next section these analogues will be discussed in a brief manner but with enough background material to hopefully provide proper insight into the problem. The use of the coupled $P-S_V$ problem for this purpose introduces unnecessary complexity and the *SH* treatment should be sufficient to introduce the concepts involved.

REFLECTIVITY AND SURFACE CONVERSION COEFFICIENT AT A THIN LAYERED SURFACE STRUCTURE

A homogeneous system of first order linear differential equations with constant coefficients describing the propagation of *SH* waves in a homogeneous space can be written as

$$\frac{d\mathbf{f}}{dz}=\mathbf{Af} \tag{7}$$

with $\mathbf{A}$ being a matrix and $\mathbf{f}$ a column vector, generally known as the displacement-stress vector. It is composed of the time and radial spatial coordinate transformed particle displacement component, u_ϕ, and shear stress, $\sigma_{r\theta}$, so that $\mathbf{f} = \left[u_\phi, \sigma_{r\theta} \right]^T$, with "$T$" indicating the transpose.

The above system of first order differential equations may be subject to boundary conditions of some form on $\mathbf{f}$. For the problem being investigated in this section, it will be convenient to specify these conditions at the lower boundary of the thin layered surface bed, as incidence from the thick layer is what is of interest. This leads to boundary conditions of the form

$$\mathbf{f}\big|_{z=z_n} = \mathbf{f}\left(z_n\right). \tag{8}$$

When considering the equation of motion for *SH* waves in a homogeneous halfspace with radial symmetry and the vertical (z) axis chosen positive downwards, the above system arises after applying a time transform and a Hankel transform of order 1 with respect to the radial coordinate (r). (Aki and Richards, 1980).

In particular if ω is the circular frequency, p the generally complex and frequency dependent ray parameter (horizontal component of the slowness vector), μ, Lame″s parameter, ρ, the volume density, and β, the generally complex and frequency dependent shear wave velocity, related to Lame″s coefficient as $\mu = \rho\beta^2$, one obtains

$$\mathbf{f} = \left[u, \sigma\right]^T, \tag{9}$$

again "T" indicating the transpose and the subscripts on u and σ have been dropped.

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & \mu^{-1} \\ \omega^2\left(\mu p^2 - \rho\right) & 0 \end{bmatrix}. \tag{10}$$

It was earlier stated that the viscoelasticity of the medium is introduced by some mechanism that allows the velocity and hence the Lame″s coefficient to be complex and frequency dependent.

The method of solution of this coupled first order differential equation problem may be found in most advanced texts on differential equations or linear algebra (Gantmacher, 1959, Hildebrandt, 1952) and will not be repeated here. What is of importance in this solution for what follows is the determination of the eigenvalues of the matrix $\mathbf{A}$ and the corresponding matrix composed of the two distinct eigenvectors. These eigenvectors are associated with the upgoing $(-)$ and downgoing $(+)$ waves are given as

$$\mathbf{y}_{\pm} = \left[1, \pm\omega\mu\eta\right]^T, \quad \text{with } \eta = \left(\beta^{-2} - p^2\right)^{1/2}. \tag{11}$$

If $\mathbf{F}(z)$ is a matrix solution of equation (7) whose columns are obtained from equation (11), then a propagator matrix (matrizant) is defined as (Aki and Richards, 1980)

$$\mathbf{P}(z_n : z) = \mathbf{F}(z)\mathbf{F}^{-1}(z_n), \tag{12}$$

such that $\mathbf{P}(z_n : z_n)$ is the identity matrix $\mathbf{I}$. The superscript "-1" on any matrix indicates the inverse. For the problem under consideration z_n corresponds to the reference depth, $z = H$; the depth from the earth's surface to the bottom of the thin layered bed.

With the knowledge that for the problem being considered

$$\mathbf{F}(z) = \begin{bmatrix} 1 & 1 \\ -i\omega\mu\eta & i\omega\mu\eta \end{bmatrix} \begin{bmatrix} e^{i\omega\eta(z_n - z)} & 0 \\ 0 & e^{-i\omega\eta(z_n - z)} \end{bmatrix} = \mathbf{E}\mathbf{\Lambda} \tag{13}$$

it may be determined that

$$\mathbf{F}^{-1}(z_n) = \begin{bmatrix} \dfrac{1}{2} & \dfrac{i}{2\omega\mu\eta} \\ \dfrac{1}{2} & \dfrac{-i}{2\omega\mu\eta} \end{bmatrix} = \mathbf{E}^{-1}. \tag{14}$$

After matrix multiplication the propagator matrix may be computed to be

$$\mathbf{P}(z_n : z) = \begin{bmatrix} \cos\left[\omega\eta(z_n - z)\right] & \dfrac{\sin\left[\omega\eta(z_n - z)\right]}{\omega\mu\eta} \\ -\omega\mu\eta\sin\left[\omega\eta(z_n - z)\right] & \cos\left[\omega\eta(z_n - z)\right] \end{bmatrix} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^{-1}. \tag{15}$$

It should be recalled at this time that equations (11) – (15) relate the value of the displacement-stress vector, $\mathbf{f}$, between two depth locations, z and z_n, in a homogeneous medium. If a similar relation is to be obtained between z_0, the free surface, and z_n, the bottom of the thin layered surface layer, a requirement for continuity across the n intervening plane boundaries must be invoked. With the aid of equation (12) this leads to the product of n propagator matrices, $\mathbf{P}_j,\ j = 1, \cdots n$. Each of the $\mathbf{P}_j$ characterizes the total *SH* wavefield within the j^{th} layer in the stack of n layers. Utilizing the properties of the propagator matrix, (Gantmacher, 1959, Aki and Richards, 1980) the following relationship is obtained

$$\begin{aligned}\mathbf{f}_{n+1}(z_n) &= \mathbf{E}_{n+1}\mathbf{w}_{n+1} \\ &= \mathbf{P}_n(z_n : z_{n-1})\mathbf{P}_{n-1}(z_{n-1} : z_{n-2})\cdots\mathbf{P}_1(z_1 : z_0)\mathbf{f}_0(z_0)\end{aligned} \tag{16}$$

where in general

$$\mathbf{w} = \mathbf{E}^{-1}\mathbf{f} \tag{17}$$

and specifically

$$\mathbf{w}_{n+1}(z_n) = \left[A_{n+1}^{+}(z_n), A_{n+1}^{-}(z_n)\right]^T = \mathbf{E}_{n+1}^{-1}(z_n)\mathbf{f}_{n+1}(z_n) \tag{18}$$

where A_{n+1}^{+} and A_{n+1}^{-} are respectively, the amplitudes of the waves reflected from, and impinging on, the bottom of the stack of thin layers at $z = z_n$ (Figure (2)). The schematic shown in Figure (2) is the surface zone shown in Figure (1) with some minor notation changes to facilitate a more compact form of solution.

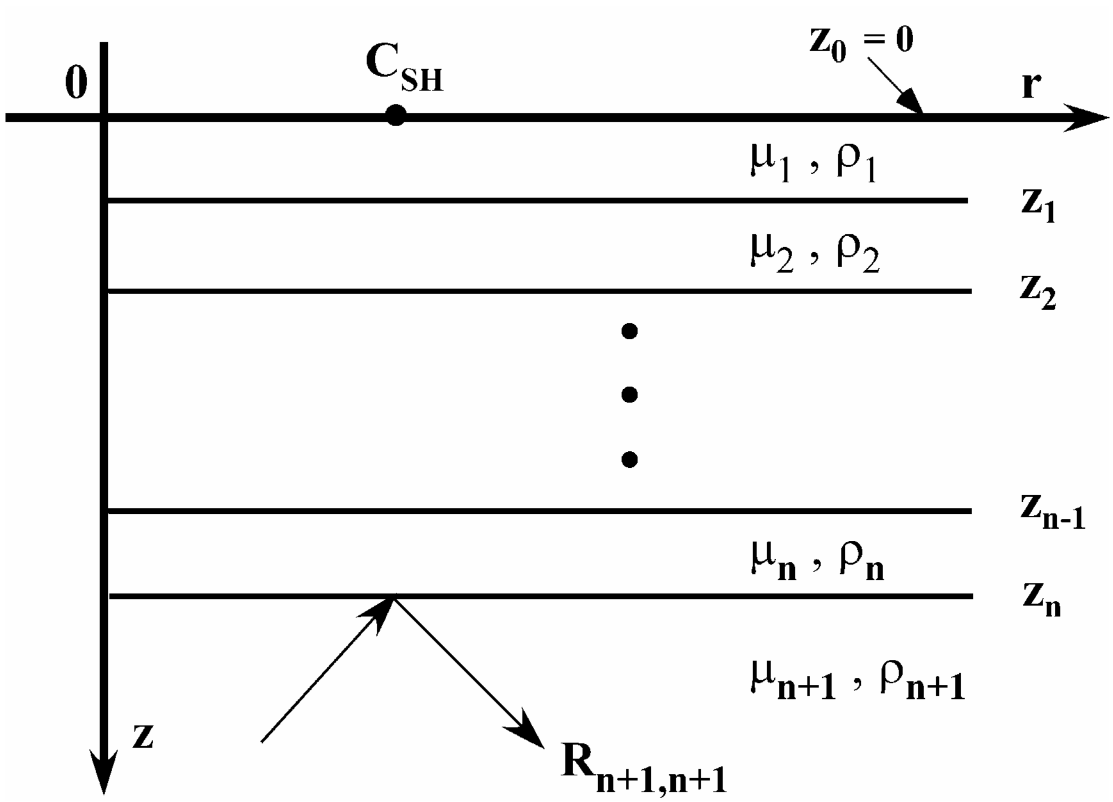


FIG. 2. Plane wave incidence from below on a stack of thin layers at a free surface. For the *SH* case the unknowns to be solved for are the reflectivity $\left(R_{n+1,n+1}\right)$ from the bottom of the zone and the conversion coefficient analogue $\left(C_{SH}\right)$ at the surface $\left(z = z_0 = 0\right)$. The quantities μ_j and ρ_j are Lame″s coefficients and densities in the individual layers.

Equations (17) and (18) emphasize the weighting characteristic of the amplitude (coefficient) vector $\mathbf{w}$ whose components in each layer are comprised of the displacement amplitudes of the downgoing and upgoing waves within a given layer.

Assuming a unit amplitude for the upgoing wave incident from the thick layer at the bottom of the stack of thin surface layers, $A_{n+1}^{-}=1$, and normalizing A_{n+1}^{+} to this unit incident amplitude, the following notation change may be introduced: $A_{n+1}^{+}=R_{n+1,n+1}$. so that

$$\mathbf{w}_{n+1}=\left[R_{n+1,n+1},1\right]^{T}. \tag{19}$$

Equation (16) provides the necessary relationship between the wave motion in the thick layer and the free surface so that the boundary condition at $z=z_0$ may be written as

$$\mathbf{f}\left(z_0\right)=\left[C_{SH},0\right]^{T} \tag{20}$$

with C_{SH} being the vector component A_0^{+} of $\mathbf{w}_0$, normalized to the incident unit amplitude A_{n+1}^{-} from the underlying thick layer. C_{SH} is the analogue of the surface conversion coefficient while $R_{n+1,n+1}$ is the reflectivity from the bottom of the stack of the thin layered surface zone.

After some manipulation of equations (16) and (17) one obtains

$$\begin{bmatrix} C_{SH} \\ 0 \end{bmatrix}=\mathbf{D}^{-1}\begin{bmatrix} R_{n+1,n+1} \\ 1 \end{bmatrix} \tag{21}$$

where the matrix $\mathbf{D}$ and its inverse $\mathbf{D}^{-1}$ are defined as follows

$$\mathbf{D}=\mathbf{E}_{n+1}^{-1}\mathbf{P}_n\,\mathbf{P}_{n-1}\cdots\mathbf{P}_1. \tag{22.a}$$

$$\mathbf{D}^{-1}=\mathbf{P}_1^{-1}\mathbf{P}_2^{-1}\mathbf{P}_2^{-1}\cdots\mathbf{P}_n^{-1}\mathbf{E}_{n+1} \tag{22.b}$$

Equation (21) may now be written in terms of the subscripted matrix of $\mathbf{D}^{-1}$, $D_{ij}\ \left(i,j=1,2\right)$ or $\mathbf{D}$, $\mathrm{D}_{ij}\ \left(i,j=1,2\right)$

$$\begin{bmatrix} C_{SH} \\ 0 \end{bmatrix}=\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}\begin{bmatrix} R_{n+1,n+1} \\ 1 \end{bmatrix}=\frac{1}{\det\left[\mathbf{D}\right]}\begin{bmatrix} \mathrm{D}_{22} & -\mathrm{D}_{12} \\ -\mathrm{D}_{21} & \mathrm{D}_{11} \end{bmatrix}\begin{bmatrix} R_{n+1,n+1} \\ 1 \end{bmatrix}. \tag{23}$$

The solution of equation (23) yields the reflectivity, $R_{n+1,n+1}$, and the surface conversion coefficient analogue, C_{SH}, in terms of the matrix elements of $\mathbf{D}^{-1}$ or $\mathbf{D}$ as

$$R_{n+1,n+1} = -\frac{D_{22}}{D_{21}} = \frac{\mathrm{D}_{11}}{\mathrm{D}_{21}} \tag{24}$$

$$C_{SH} = -\frac{\det\left[\mathbf{D}^{-1}\right]}{D_{21}} = \frac{1}{\mathrm{D}_{21}} \tag{25}$$

Returning to equation (4), the product of the reflection coefficients from the thick layer – halfspace boundary is given by

$$R_{SH}\left(p\right) = \frac{\mu_I\eta_I - \mu_B\eta_B}{\mu_I\eta_I + \mu_B\eta_B} \tag{26}$$

(taken to the appropriate power of *k*) together with the free surface reflectivity, $\left(R_{n+1,n+1}\right)^{m-1}$ and the analogue of the free surface conversion coefficient, C_{SH}, which make up the quantity $R\left(p,\omega\right)$ in equation (4) have now all been determined. The latter two have been derived for a generally viscoelastic thin zone surface structure. Numerical examples, which require the use of these derived formulae, will be presented in the next section.

NUMERICAL RESULTS

The models chosen for the computation of numerical results are variations on a medium consisting of a single thin low velocity layer of varying thickness and degree of viscoelasticity overlying a 1000 *m* layer and a halfspace. The shear wave velocities in the isotropic homogeneous elastic thick layer and halfspace are 2500 *m/s* and 3500 *m/s*, respectively, with their volume densities being 1.9 g/cm^3 and 2.0 g/cm^3. The critical offset corresponding to the layer – halfspace velocity contrast is approximately 2040 *m*.

Defining the quality factor $Q\left(\omega\right)$ in terms of the complex Lame' coefficient

$$\mu\left(\omega\right) = \rho\beta^2\left(\omega\right) = \left[\mu\left(\omega\right)\right]_{\text{Re}} - i\left[\mu\left(\omega\right)\right]_{\text{Im}} \tag{27}$$

results in

$$1/Q\left(\omega\right) = \left[\mu\left(\omega\right)\right]_{\text{Im}} / \left[\mu\left(\omega\right)\right]_{\text{Re}}. \tag{28}$$

Assume that at some reference circular frequency, $\omega_R = 2\pi f_R$ a reference absorption(quality) factor, $Q_R = Q\left(\omega_R\right)$, and reference *SH* – wave velocity, $\beta_R\left(\omega_R\right)$, are known. These quantities are real valued. At some other frequency, ω, the values of $Q\left(\omega\right)$ and $\beta_R\left(\omega\right)$ are given by the relations

$$Q(\omega) = Q(\omega_R)\left[1 - \frac{1}{\pi Q\left(\omega_R\right)}\ln\left(\frac{\omega}{\omega_R}\right)\right] \tag{29}$$

and

$$\beta_R(\omega) = \beta_R(\omega_R)\frac{Q(\omega_R)}{Q(\omega)}. \tag{30}$$

The two values $Q(\omega)$ and $\beta_R(\omega)$ obtained above are also real. Viscoelasticity or absorption is introduced into a medium through a complex velocity obtained by an analysis of the attenuating mechanism. The high frequency expression for this complex velocity, $\beta(\omega)$, in terms of the real parameters $Q(\omega)$ and $\beta_R(\omega)$ used in this paper is defined by

$$\frac{1}{\beta(\omega)} = \frac{1}{\beta_R(\omega)}\left[1 + \frac{i}{2Q(\omega)}\right]. \tag{31}$$

Equations (29)-(31) are the same as those presented in Zharadnik. et al. (2002).

This simple model consisting of a single low velocity surface layer was chosen so that analytical expressions for the quantities $R_{n+1,n+1}$ and C_{SH} could be obtained in forms that are not cumbersome to manipulate. The motivation for this is to obtain some insight regarding the behaviour of the surface conversion coefficient analogue, in particular, with varying layer thickness, degree of viscoelasticity, ray parameter and frequency. The equations for the reflectivity and surface conversion coefficient analogue may be written as

$$R_{n+1,n+1} = \left(\mu\eta\sin[\omega\eta H] + i\mu_I\eta_I\cos[\omega\eta H]\right)/\Delta \tag{32}$$

and

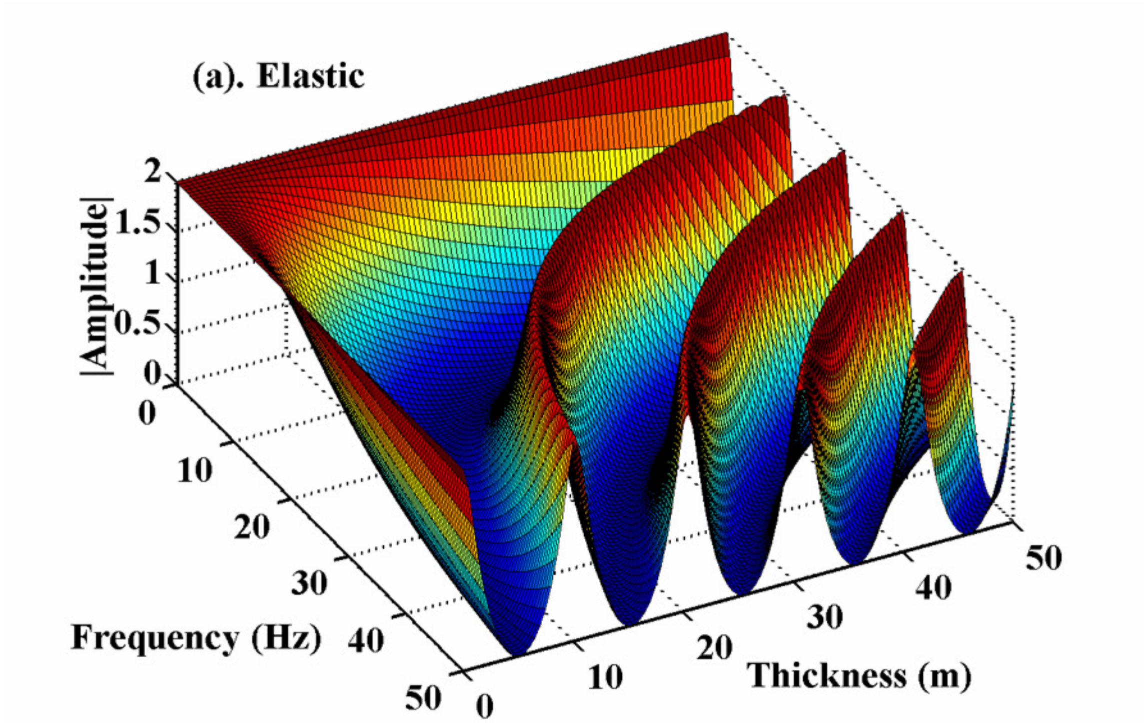
$$C_{SH} = 2i\mu_I\eta_I/\Delta \tag{33}$$

where

$$\Delta = -\mu\eta\sin[\omega\eta H] + i\mu_I\eta_I\cos[\omega\eta H]. \tag{34}$$

The subscripted quantities refer to the thick layer and the unsubscripted ones are related to the thin surface layer, which may be viscoelastic. It may be inferred from equation (33) that for a low velocity surface layer model the minimum values of $|C_{SH}|$ occur at $\omega H\eta = \ell\pi, (\ell = 0,1,2,\ldots)$, and the maximums of $|C_{SH}|$ when $\omega H\eta = \ell\pi/2$, $(\ell = 1,3,5,\ldots)$. In the limit as $H \to 0$, the surface conversion coefficient C_{SH} tends to 2, which is the constant value of the conventional *SH* surface conversion coefficient.

For a preliminary investigation of the behaviour of C_{SH} the thickness of the surface layer will be varied from 0 to 50 *m* and the frequency from 0 to 50 *Hz*. The angle of the incident *SH* wave from the underlying layer is held constant at 30°. As the shear wave velocity in the thick layer is set to 2500 *m/s*, the corresponding horizontal slowness (ray parameter) is $p = \sin 30^\circ/(2500\,m/s) = 0.0002\,s/m$. The velocity in the thin surface layer



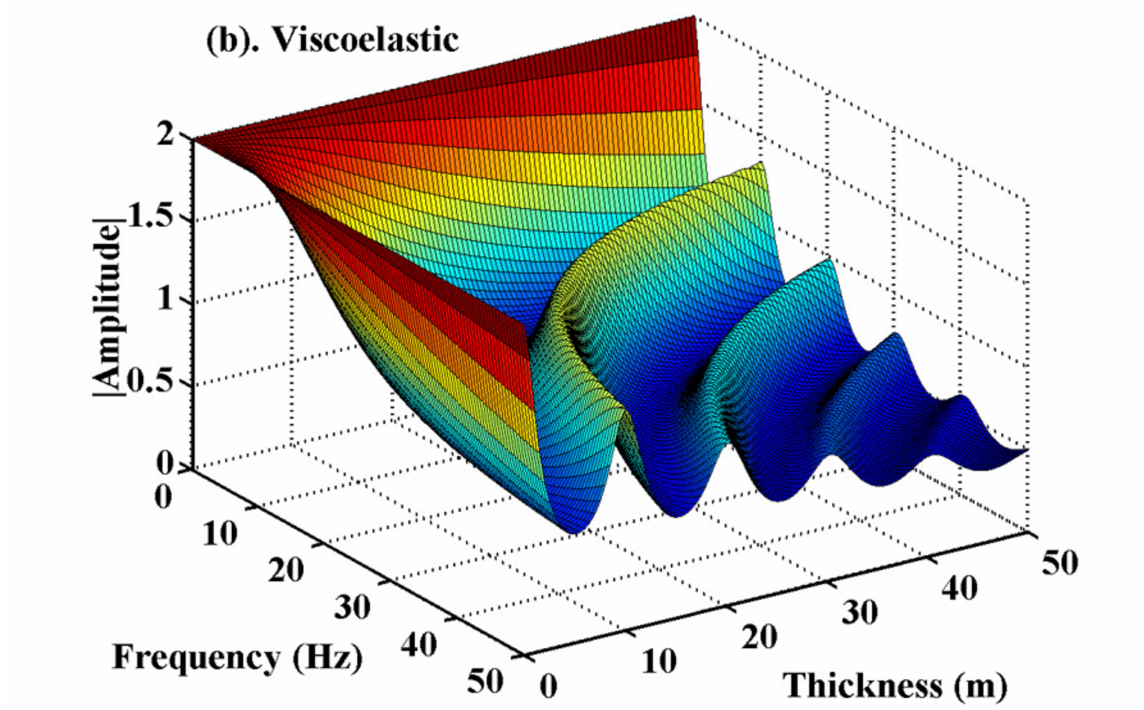


FIG. 3. The surface conversion coefficient analogue computed with a constant angle of incidence from the underlying thick layer. The amplitude $\left|C_{SH}\right|$ is plotted against the thin surface layer thickness, from 0*m* to 50*m* and a range of frequencies from 0*Hz* to 50*Hz*. In the upper plot the surface layer is elastic while in the lower plot it is viscoelastic with $Q(\omega_R) = 20$ at $\omega_R = 2\pi f_R$, $f_R = 30Hz$.

is 1000 *m/s* at the reference frequency $f_R = 30Hz$ and the quality factor in the surface layer is given as $Q(\omega_R) = 20.0$. The densities in the thick and surface layer are $1.9\,g/cm^3$ and $1.5\,g/cm^3$, respectively. Surface plots of the modulus of C_{SH} for both the elastic and viscoelastic cases are shown in Figures (3.a) and (3.b).

As the surface conversion coefficient at the interface between a solid halfspace and a vacuum is 2.0 for all angles of incidence, it would be expected that in the analogue case the maximum value that C_{SH} would attain is 2.0, which is the case for the elastic case shown in Figure (3.a). The viscoelastic effects are quite evident in Figure (3.b), with the amplitude of the surface conversion analogue decaying with increasing surface layer thickness and increasing frequency.

Synthetic seismograms are shown for a variety of parameter variations involving both an elastic and viscoelastic surface layer in Figures (4) through (6). The time dependence of the source wavelet used in all of the synthetic traces is

$$g(t) = \sin\left[2\pi f_0\left(t - t_h\right)\right]\exp\left[-\left(\frac{2\pi f_0\left(t - t_h\right)}{\gamma}\right)^2\right] \quad , \quad 0 < t < 2t_h , \qquad (35)$$

where γ, a dimensionless factor controlling the pulse width and side lobes, was chosen to be 4.0 and f_0, the predominant frequency of the pulse, was set equal to $50Hz$. A frequency of $30Hz$ is used to define the reference circular frequency, $\omega_R = 2\pi f_R$ used in equations (29) and (30). The quantity $2t_h$ is the time duration of the pulse in the time domain and is approximately equal to γ/f_0. The source is located below the surface at the top of the thick layer. All phenomena associated with the direct arrival at the receivers located on the surface have not been included in the synthetic traces.

The amplitude and phase of this arrival is dependent, apart from the frequency independent geometrical spreading in the thick layer and reflection coefficient at the thick layer-halfspace boundary, on the surface conversion coefficient analogue. This coefficient is dependent on frequency, thickness of the surface layer and angle of incidence.

The first example pushes the theory to the limit as a 50*m* surface layer is assumed. When compared with the predominant frequency of the source pulse and the shear wave velocity in the incident medium this layer thickness is equal to one wavelength (WL). In the elastic surface layer a wavelength is 20*m*, or $2.5WL$ if the elastic velocity in the surface is used for this determination. This model was used to compare the surface multiples produced by the matrix method with ray theory in the elastic case where the surface layer is treated as a thick layer and the interbed multiples should be fairly well separated on the traces with this layer thickness. The results are displayed in Figure (4) where offsets vary from 800 *m* to 1200 *m* at 50 *m* increments. Near vertical offset ranges are avoided as the reflected *SH* particle displacement is zero at zero offset as may be seen from viewing equation (4). In the ray approach the first, second, and third order multiples

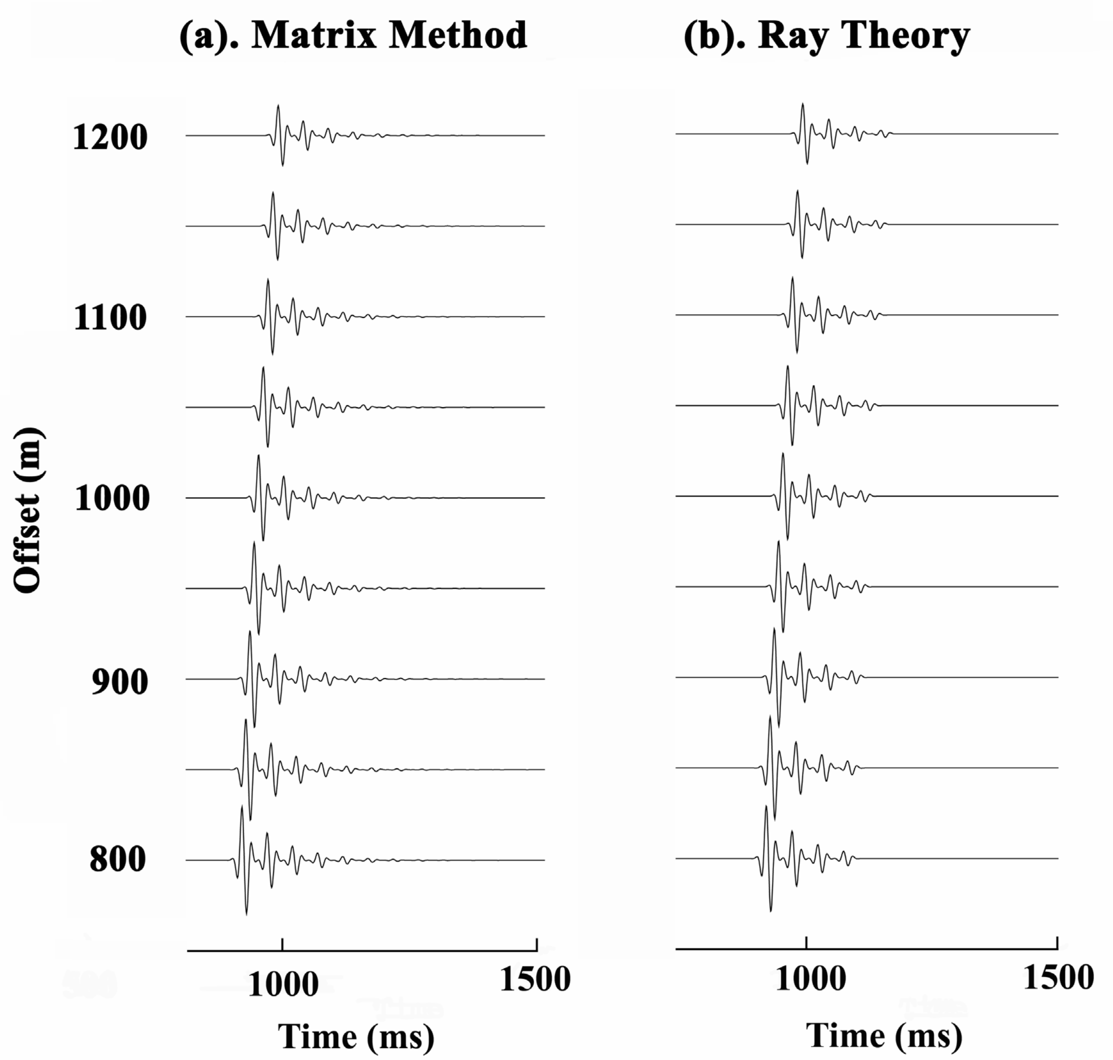


FIG. 4. An elastic 50*m* surface layer model for comparing the conversion coefficient analogue with the ray method. Apart from the primary arrival, first to third order interbed multiples are included in the ray synthetics.

have been introduced in the surface layer for comparison with the synthetic traces produced by matrix based conversion coefficient method. Upon viewing this figure it becomes apparent that the two methods produce similar results, the difference being that all multiple reflections within the surface layer are included in the propagator matrix method synthetic.

In the second example the matrix method is used to produce two sets of traces; one where the reference (elastic) model has $Q = \infty$ and the other having $Q = 20$ at the reference frequency $f_R = 30Hz$. The surface layer thickness is $20m$ ($1WL$). The effect of

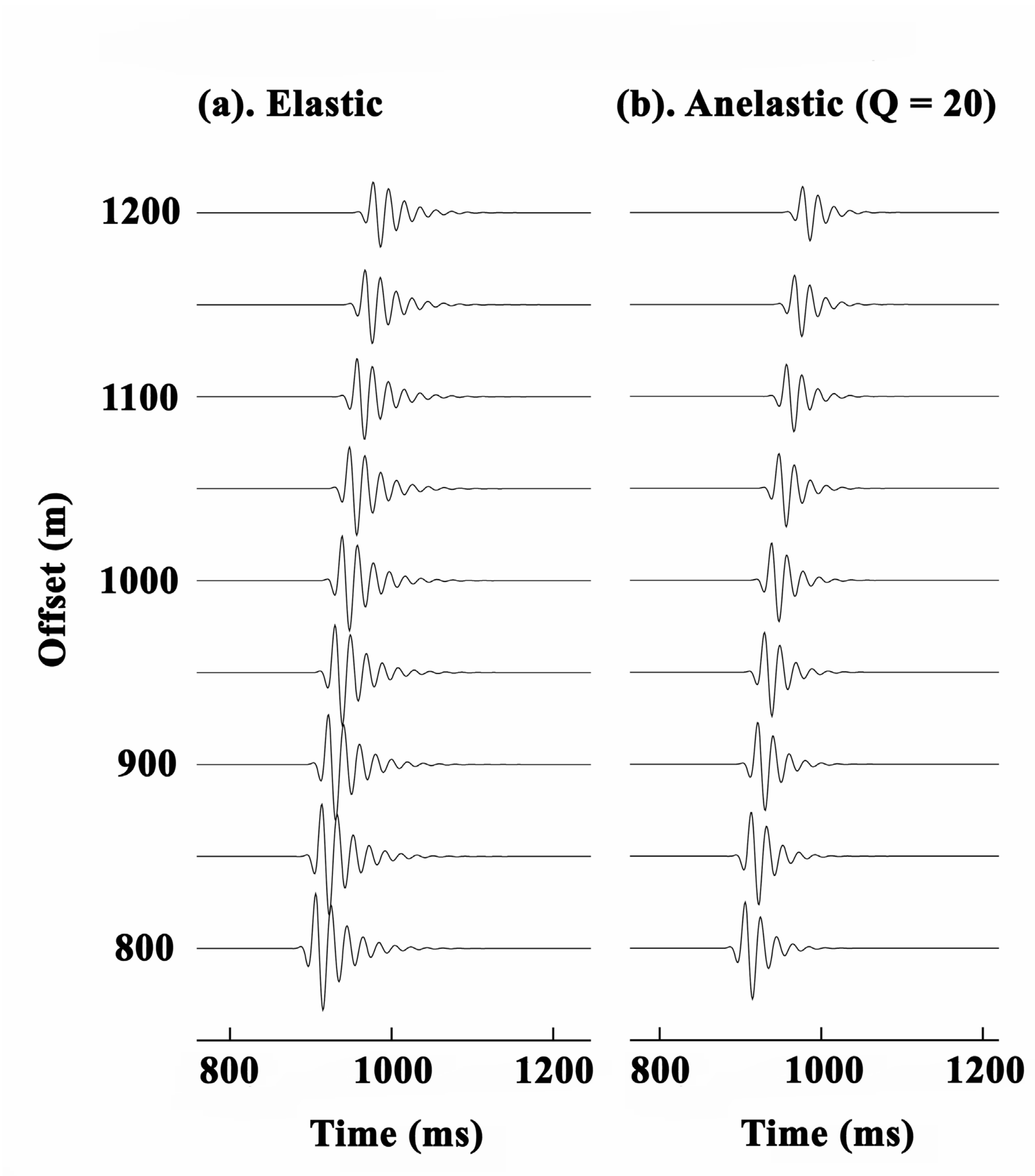


FIG. 5. The comparison of the seismic response of an elastic surface layer with a viscoelastic surface layer. In the elastic case zero order asymptotic ray theory is used to generate the reverberations in the surface layer, while in the viscoelastic case matrix methods are employed in the surface zone. The thickness of the surface layer is 10*m* and the value of *Q* at the reference frequency, $f_R = 30Hz$, is 20.

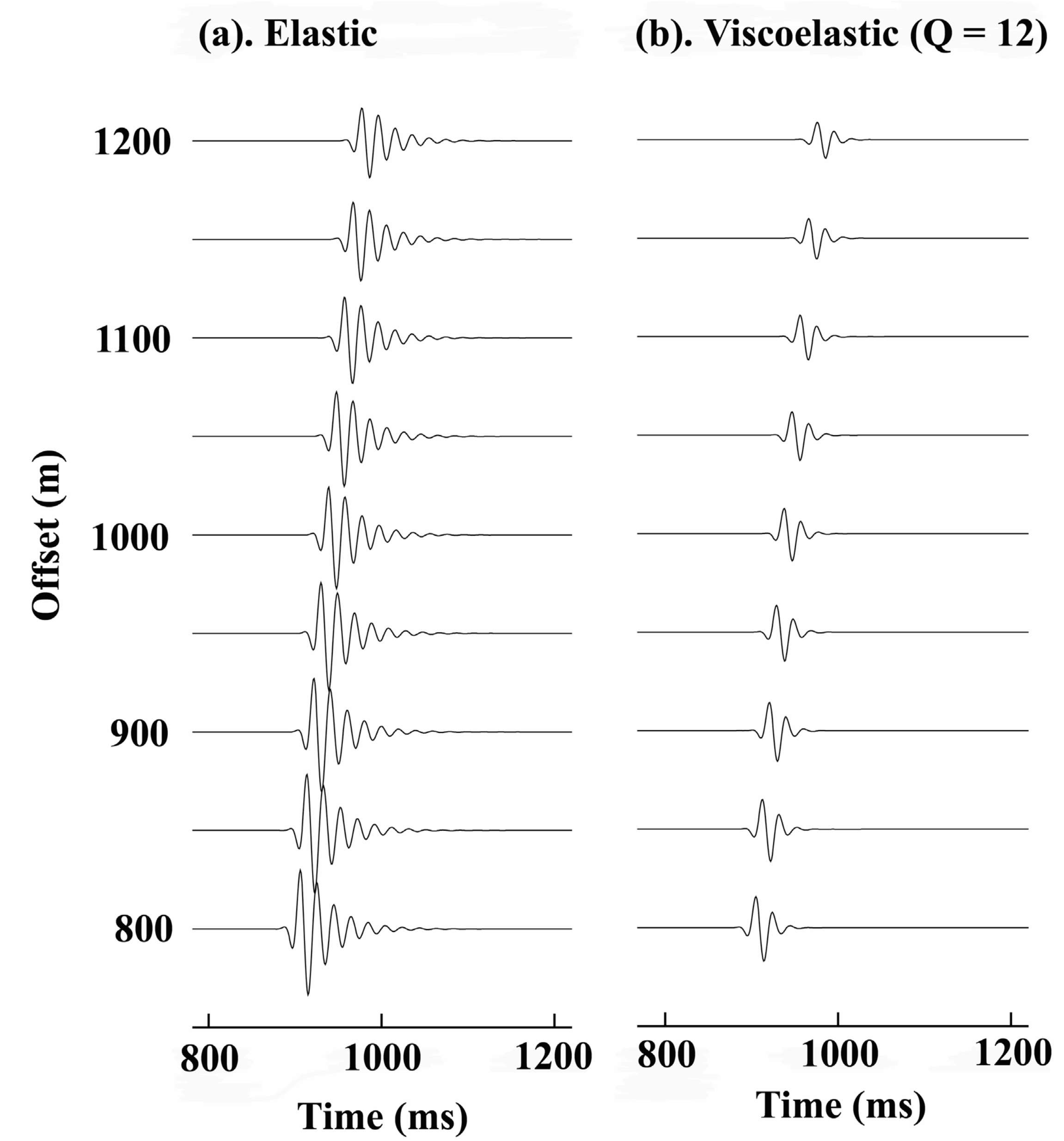


FIG. 6. An example similar to that shown in Figure (5) with the exception that the value of Q in this case is 15 at the reference frequency.

introducing attenuation in the surface layer is shown in Figure (5) where these two models are displayed.

The synthetic traces shown in Figure (6) are similar to those in the previous figure with the exception that the degree of attenuation has been increased by setting $Q = 15$ in the surface layer. The decreased ringing in the wave train is a result of this.

As the velocities in the surface layer are frequency dependent this effect should be noticeable in the spectrum of two traces at the same offset but with differing degrees of attenuation in the surface layer. Figure (7) compares the frequency spectrum of two traces at an offset of 1200*m*; one elastic and the other viscoelastic with Q = 20 at $f_R = 30Hz$. The overall loss of energy in the viscoelastic trace is the first thing that becomes apparent when viewing this figure. The second is the loss of high frequency content in this trace, which is consistent with what is observed in actual field data.

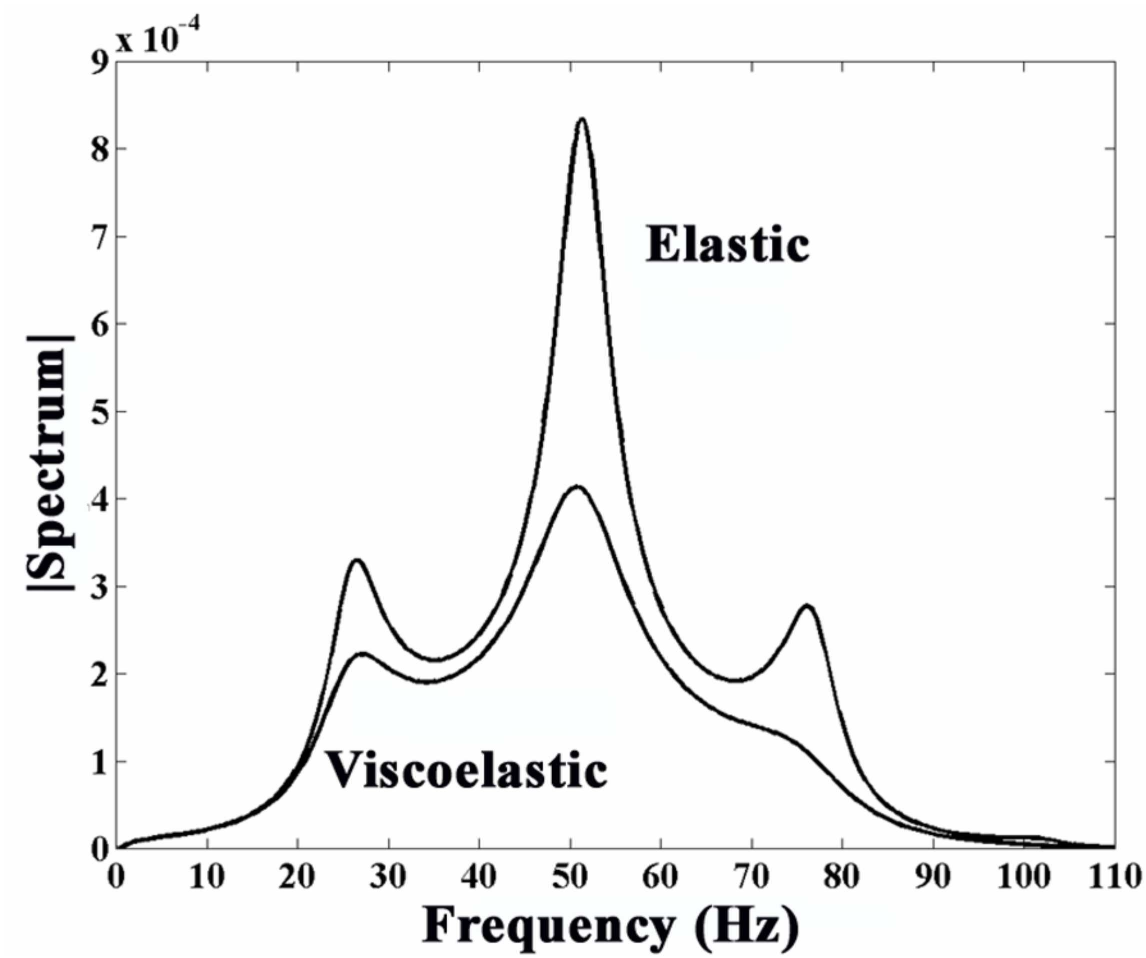


FIG. 7. A comparison of the amplitude spectrums of synthetic traces for a model with a 10*m* surface layer at an offset of 1200*m*. The elastic and viscoelastic spectrums are indicated in the figure. The viscoelastic surface layer has Q = 20 at $f_R = 30Hz$.

CONCLUSIONS

A more appropriate, from a mathematical perspective, manner of investigating the effects of a thin, possibly viscoelastic, surface or weathering layer has been presented. As this layer is often "thin" when compared with the predominant wavelength in the structure, the results produced using asymptotic ray theory or other high frequency ray based approximations are suspect due to the nature of the assumptions upon which they are formulated. As an alternative a matrix based method has been employed for determining the surface effects introduced through the surface conversion coefficient, or more correctly a surface conversion analogue. When the surface zone is composed of a single layer, asymptotic ray theory predicts the seismic response quite adequately for the elastic case. When a number of thin layers, some or all of which may be viscoelastic, are used to define the surface layer this agreement with asymptotic ray theory deteriorates due to both the viscoelasticity and the number of rays which must be generated to achieve the total seismic response inherent in the matrix formulation. The method presented here for the solution of this seismic problem that is known to occur in practice is that, given a reasonable knowledge of the viscoelastic properties of the near surface zone, a technique may possibly be devised to recover some of attenuated high frequency content of surface recorded seismic traces.

The decay of the reverberations when the elastic synthetics are compared with the viscoelastic traces is a point of note. These arguably could have been predicted a priori but a reasonably rigorous derivation from accepted mathematical and physical bases indicates that the method of treating the surface layer presented in this paper has possible potential in the numerical modeling of structures of this geological type. A more convincing argument for implementation of this type of analysis could probably be achieved by comparing $P-SV$ field data with the equivalent modeling presented here.

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