Estimation of Q

Pavan Elapavuluri and John Bancroft

ABSTRACT

A simple crosscorrelation method to estimate the Quality Factor Q is described. Q-wavelets are used for this crosscorrelation. The technique is applied on a synthetic seismic trace that has been Q-filtered with Q value of 70. It is shown that the Q-wavelet (with a Q value of 70) gives highest crosscorrelation coefficients.

INTRODUCTION

The decrease in the strength of seismic signal in the course its propagation thru the medium is defined as the seismic wave attenuation (Sheriff, 1991). Absorption of Seismic energy which is quantized by the quality factor (Q) is one of the mechanisms by which seismic waves are attenuated in an inelastic earth.

The estimation of attenuation and correcting for it is one of the challenges in the seismic data processing. Dasgupta and Clark (1998) state that there are not many techniques to estimate Q from the surface seismic data.

VSP data is frequently used to estimate Q (Tonn, 1991).

Usually Q is estimated using method of spectral ratios. This involves determining the frequency spectrum of a waveform at different receiver locations. The ratios of these spectra then can be used to estimate Q.

A crosscorrelation technique between the Q-wavelet at various times and various Q values over a seismic trace is proposed to estimate the Value of Q.

THEORY

The Quality Factor 'Q'

The quality factor, Q, is defined as the loss of energy per unit cycle (Aki and Richards, 1980).

Let $-\Delta E$ and $-\Delta A$ be the energy and amplitude, respectively, lost in each energy cycle due to inelasticity, then the Q can be written as:

$$\frac{1}{Q(\omega)} = \frac{-\Delta E}{\left(2\pi E\right)} \text{ and } \frac{1}{Q(\omega)} = \frac{-\Delta A}{\left(2\pi A\right)} \tag{1}$$

In a perfectly elastic medium Q=∞ and the loss factor 1/Q=0 (Aki and Richards, 1980).

Q is dependent on frequency, but it has been shown that it varies a little in the range of seismic frequencies (Aki and Richards, 1980).

Convolution Model

There are various simplifying assumptions involved in the deconvolution theory and they are encapsulated and referred as convolution model (Sheriff and Geldart, 1995).

This model can be expressed mathematically as

$$S(t) = Ir(t) * Ws(t), \tag{2}$$

where,

$I_r(t)$ is the earth impulse response,

W_s is the source waveform, and

$S(t)$ is the earth response to the source waveform.

$I_r(t)$, the impulse response can be written as follows, shown by Sheriff and Geldart (1995) as

$$Ir(t) =ns(t)*p(t)*e(t), \tag{3}$$

where,

$n_s(t)$ represents the near-surface effects beneath the source and receiver,

$p(t)$ represents the effects such as multiples, absorption, mode conversions etc which cant be included in the earth model and

$e(t)$ is the reflectivity of the earth, or the impulse response of the target reflectors.

In this paper we neglect the near-surface effects, therefore equation (3) can be written as:

$$Ir(t) = p(t)*e(t). \tag{4}$$

In an attenuating medium, equation (4) can be written using (3) as

$$S(t) = e(t)*p(t) * Ws(t). \tag{5}$$

In this paper, we assume that the factor $p(t)$ represents only the absorption of the medium ignoring all the other effects like multiples and mode conversions etc.

Now equation can be written as:

$$S(t) = e(t)*Wsa(t), \tag{6}$$

where $W_{sa}(t)$ is the attenuated wavelet.

The amplitude spectrum of $W_{sa}(t)$ can be written as follows,

$$W_{sa}\left(f,\tau\right)=W_{sa}\left(f\right)e^{-\frac{\pi f\tau}{Q}}, \qquad (7)$$

(Zhang and Ullrych, 2002), where,

Q is the quality factor,

f is the frequency,

τ is the median time of the time window ,

It is apparent from the above expression that, absorption increases with time.

Crosscorrelation

The crosscorrelation of two digital signals s and r can be written as:

$$c_j=\sum_k s_k r_{k+j}\ . \qquad (8)$$

Normalized Crosscorrelation

$$c_j^{norm}=\frac{\sum_k s_k r_{k+j}}{\sqrt{s_k^2\quad r_j^2}} \qquad (9)$$

Q ESTIMATION

A seismic trace is crosscorrelated with Q-filtered versions of the source wavelets which are formed at different times and with different values of Q.

The Q-filtered wavelets are generated using the equation (7)

Methodology

The technique discussed above will be used to estimate Q over a synthetic seismic trace.

The first step in this methodology is to form Q-filtered wavelets. The source wavelet in this paper is considered to be a Ricker wavelet with dominant frequency of 50 Hz. The ricker wavelet is shown in Figure 1.

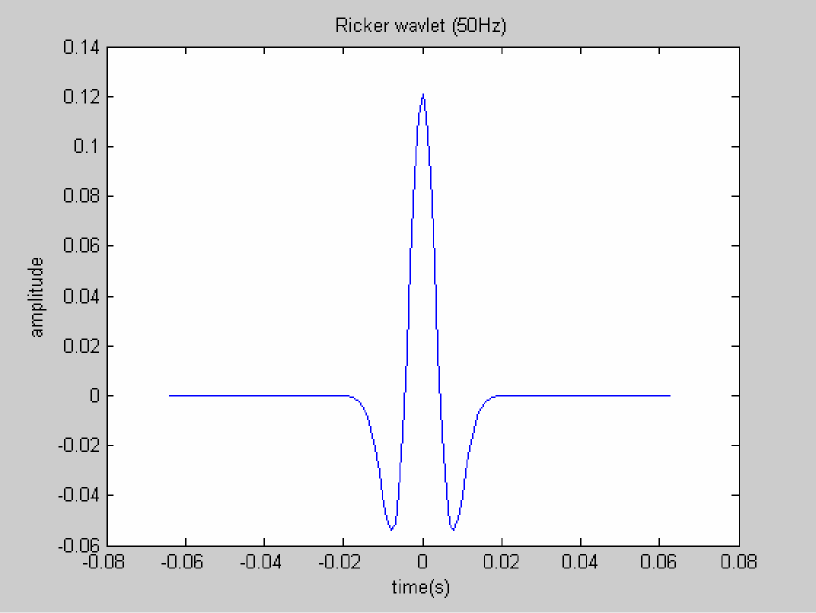

FIG. 1. Ricker wavelet with a dominant frequency of 50 Hz.

This ricker wavelet is used to form Q-filtered wavelets at 1, 3 and 5 seconds with Q values of 100, 70 and 50 respectively.

These wavelets are shown in Figures 2.

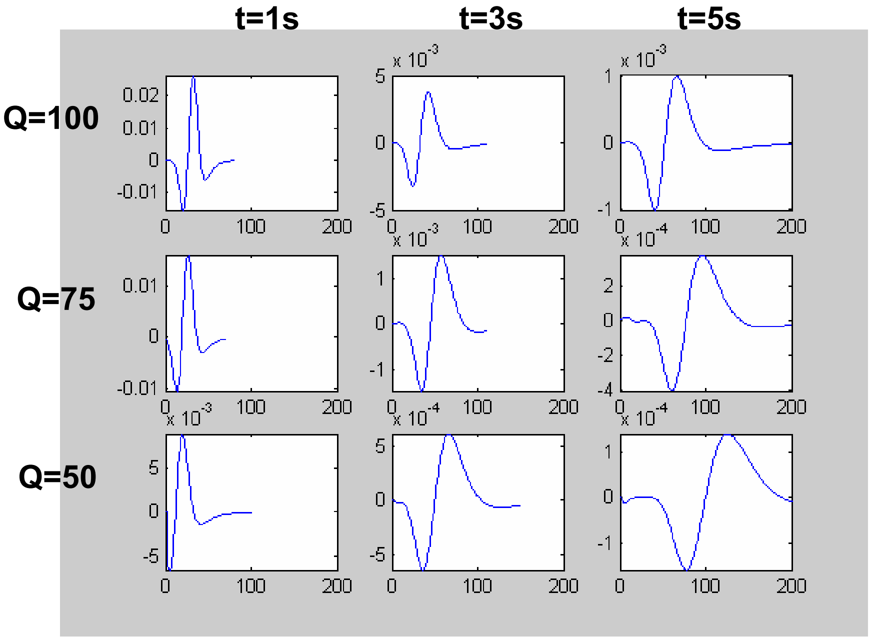

FIG. 2. Ricker wavelet with different times with different Q Values

These wavelets will be used for the crosscorrelation of the Q-filtered seismic trace.

TEST CASES

We tested this method on synthetic traces formed over two kinds of reflectivity series.

Case1

It has three spikes with a very little noise added to it. The three spikes are at 1 s, 3 s and 5 s. This reflectivity is shown in Figure 3.

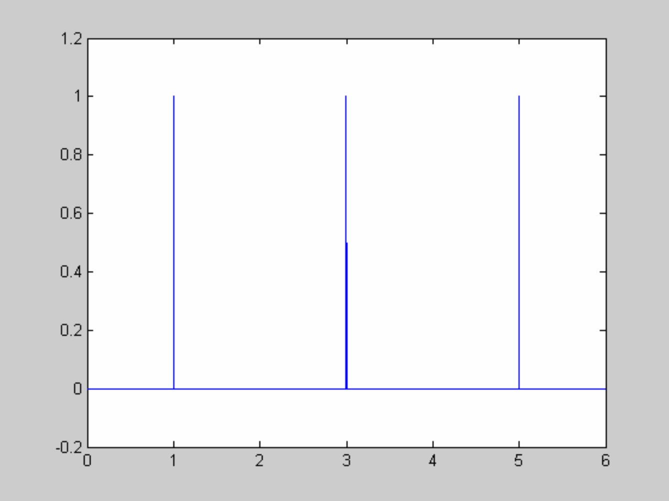

Fig.3. The reflectivity series with spikes at 1s, 3s, and 5s.

The seismic trace formed by convolving with source wavelet is shown in Figure 4.

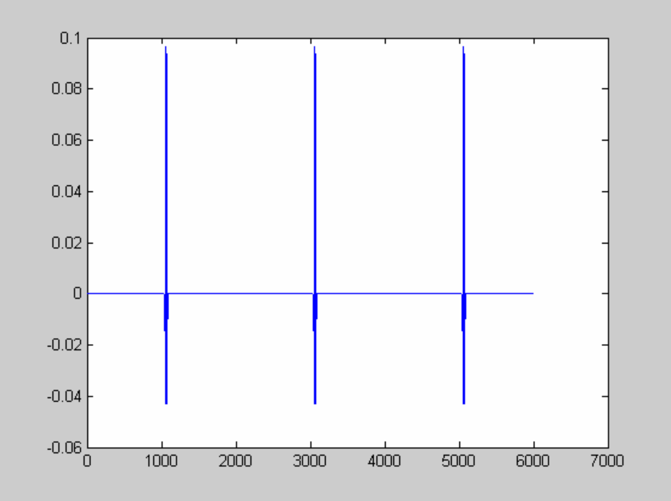

FIG.4. The seismic trace formed by convolving 50 Hz Ricker wavelet with the spiked reflectivity.

This seismic trace is then Q filtered. with a Q value of 70. The Q filtered trace is shown in Figure 5.

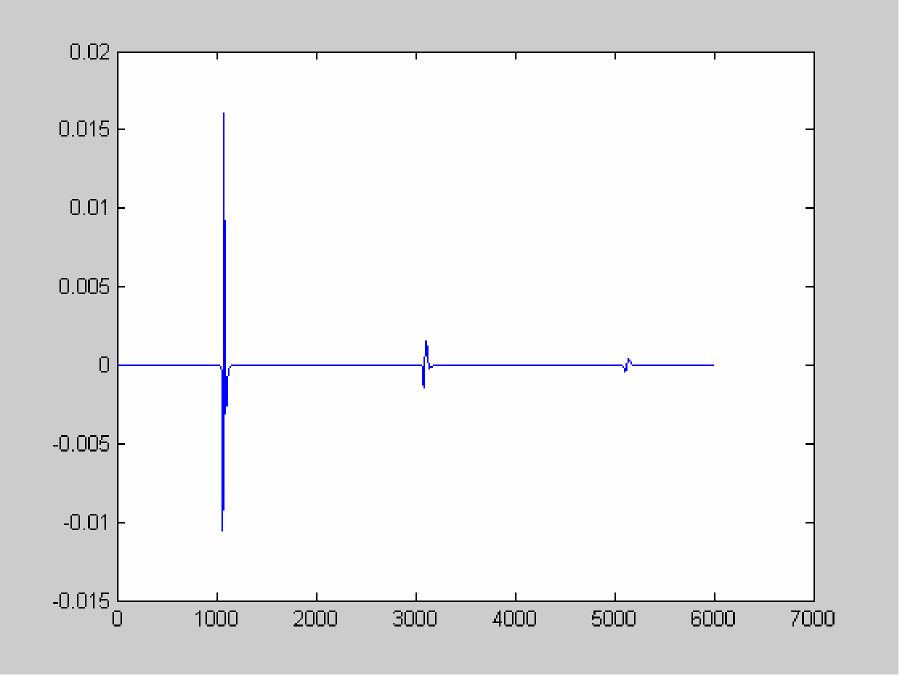

FIG. 5. The Q-filtered seismic trace with Q=70.

This Q filtered seismic trace is crosscorrelated with the 9 Q-filtered wavelets which were discussed earlier. The plots of the crosscorrelation coefficients are shown in Figure 6.

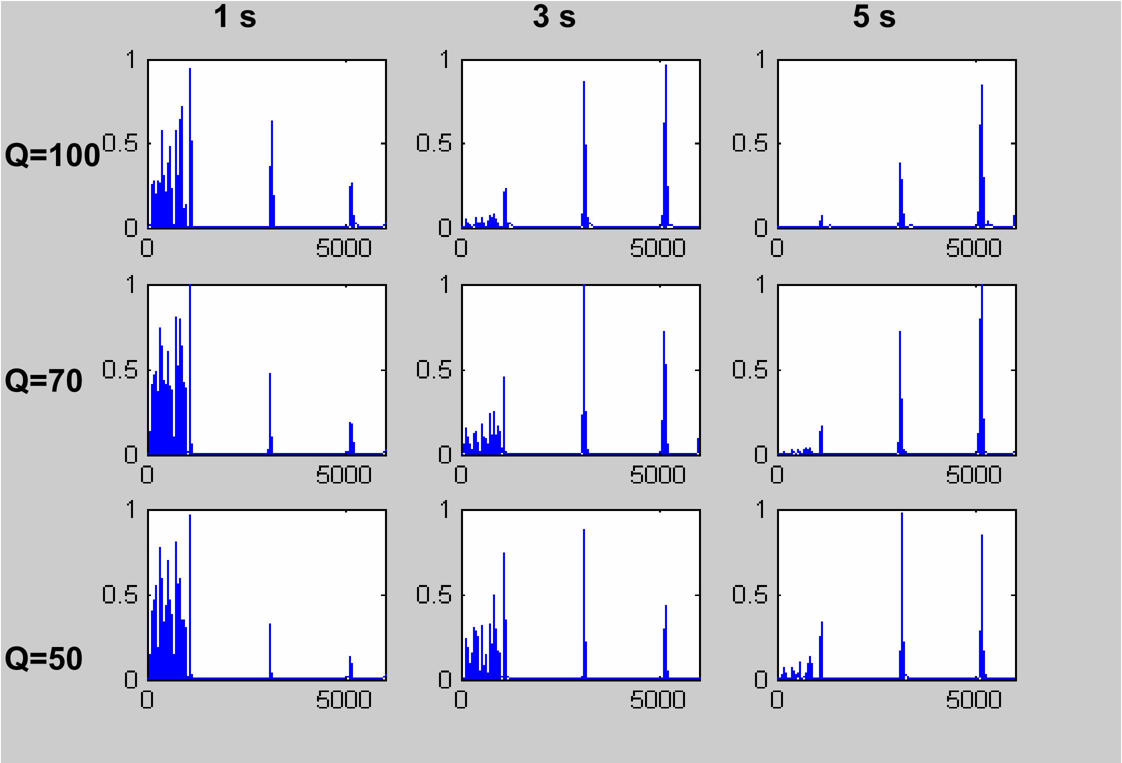

FIG.6. The plot of cross correlation coefficients.

It is obvious from the Figure 6 that the crosscorrelation with the wavelet (with q=70) gives the maximum cross correlation coefficient at respective times. The crosscorrelation coefficient is 1 at 1 s when the trace is correlated the wavelet (Q=70, 1s), the crosscorrelation coefficient is 1 at 3 s when the trace is correlated the wavelet (Q=70, 3s) and the crosscorrelation coefficient is 1 at 5 s when the trace is correlated the wavelet (Q=70, 5s).

Case 2

We then tested this method over a pseudo-reflectivity; this reflectivity is shown in Figure 7.

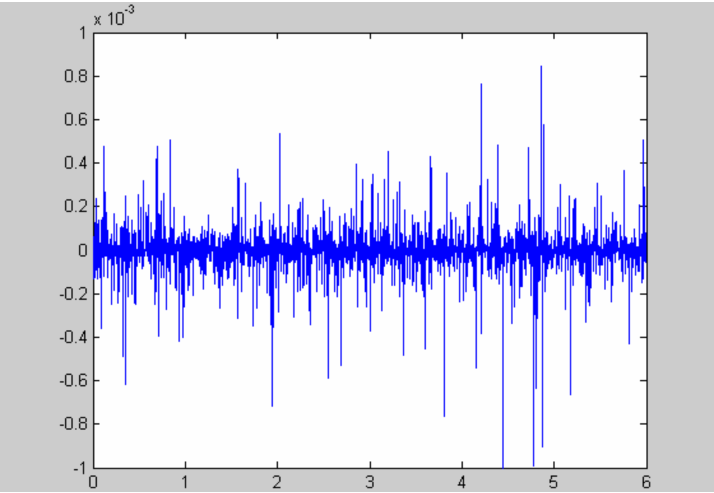

Fig.7. The reflectivity series with spikes at 1s, 3s, and 5s.

The seismic trace formed by convolving with source wavelet is shown in Figure 8.

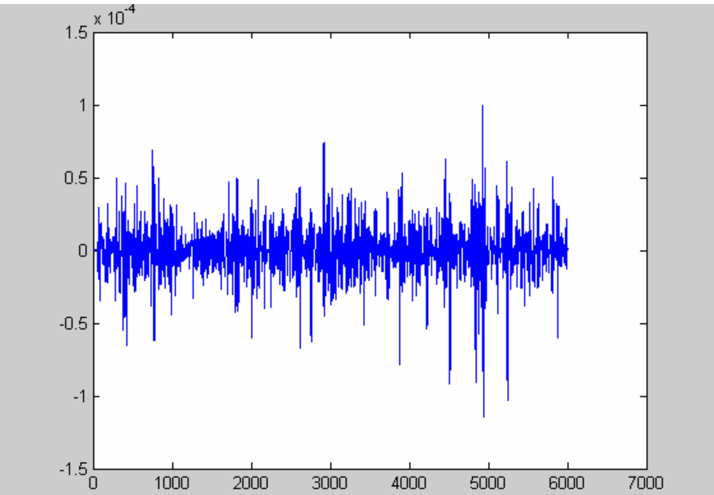

FIG.8. The seismic trace formed by convolving 50 Hz Ricker wavelet with random reflectivity.

This seismic trace is then Q filtered. with a Q value of 70. The Q filtered trace is shown in Figure 9.

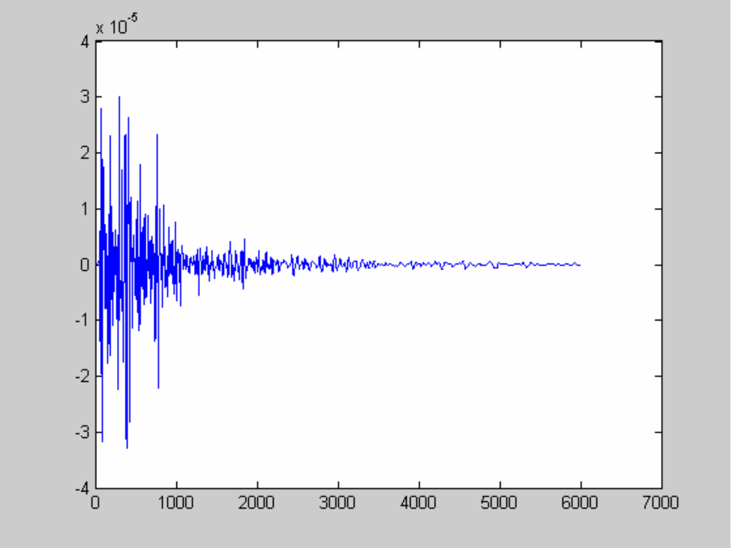

FIG. 9. The Q-filtered seismic trace with Q=70.

This Q filtered seismic trace is crosscorrelated with the 9 Q-filtered wavelets which were discussed earlier. The plots of the crosscorrelation coefficients are shown in Figure 10.

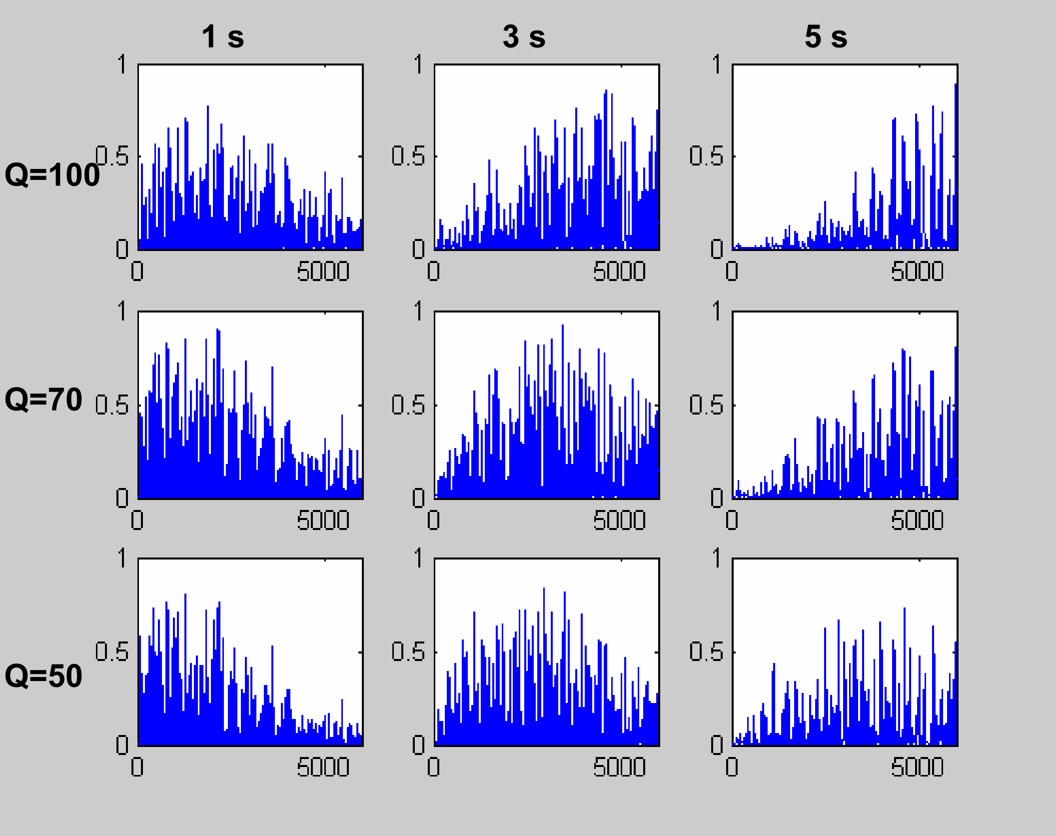

FIG.10. The plot of cross correlation coefficients.

Figure 10 indicates that the crosscorrelation with the wavelet (with Q = 70) gives the maximum cross correlation coefficient at respective times. The crosscorrelation coefficient is 0.9 at 1 s when the trace is correlated the wavelet (Q=70, 1s), the crosscorrelation coefficient is 0.98 at 3 s when the trace is correlated the wavelet (Q=70, 3s) and the crosscorrelation coefficient is 0.8 at 5 s when the trace is correlated the wavelet (Q=70, 5s).

CONCLUSION

An intuitive and simple method to estimate Q is proposed in this paper. The measure of crosscorrelation of a Q-filtered wavelet with a seismic trace was used to estimate Q.

This method was applied to two synthetic traces that had Q attenuation applied. The resulting simplified Q analysis indicated that the Q could be estimated by the cross-correlation process.

It was observed that estimating Q at shallow depths is not very accurate as the Q-filter has little effect of shape of the wavelet.

The results were applied to a single trace. Application of the method to real data would increase the reliability as the Q would be estimated from many traces.

The value of Q estimated thus may be used as an initial estimate in more sophisticated algorithms which need an initial guess of Q.

One of the main assumptions of this paper is the knowledge of the source wavelet and this may be a major disadvantage of this technique.

ACKNOWLEDGEMENTS

We acknowledge the CREWES sponsors for their generous support.

REFERENCES

Aki, K., and Richards, P. G., 1980, Quantitative Seismology: Theory and methods, v.1: W. H. Freeman and Co.

Dasgupta, R. and Clark, R. A., 1998, Estimation of Q from surface seismic reflection data: Geophysics, Soc. of Expl. Geophys., **63,** 2120-2128.

Sheriff, R.E. and Geldart, L.P, 1995, Exploration seismology, Cambridge University press.

Sheriff, R. E., 1991, Encyclopedic Dictionary of Exploration Geophysics,: Soc. of Expl. Geophys., 384.

Tonn, R., 1991, The determination of the seismic quality factor Q from VSP data: A comparison of different computational methods: Geophys. Prosp., Eur. Assn. Geosci. Eng., **39,** 1-28.

Zhang, C., and Ulrych, T., 2001, Estimation of Quality factors: an analytical approach, Ann. Mtg: Can. Soc. Of Expl Geophys.