The power of stacking, Fresnel zones, and prestack migration

John C. Bancroft and Shuang Sun

ABSTRACT

The stacking of common midpoint (CMP) gathers assumes the presence of specula reflection energy and is standard practice in seismic processing. Stacking may also be applied when forming prestack migration gathers.

INTRODUCTION AND REVIEW

The stacking of CMP gathers assumes the presence of specula reflection energy in all offset traces. After moveout correction, these traces are summed and then divided by the fold: i.e., the number of traces that contribute energy at that particular time. In essence, stacking averages the energy in the contributing traces. The power of averaging this energy is usually not recognised, but it is this process that has enabled the forming of stacked sections in areas where the acquisition geometry is discontinuous. The fold can vary greatly between neighbouring traces, but the stacking tends to compensate and produce a stacked section that appears to have continuous reflection amplitude.

In contrast, a Kirchhoff time migration can only gather specula energy that is tangent to its diffraction window. The location of this tangential energy on the diffraction is dependent on the unknown dip of the reflector. Consequently, we sum over the entire diffraction in anticipation of collecting any tangential energy. An amplitude weighting is applied to the data before summation to "balances" the summed specula energy with noise that is summed in other areas of the diffraction window. Traces that are missing harm this process. Consequently, extensive effort is expended in attempts to compensate or fill in these missing traces.

When viewed from a prestack migration vantage point, stacking CMP gathers is part of a prestack migration process, and energy on these traces should be weighted accordingly when stacked. However, the power of stacking has empowered the conventional processing of moveout correction, stacking, and poststack migration, so that it has endured for nearly forty years. Even when the final objective is a prestack migration, the power of stacking is a useful tool for identifying geometry errors, evaluating statics, and providing initial estimates of velocity.

The power of the stacking process is now applied to other data in the prestack volume. Identifying the areas of specula energy in the prestack volume and stacking their energy may simplify the prestack migration process, balance amplitudes independent of the acquisition geometry, and aid in forming gathers for amplitude versus offset (AVO) analysis.

CHEOPS PYRAMID

Prestack traces of 2D data may be identified by their CMP location, x, and half their source-receiver offset, h. These two parameters define the surface of a volume where the additional dimension is the recording time of a trace. Traces in this volume of (x, h, t) may be organized in CMP gathers, constant offset sections, or even their recording order of source gathers. However, identifying them in volume aids in locating where reflection and diffraction energy lies. Energy from one scatterpoint will be theoretically contained in all input traces. Its location in time T on a trace is defined by the double square root equation

$$T = \sqrt{\frac{T_0^2}{4} + \frac{(x+h)^2}{V_{rms}^2}} + \sqrt{\frac{T_0^2}{4} + \frac{(x-h)^2}{V_{rms}^2}}\ , \tag{1}$$

where T_0 is the vertical two-way time, and V_{rms} is the RMS velocity at the scatterpoint location. This equation defines a surface in the prestack volume that is referred to as Cheops pyramid and is illustrated in Figure 1. Energy from one scatterpoint will lie on all traces that intersect this surface. Prestack migration sums and weights all the energy on this surface and places it at the location of the scatterpoint.

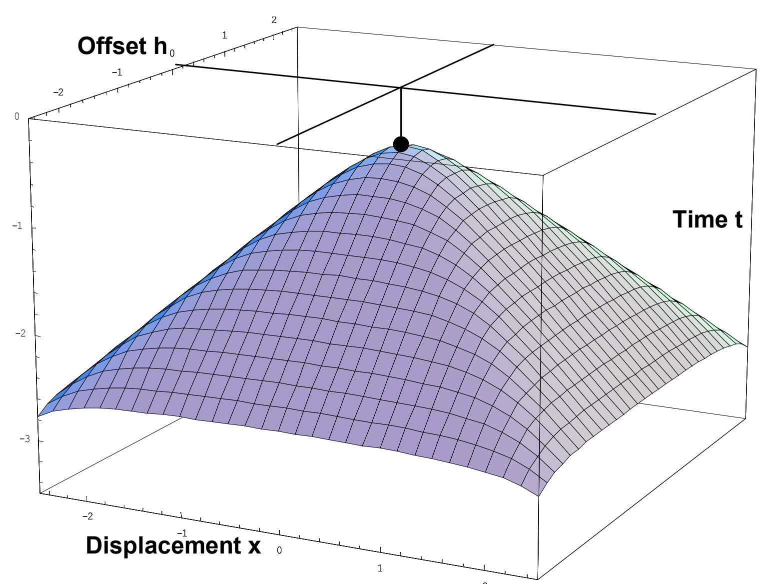


FIG. 1. The traveltimes from one scatterpoint are define by this surface that is referred to as Cheops pyramid.

The scatterpoint principle assumes a 2D reflector can be composed of scatterpoints. Theses scatterpoints can then be replaced with Cheops pyramids to create prestack seismic data modelled into the prestack volume (*x, h t*). Two prestack surfaces are shown in Figure 2 that represent the specula reflection energy from horizontal and dipping reflectors. In the prestack volume, the surface in (a) is a hyperbolic cylinder and in (b) a hyperbolic surface. Both these surfaces have moveout that is exactly hyperbolic. The moveout for the horizontal reflector is

$$T^2 = T_0^2 + \frac{4h^2}{V_{rms}^2}, \tag{2}$$

and that for the dipping reflector is

$$T^2 = T_0^2 + \frac{4h^2 \cos^2 \beta}{V_{rms}^2} = T_0^2 + \frac{4h^2}{V_{stk}^2} \tag{3}$$

where β the dip of the reflector and V_{stk} the stacking velocity. Even though this is a very convenient equation and may simplify the stacking process, it assumes the reflection energy lies on the hyperbolic path of the CMP gather. That is not the case.

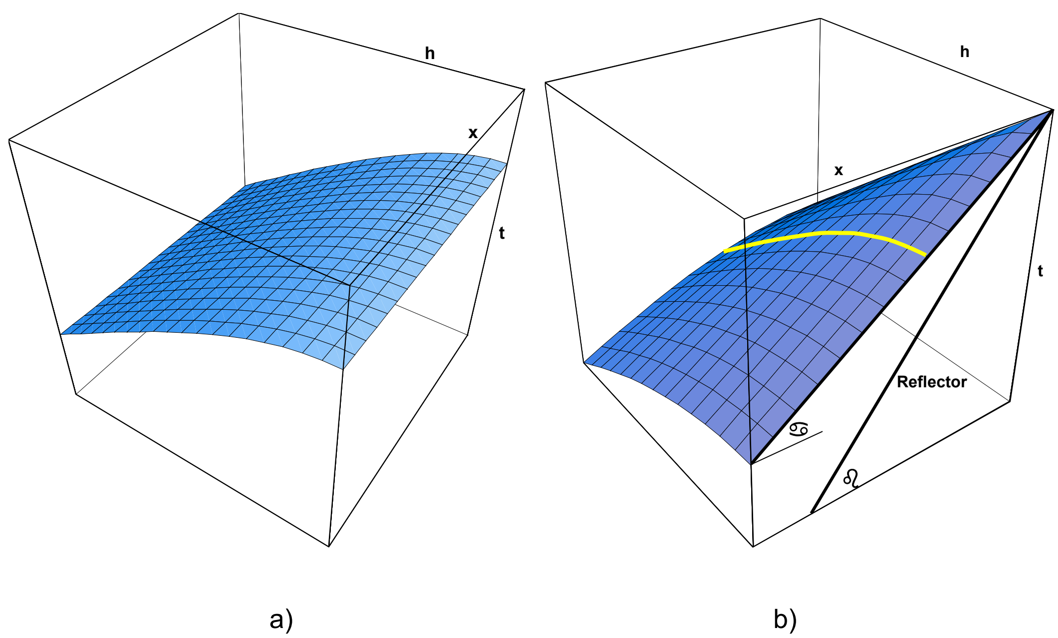


FIG. 2. Prestack surfaces of a) horizontal and b) dipping reflector.

Consider Figure 3 that shows raypaths for a zero offset and a fixed offset that have the same midpoint. Note that the location of the reflection point moves up–dip from the zero–offset location. Therefore, stacking energy along this hyperbolic path (yellow in Figure 2b) will sum energy from different reflection points, causing a smear of energy along the reflector.

Now consider Figure 4, that is similar to Figure 3, but now the offset rays have been positioned to have the same reflection point. Note that the midpoint location moves down–dip as the offset is increased.

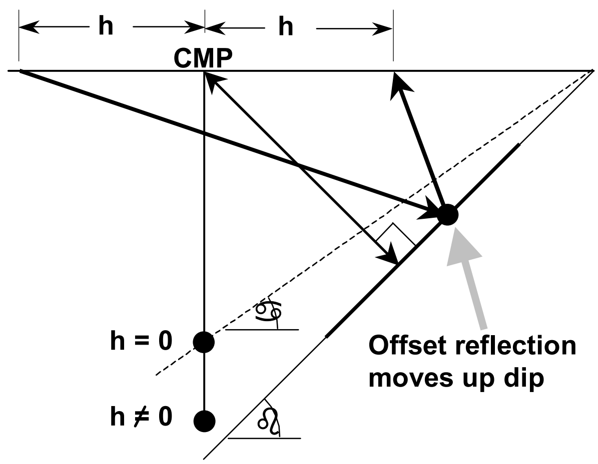


FIG. 3. Reflector location moves up dip with increase offset.

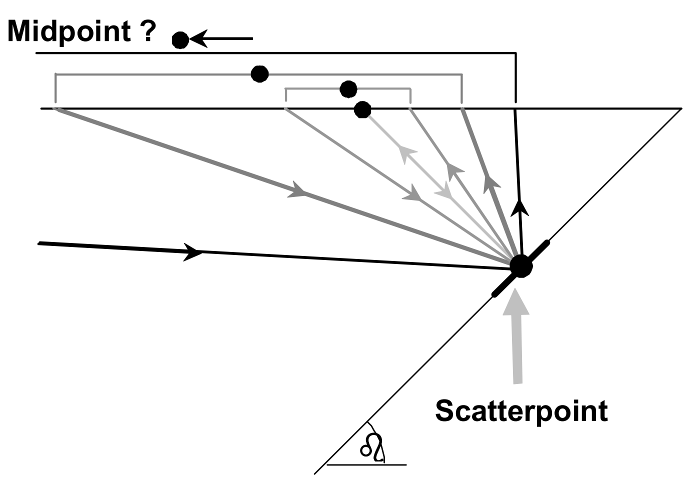
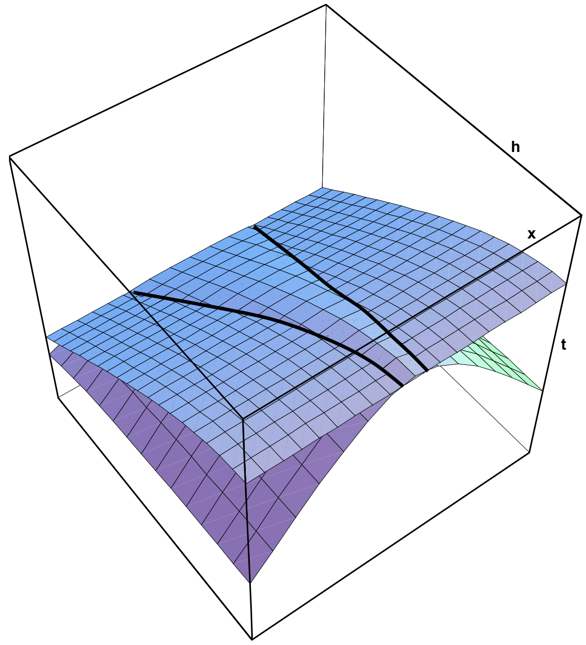
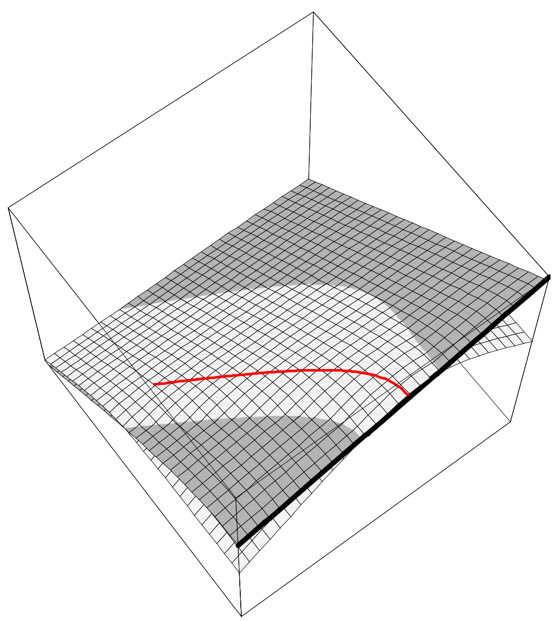


FIG. 4. Midpoint move down dip as the offset is increased.



a)

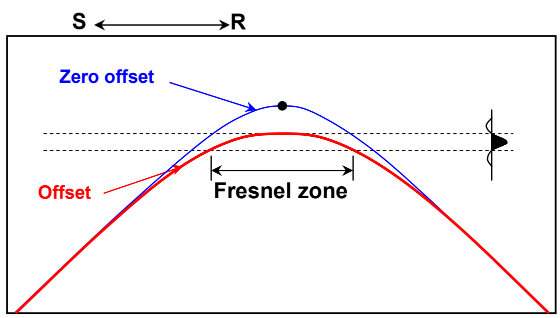


b)

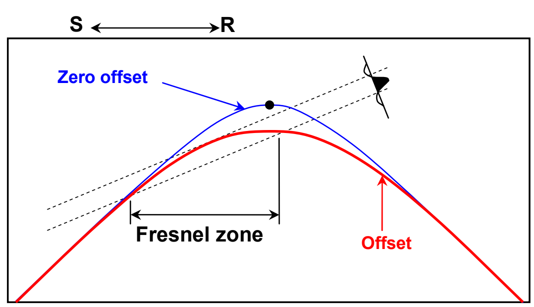
FIG. 5. Intersections of Cheops pyramid with a) the hyperbolic cylinder for a horizontal reflector, and b) the hyperbolic surface of a dipping reflector. Note the location of the dipping energy moves down dip as the offset is increased.

FRESNEL ZONES

Prestack migration sums the energy that lies on the surface of Cheops pyramid. It is the tangential energy from specula reflections that is summed to the migration point. The midpoint and offset locations for a single reflecting element can be mapped onto the hyperbolic surface or on Cheops pyramid, defining the location of tangency between the two surfaces. This line of tangency is only valid for infinite frequencies. A more practical description is obtained when the hyperbolic surface is raised slightly to allow Cheops pyramid to protrude the hyperbolic surface that contains the intersecting surfaces for a horizontal and dipping reflector, as illustrated in Figure 5. When the amount of shift is a half–wavelet, the protruding area defines an area of tangency that is defined as the Fresnel zone. In Figure 6a, this area is identified by the two black lines on the surface; in Figure 6b, the red line defines the tangential surface when there is no shift, and the grey band defines the fresnel zone when there is a half–wavelength shift.



a)



b)

FIG. 6. Fresnel zones for a) a horizontal reflector and b) a dipping reflector.

The areas of tangency between a diffraction and specula energy are used to define the size of the Fresnel zone. The energy defined by this intersection is the actual reflection energy, that gets “picked up” when summing over a diffraction or Cheops pyramid. Constructions that help define the size of the Fresnel zone for a horizontal reflector with constant offset, is shown in Figure 6a, and for a dipping reflector, in Figure 6b. The half wavelength is represented by the dashed lines in Figures 6a and b. The intersection of the lower dashed line with the diffraction defines the spatial extent of the Fresnel zone. We define the wavelength as ↔, and the vertical two–way time to the scatterpoint as t_0. The size of the Fresnel zone x_f can be derived for the horizontal reflector as

$$x_f^2 = \frac{t_0 \tau v^2}{4} \frac{\left(\sqrt{1 + t_0^2 \frac{4h^2}{v^2}} + \frac{\tau}{2t_0} \right)}{\left(1 - \frac{4h^2}{t_0^2 v^2 \left(\sqrt{1 + \frac{4h^2}{t_0^2 v^2}} + \frac{\tau}{t_0} \right)^2} \right)} \tag{4}$$

If the wavelength ↔ is assumed to be much smaller than the time t_0, we get an approximate Fresnel zone size of

$$x_f \approx \frac{v}{2} \sqrt{t_0 \tau} \left(1 + \frac{4h^2}{t_0^2 v^2} \right)^{3/4} \tag{5}$$

The equations for the Fresnel zone size of dipping reflectors is more complex and is contained in Sun (2003).

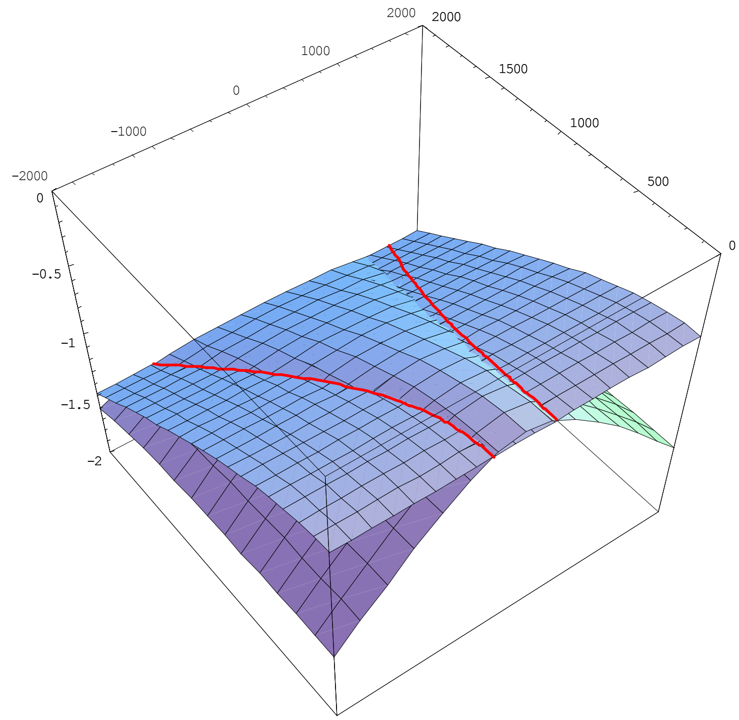


FIG. 7. Fresnel zones defined from the above equations with the red line on the left using the exact equation (4) and the red line on the right using the approximate equation (5).

The red line on the left of Figure 7 is defined by the accurate definition of the Fresnel zone and occurs at the line of intersection between the two surfaces. The red line on the right represents the approximate definition of equation (5).

Common Scatterpoint gathers

An intermediate step of the equivalent offset method (EOM) of prestack migration (Bancroft et al. 1998) forms common scatterpoint (CSP) gathers. Moveout correction and stacking completes the prestack migration. These gathers leave the data at the original input times where the reflection energy is focused on paths that are hyperbolic with offset, similar to CMP gathers. Prestack AVO information can be extracted directly off these gathers.

Specula energy and fold division

The Fresnel zones in the previous figures identify area of tangential specula energy. For example in Figure 7, the Fresnel zone for horizontal data identifies the area where a super CMP gather may be formed for velocity analysis. If the data was flat, the super gather would be “almost” suitable as a prestack migration gather. The correct moveout and stacking of this data would approximate a prestack migration. The same concept applies to the dipping reflection data in Figure 5b. Moveout correcting and “stacking” the energy in this band will also approximate a prestack migration. This method of stacking will work well in areas where there are missing traces due to geometry problems.

DATA EXAMPLES

The data in the following examples was gathered using a prestack aperture that was half the size of the Fresnel zone. Offset traces in the CSP gather were formed by summing the input traces, and a time and offset varying map of the fold was maintained. After all the input traces were summed into the gather, the gather was divided by the time and offset varying fold.

Energy within the half Fresnel zone was considered to be specula, and that fold division would preserve the equivalent offset amplitude, independent of variations in the acquisition geometry, similar to the stacking of CMP gathers.

Prestack modelled data was created using Ostrander's gas sand model (1984), and processed to CSP gathers. The amplitudes extracted from a CSP gather, along with the theoretical amplitudes, are displayed in Figure 8. Real data is from the Blackfoot area in Alberta Canada. A sand channel is identified by the arrows in Figure 9.

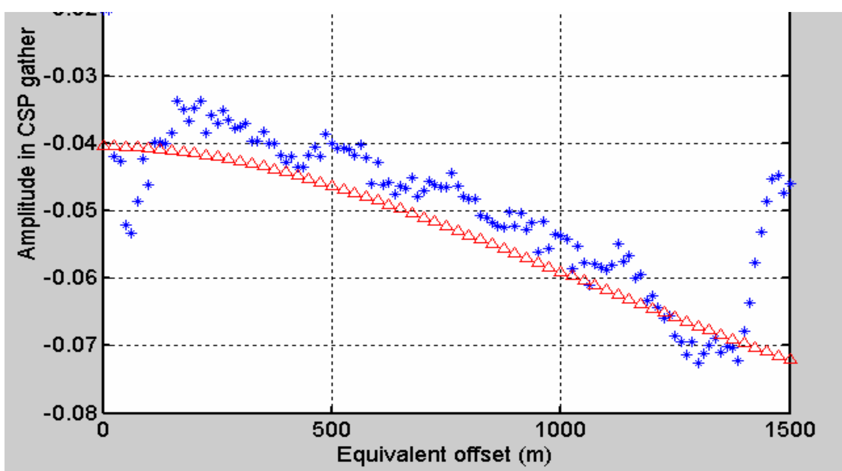
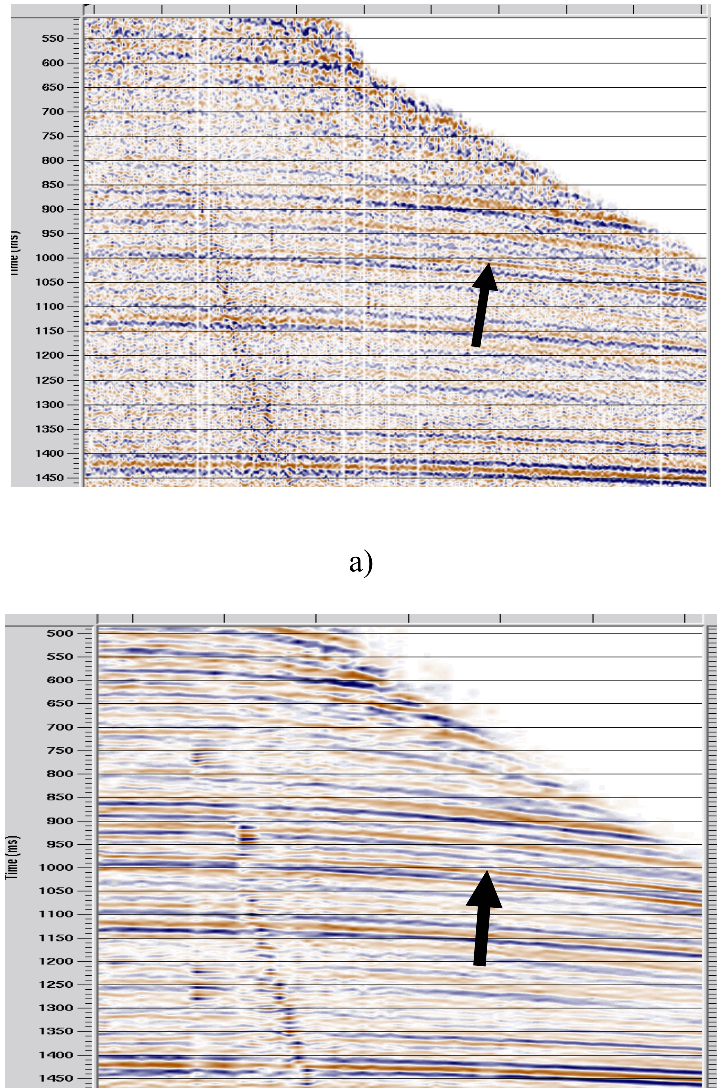


FIG. 8. Example of amplitudes extracted from a CSP gather from modelled data.



a)

b)

FIG. 9. Gathers from the Blackfoot area with a) a Super CMP gather and b) a CSP gather.

COMMENTS AND CONCLUSIONS

A generalized definition for the Fresnel zone was presented for the application of a limited–aperture prestack migration. It must be noted that any limited–aperture migration must be used with utmost caution as any errors in the velocity model will tend to be supported by the process. This process assumes that sufficient prestack migrations have accurately defined the geological structure.

Prestack migrations can be accomplished by "stacking" specula data in a band that is defined by the Fresnel zone. This method speeds the prestack process, balances amplitudes for AVO analysis, and is especially valuable in areas where there are geometry problems

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REFERENCES

Sun, S., 2003, The use of common scatterpoint gathers to detect amplitude variation with offset: M.Sc. Thesis, University of Calgary.

Bancroft, J.C., Geiger, H.D., and Margrave, G.F., 1998, The equivalent offset method of Prestack time migration: Geophysics, **63**, 2042–2053

Claerbout, J.F., 1985, Imaging the Earth's Interior: Blackwell Scientific Publications.

Ostrander, W.J., 1984, Plane-wave reflection coefficients for gas sands at non-normal angles of incidence: Geophysics, **49**, 1637–1648