Testing pseudo-linear Zoeppritz approximations: Analytical error expressions

Charles P. Ursenbach

ABSTRACT

Analytical expressions are derived for the error of contrasts estimated by AVO inversion. This is carried out for three approximations to the Zoeppritz equations, and for both conventional and converted-wave data. The expressions are tested against errors obtained by explicit numerical inversion. The formulae obtained show that errors in P-wave impedance contrasts are an order of magnitude smaller than those of other contrasts. The formulae for errors in shear-wave impedance contrast and density contrast are particularly accurate for linear theories and multicomponent data.

INTRODUCTION

In two earlier reports (Ursenbach, 2003a,b, hereafter referred to as Papers I and II) it was found that contrasts estimated by approximations to the Zoeppritz equations have errors which correlate strongly with $\Delta\beta/\beta$ and with $\Delta I_S/I_S$. Empirical relations were derived in Paper I to represent this correlation. In this paper we pursue the issue further by formally deriving the lowest-order terms in an expansion of the error. We then compare these expressions against the actual error, and explore in a preliminary way their potential use in correcting the results of AVO inversions.

METHOD

In Papers I and II, inversions were carried out using synthetic data at incident angles of $\theta = 0°, 1°, 2°, \ldots, 30°$. In separate calculations (not shown) it was found that contrasts estimated using only three angles ($\theta = 0°, 15°, 30°$) were very similar to those using more densely sampled data. From this observation it was concluded that it would be feasible to perform inversion on a semi-analytical data set, i.e., one in which $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$, and β/α remain in symbolic form, but the angle of incidence is specified at the three values mentioned above. Values of $\sin\theta$ are then 0, $\sqrt{2}(\sqrt{3}-1)/4$, and ½ respectively, and from these the values of R_{PP} and R_{PS} can be determined analytically as well. (For the converted-wave case we use only $\theta = 15°$ and $30°$, as the normal incidence reflectivity is always zero.)

As discussed in the earlier papers, Zoeppritz approximations, though expressed in apparently linear form in the contrasts, are inherently non-linear as the coefficients are weakly dependent on the contrasts. (In this instance we use "weakly dependent" to mean that the coefficients do not vanish when the contrasts are set to zero.) In practical inversions, some estimated background value of the contrasts is used in the coefficients, such as using velocity analysis results for raytracing. In papers I and II the contrasts were initially set to zero in the coefficients, and then replaced by the resulting estimate in an iterative fashion until a converged result was obtained. For the purpose of the present analytical inversion, the contrasts in coefficients will be assumed to take on their exact values. This will result in a tractable problem, and the resulting estimates will be viewed as a lower bound to the errors obtained in practical schemes. As the results below illustrate, it provides a fairly accurate representation of errors, suggesting that the errors are more sensitive to the theoretical expression than to the precise values of contrasts employed in the coefficients.

We carried out the inversions proposed above using symbolic mathematics software (Maple 7). The resulting estimated contrasts were expanded to third order in the exact contrasts. Each term in this expansion has a coefficient which depends on β/α, usually in a highly complex expression. To simplify, we have expanded each coefficient about $\beta/\alpha = 1/2$, and evaluated the numerical parts in decimal notation. This results in an approximate coefficient of the form A + B×β/α, which is exact for $\beta/\alpha = 1/2$, and should be accurate for values in that vicinity. (Trailing zeros which may appear in the A and B quantities below are not generally significant.)

In the Results section, we present the derived formula and accompanying graphs for the various combinations of

i) **estimated contrast** – $\Delta I_P/I_P$, $\Delta I_S/I_S$, or $\Delta\rho/\rho$ (from which other contrasts may be derived)

ii) **theoretical method** – Aki-Richards (AR), Pseudo-Linear (PL), or Pseudo-Quadratic (PQ) (For definitions of these, see Papers I and II.)

iii) **data source** – PP (conventional) data or PS (converted-wave) data

The graphs compare the errors predicted by the derived formulae to actual errors obtained from inversion of data for 125 interfaces described in Papers I and II. Then, with an eye to potential application of these expressions, we also perform this comparison using the estimated contrasts in the formulae instead of the exact contrasts. (For PS inversion formulae, the exact $\Delta\alpha/\alpha$ is used in both cases, as its value is not estimated.) Obviously the exact contrasts are not known in real problems and it would be of value to see whether the formulae provide useful corrections when only approximate values are used in them.

RESULTS

A number of trends may be noted in the results displayed below:

- The Aki-Richards and Pseudo-Linear theories yield errors which are at least of quadratic order. This is consistent, both with empirical observations in Papers I and II, and with the fact that they are linear approximations.

- The Pseudo-Quadratic theory yields errors which are at least of cubic order, again consistent with expectations.

- The errors in $\Delta I_P/I_P$ are at least one order more favorable than for other contrasts. That is, the Aki-Richards and Pseudo-Linear errors are cubic, and the Pseudo-Quadratic errors are at least quartic (i.e., they were zero in the expressions derived here, which are cubic). It has been noted frequently before that $\Delta I_P/I_P$ estimations are more accurate than other contrasts, and this result confirms such observations with a clear mathematical statement.

- The errors in $\Delta I_P/I_P$ are also unusual in that they are independent of β/α, and possess very simple coefficients, at least to the cubic order investigated here.

With these general trends in mind, let us consider the individual cases.

$\Delta I_P/I_P$ from P-P data by the Aki-Richards method

As noted above, the error in this case is of cubic order (i.e., the quadratic errors in $\Delta\alpha/\alpha$ and $\Delta\rho/\rho$ cancel each other) and the coefficient is independent of β/α.

$$\left(\frac{\Delta I_P}{I_P}\right)_{AR}^{PP} - \left(\frac{\Delta I_P}{I_P}\right)_{exact}^{PP} = -\frac{1}{4}\left(\frac{\Delta\alpha}{\alpha}\right)\left(\frac{\Delta\rho}{\rho}\right)\left[\left(\frac{\Delta\alpha}{\alpha}\right)+\left(\frac{\Delta\rho}{\rho}\right)\right] \tag{1}$$

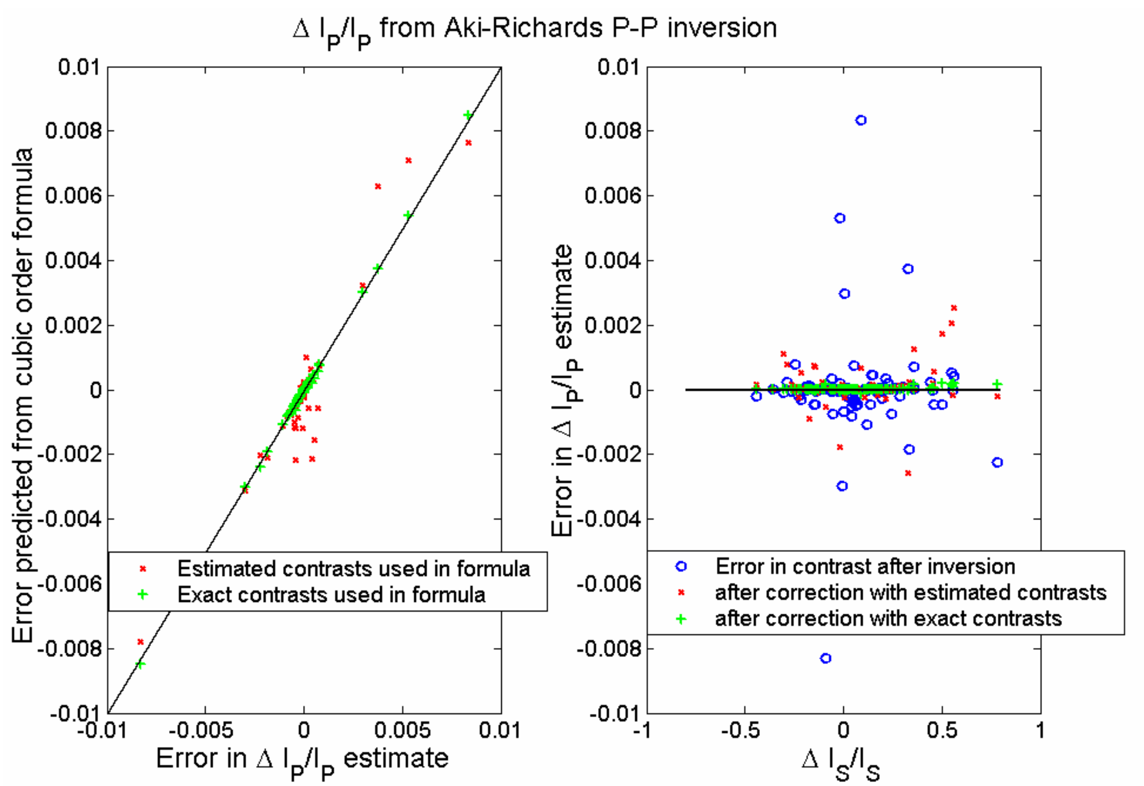


FIG. 1. The first panel compares the actual errors in the $\Delta I_P/I_P$ estimate to those predicted by Equation (1). The green points employ the correct contrasts in Equation (1), and the red points use the estimated contrasts instead. The second panel displays the estimated contrasts plotted against the shear-impedance contrast, and compares them to the same quantity after correction by Equation (1), using both the estimated contrasts and then the exact contrasts.

$\Delta I_P/I_P$ from P-P data by the Pseudo-Linear method

Although the error of the Pseudo-Linear method differs from the Aki-Richards error, the same general comments apply. We have also given the Pseudo-Quadratic error here, but we have not plotted results for it, as it is simply zero to cubic order.

$$\left(\frac{\Delta I_P}{I_P}\right)_{PL}^{PP} - \left(\frac{\Delta I_P}{I_P}\right)_{exact}^{PP} = -\frac{1}{4}\left(\frac{\Delta\alpha}{\alpha}\right)\left(\frac{\Delta\rho}{\rho}\right)^2 \tag{2}$$

$$\left(\frac{\Delta I_P}{I_P}\right)_{PQ}^{PP} - \left(\frac{\Delta I_P}{I_P}\right)_{exact}^{PP} = O\left[\left(\frac{\Delta\alpha}{\alpha}\right)^l\left(\frac{\Delta\beta}{\beta}\right)^m\left(\frac{\Delta\rho}{\rho}\right)^n\right], \quad l+m+n \geq 4$$

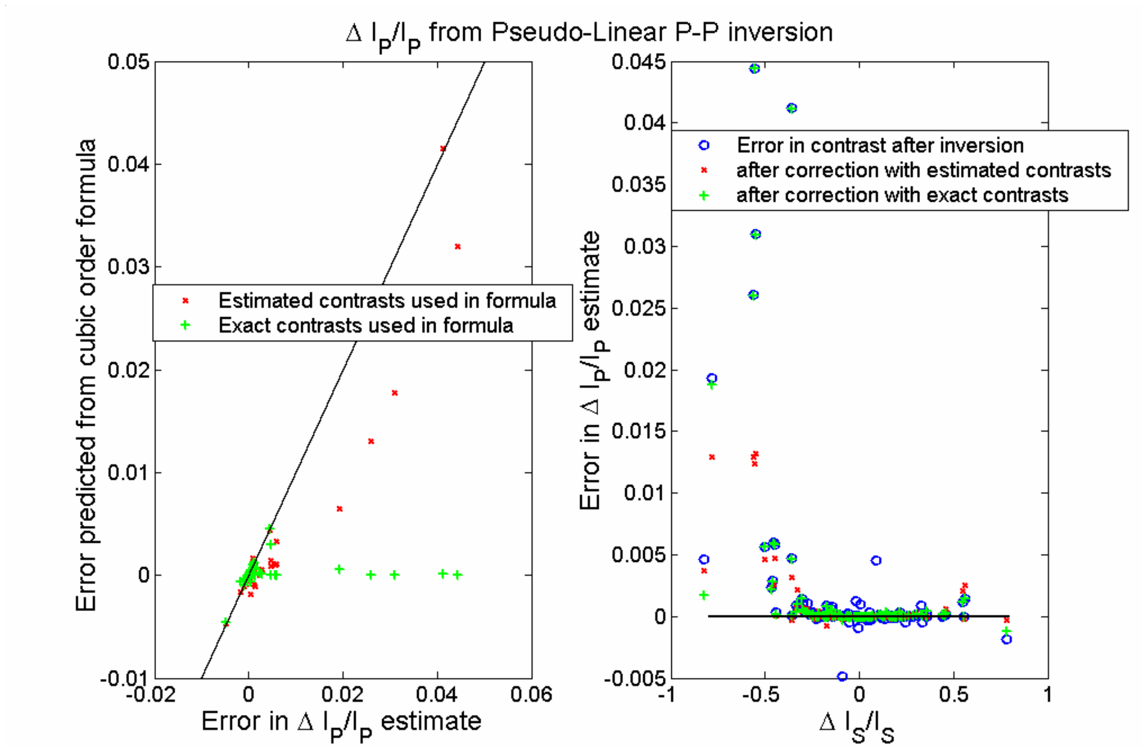


FIG. 2. This figure describes the errors of $\Delta I_P/I_P$ estimation by the Pseudo-linear method from P-P data, and their comparison with Equation (2). (See caption of Figure 1 for detailed explanation.)

$\Delta I_S/I_S$ from P-P data by the Aki-Richards method

$$
\begin{aligned}
\left(\frac{\Delta I_S}{I_S}\right)_{AR}^{PP} - \left(\frac{\Delta I_S}{I_S}\right)_{exact}^{PP} &= \left(-0.235255+0.71453\gamma\right)(\Delta\alpha/\alpha)^3 \\
&+\left(1.45853-3.4051\gamma\right)(\Delta\alpha/\alpha)^2(\Delta\beta/\beta) \\
&-0.98803\left(\gamma-1/2\right)(\Delta\rho/\rho)(\Delta\alpha/\alpha)^2 \\
&+\left(-1.10649+2.7236\gamma\right)(\Delta\alpha/\alpha)(\Delta\beta/\beta)^2 \\
&+\left(0.24502\gamma-0.369450\right)(\Delta\rho/\rho)(\Delta\alpha/\alpha)(\Delta\beta/\beta) \\
&+\left(0.008120-0.26532\gamma\right)(\Delta\rho/\rho)^2(\Delta\alpha/\alpha) \\
&+\left(-0.84513\gamma+0.4201669000\right)(\Delta\beta/\beta)^3 \\
&+\left(0.64719\gamma-0.199075\right)(\Delta\rho/\rho)(\Delta\beta/\beta)^2 \\
&+\left(-0.5217835000+0.049387\gamma\right)(\Delta\beta/\beta)^2 \\
&+\left(0.50298\gamma-0.376570\right)(\Delta\rho/\rho)^2(\Delta\beta/\beta) \\
&+\left(-0.086490-0.82484\gamma\right)(\Delta\rho/\rho)(\Delta\beta/\beta) \\
&+\left(-0.000368004+0.00070693\gamma\right)(\Delta\rho/\rho)^3 \\
&+\left(0.1959528800-0.39145\gamma\right)(\Delta\rho/\rho)^2
\end{aligned}
\tag{3}
$$

Here $\gamma \equiv \beta/\alpha$.

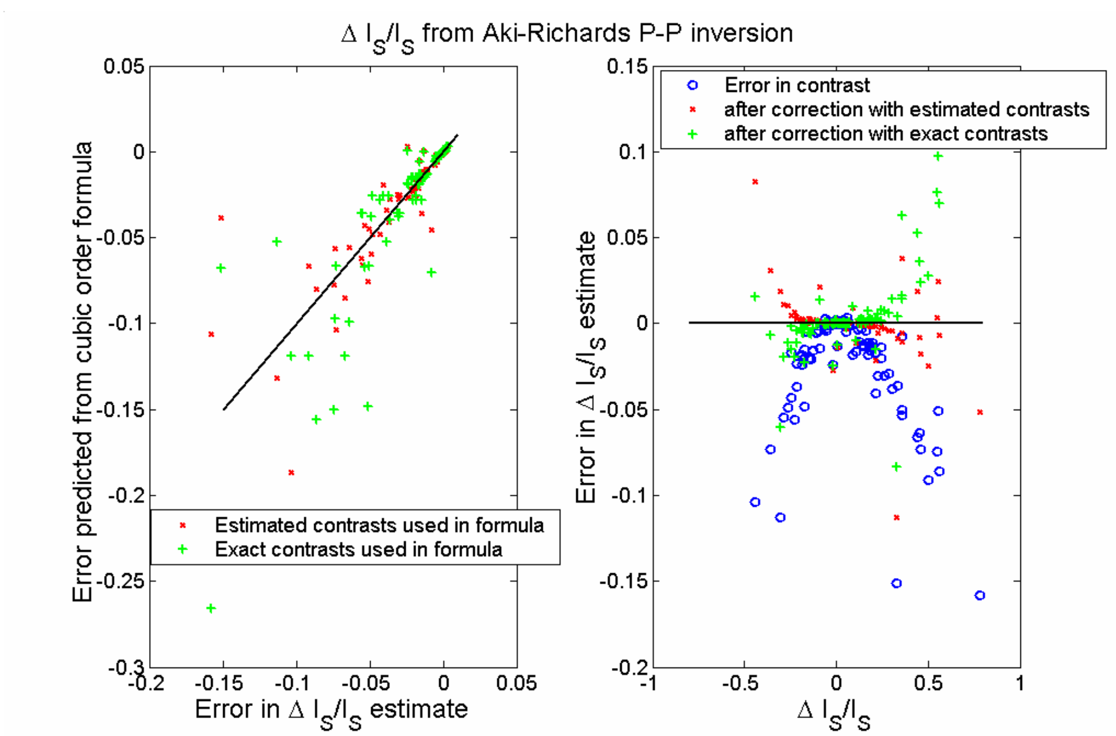


FIG. 3. This figure describes the errors of $\Delta I_S/I_S$ estimation by the Aki-Richards method from P-P data, and their comparison with Equation (3). (See caption of Figure 1 for detailed explanation.)

$\Delta I_S/I_S$ from P-S data by the Aki-Richards method

$$
\begin{aligned}
\left(\frac{\Delta I_S}{I_S}\right)_{AR}^{PS} - \left(\frac{\Delta I_S}{I_S}\right)_{exact}^{PS} &= \left(0.490439 - 0.94849\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha)^2 \\
&+ \left(0.228320 - 0.70856\gamma\right)(\Delta\rho/\rho)(\Delta\alpha/\alpha)^2 \\
&+ \left(-0.0379881 + 0.082764\gamma\right)(\Delta\beta/\beta)^2(\Delta\alpha/\alpha) \\
&+ \left(-0.2831175800 + 0.56579\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha)(\Delta\rho/\rho) \\
&+ \left(-0.196180 - 0.59658\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha) \\
&+ \left(-0.178345 + 0.10753\gamma\right)(\Delta\alpha/\alpha)(\Delta\rho/\rho)^2 \\
&+ \left(-0.412965 + 0.32041\gamma\right)(\Delta\alpha/\alpha)(\Delta\rho/\rho) \\
&+ \left(0.0306486 - 0.065697\gamma\right)(\Delta\beta/\beta)^3 \\
&+ \left(0.043955 - 0.33627\gamma\right)(\Delta\beta/\beta)^2(\Delta\rho/\rho) \\
&+ \left(1.0374\gamma - 0.5163145000\right)(\Delta\beta/\beta)^2 \\
&+ \left(-0.1590305000 + 0.067181\gamma\right)(\Delta\beta/\beta)(\Delta\rho/\rho)^2 \\
&+ \left(-0.055185 - 0.38603\gamma\right)(\Delta\beta/\beta)(\Delta\rho/\rho) \\
&+ \left(0.000855638 - 0.0016825\gamma\right)(\Delta\rho/\rho)^3 \\
&+ \left(0.2264830900 - 0.45236\gamma\right)(\Delta\rho/\rho)^2
\end{aligned}
\tag{4}
$$

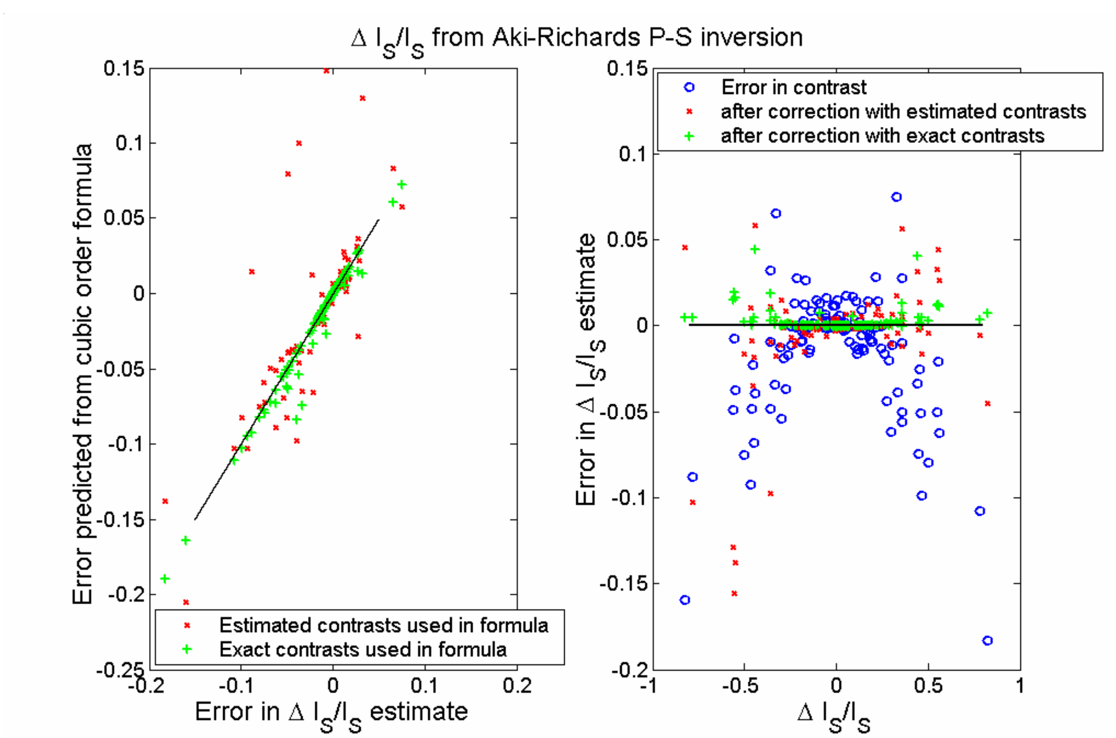


FIG. 4. This figure describes the errors of $\Delta I_S/I_S$ estimation by the Aki-Richards method from P-S data, and their comparison with Equation (4). (See caption of Figure 1 for detailed explanation.)

$\Delta I_S/I_S$ from P-P data by the Pseudo-Linear method

$$\begin{aligned}\left(\frac{\Delta I_S}{I_S}\right)_{PL}^{PP}-\left(\frac{\Delta I_S}{I_S}\right)_{exact}^{PP} &= \left(-1.106490000+2.7236\gamma\right)(\Delta\alpha/\alpha)(\Delta\beta/\beta)^2\\ &+\left(0.24502\gamma-0.3694500000\right)(\Delta\rho/\rho)(\Delta\alpha/\alpha)(\Delta\beta/\beta)\\ &+\left(0.0081200000-0.26532\gamma\right)(\Delta\rho/\rho)^2(\Delta\alpha/\alpha)\\ &+\left(-0.84513\gamma+0.4201669000\right)(\Delta\beta/\beta)^3\\ &+\left(0.64719\gamma-0.1990750000\right)(\Delta\rho/\rho)(\Delta\beta/\beta)^2\\ &+\left(-0.5217835000+0.049387\gamma\right)(\Delta\beta/\beta)^2\\ &+\left(0.50298\gamma-0.3765700000\right)(\Delta\rho/\rho)^2(\Delta\beta/\beta)\\ &+\left(-0.0864900000-0.82484\gamma\right)(\Delta\rho/\rho)(\Delta\beta/\beta)\\ &+\left(-0.0003680040000+0.00070693\gamma\right)(\Delta\rho/\rho)^3\\ &+\left(0.1959528800-0.39145\gamma\right)(\Delta\rho/\rho)^2\end{aligned}\tag{5}$$

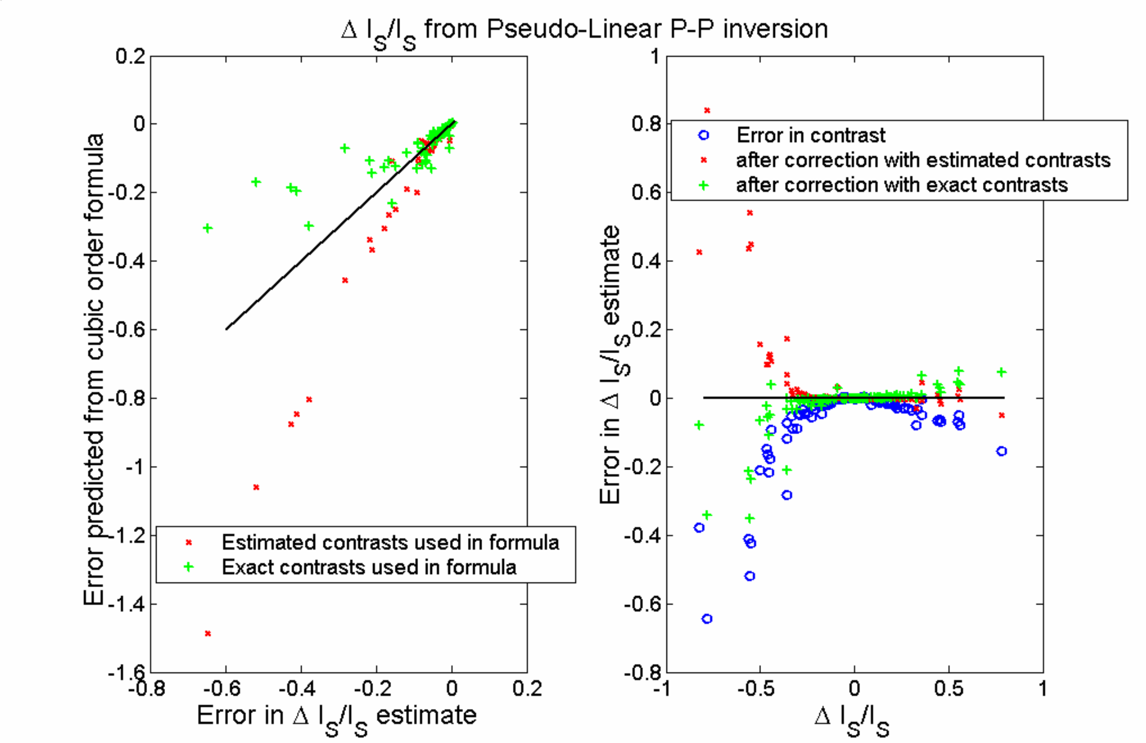


FIG. 5. This figure describes the errors of $\Delta I_S/I_S$ estimation by the Pseudo-linear method from P-P data, and their comparison with Equation (5). (See caption of Figure 1 for detailed explanation.)

$\Delta I_S/I_S$ from P-S data by the Pseudo-Linear method

$$\begin{aligned}\left(\frac{\Delta I_S}{I_S}\right)^{PS}_{PL}-\left(\frac{\Delta I_S}{I_S}\right)^{PS}_{exact} &= \left(0.664050-1.8209\gamma\right)(\Delta\beta/\beta)^2(\Delta\alpha/\alpha)\\
&+\left(0.071160-0.44218\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha)(\Delta\rho/\rho)\\
&+\left(-0.0979955-0.075209\gamma\right)(\Delta\alpha/\alpha)(\Delta\rho/\rho)^2\\
&+\left(0.0308484-0.066050\gamma\right)(\Delta\beta/\beta)^3\\
&+\left(0.043070-0.33464\gamma\right)(\Delta\beta/\beta)^2(\Delta\rho/\rho)\\
&+\left(1.0374\gamma-0.5163145000\right)(\Delta\beta/\beta)^2\\
&+\left(-0.1590305000+0.067181\gamma\right)(\Delta\beta/\beta)(\Delta\rho/\rho)^2\\
&+\left(-0.055185-0.38603\gamma\right)(\Delta\beta/\beta)(\Delta\rho/\rho)\\
&+\left(0.000855638-0.0016825\gamma\right)(\Delta\rho/\rho)^3\\
&+\left(0.2264830900-0.45236\gamma\right)(\Delta\rho/\rho)^2\end{aligned} \tag{6}$$

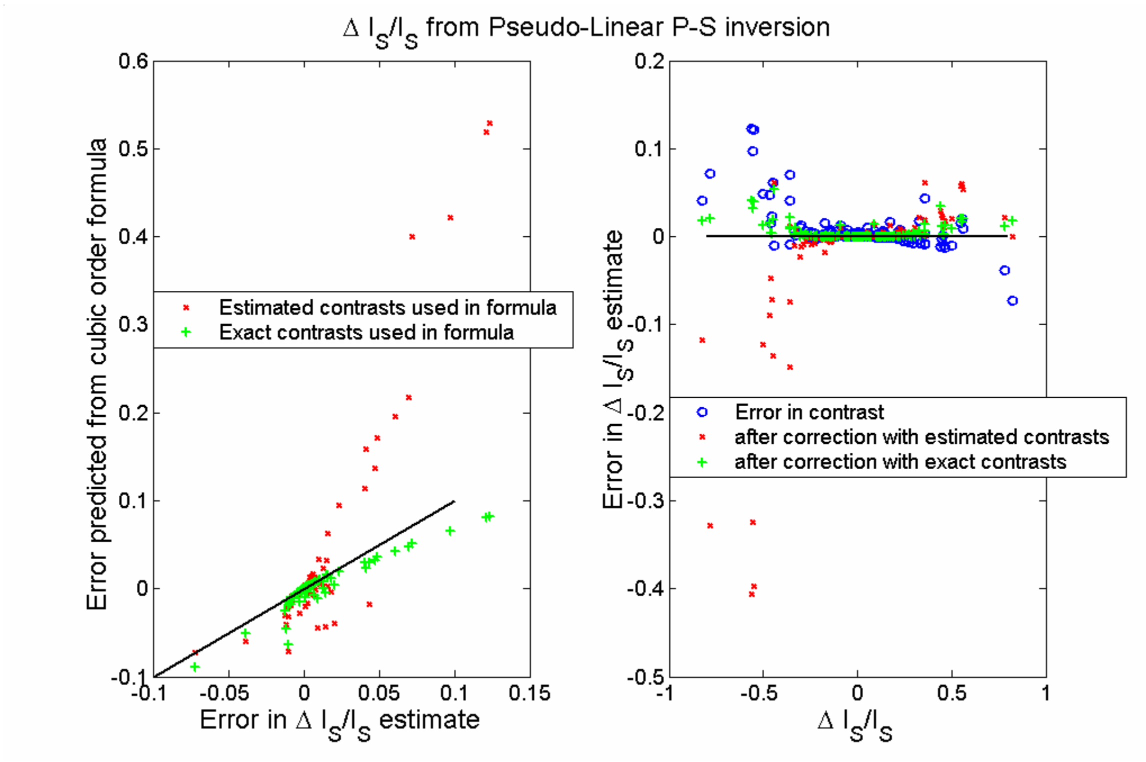


FIG. 6. This figure describes the errors of $\Delta I_S/I_S$ estimation by the Pseudo-linear method from P-S data, and their comparison with Equation (6). (See caption of Figure 1 for detailed explanation.)

$\Delta I_S / I_S$ from P-P data by the Pseudo-Quadratic method

$$
\begin{aligned}
\left(\frac{\Delta I_S}{I_S}\right)_{PQ}^{PP} - \left(\frac{\Delta I_S}{I_S}\right)_{exact}^{PP} &= \left(-0.84513\gamma + 0.4201669000\right)\left(\Delta\beta/\beta\right)^3 \\
&+ \left(0.64719\gamma - 0.199075\right)\left(\Delta\rho/\rho\right)\left(\Delta\beta/\beta\right)^2 \\
&+ \left(0.50298\gamma - 0.376570\right)\left(\Delta\rho/\rho\right)^2\left(\Delta\beta/\beta\right) \\
&+ \left(-0.000367998 + 0.00070689\gamma\right)\left(\Delta\rho/\rho\right)^3
\end{aligned}
\tag{7}
$$

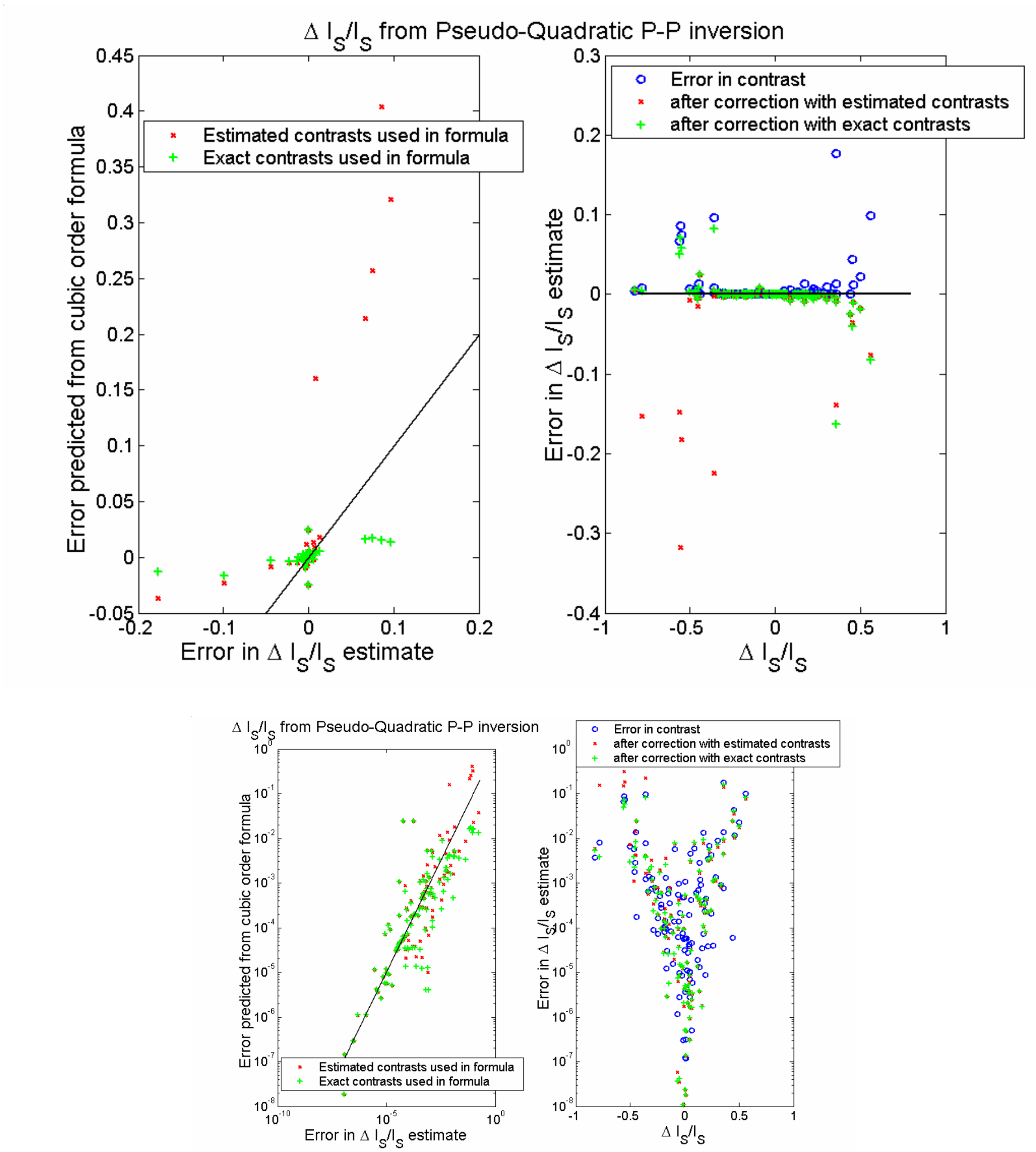


FIG. 7. This figure describes the errors of $\Delta I_S/I_S$ estimation by the Pseudo-quadratic method from P-P data, and their comparison with Equation (7). (See caption of Figure 1 for detailed explanation.) The lower panels display the same data but on log-log and semilog plots.

$\Delta I_S/I_S$ from P-S data by the Pseudo-Quadratic method

$$\left(\frac{\Delta I_S}{I_S}\right)_{PQ}^{PS}-\left(\frac{\Delta I_S}{I_S}\right)_{exact}^{PS}=\left(0.0308484-0.066050\gamma\right)(\Delta\beta/\beta)^3$$
$$+\left(0.043070-0.33464\gamma\right)(\Delta\beta/\beta)^2(\Delta\rho/\rho)$$
$$+\left(-0.1590305000+0.067181\gamma\right)(\Delta\beta/\beta)(\Delta\rho/\rho)^2$$
$$+\left(0.000855640-0.0016825\gamma\right)(\Delta\rho/\rho)^3 \tag{8}$$

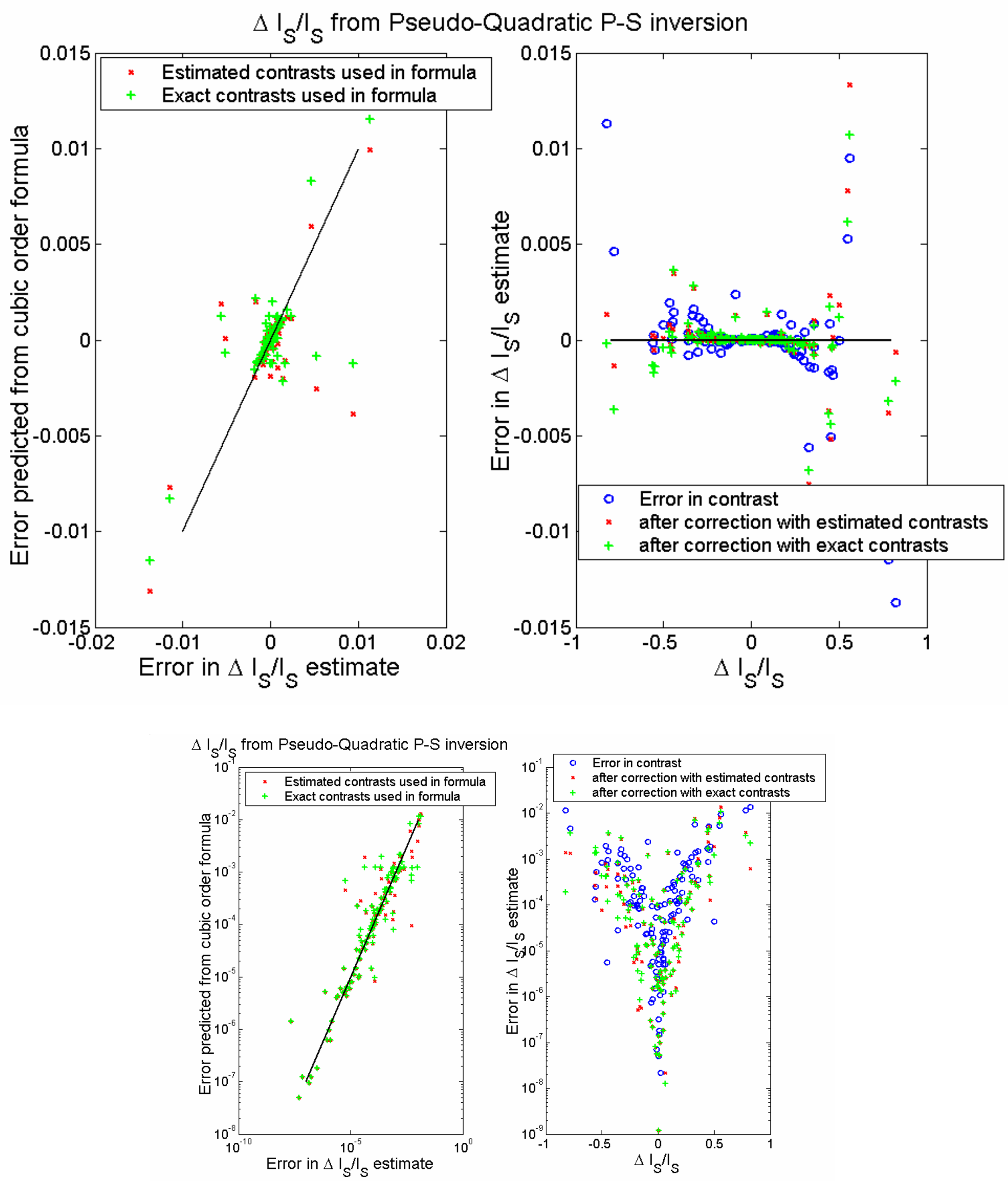


FIG. 8. This figure describes the errors of $\Delta I_S/I_S$ estimation by the Pseudo-quadratic method from P-S data, and their comparison with Equation (8). (See caption of Figure 1 for detailed explanation.) The lower panels display the same data but on log-log and semilog plots.

Δρ/ρ from P-P data by the Aki-Richards method

$$\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{AR}}^{\mathrm{PP}}-\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{exact}}^{\mathrm{PP}}=0.60128(\Delta\alpha/\alpha)^3$$
$$+(-6.8103\gamma+1.7026)(\Delta\alpha/\alpha)^2(\Delta\beta/\beta)$$
$$+(-3.4051\gamma+1.4526)(\Delta\rho/\rho)(\Delta\alpha/\alpha)^2$$
$$+(-3.2353+8.6936\gamma)(\Delta\alpha/\alpha)(\Delta\beta/\beta)^2$$
$$+(3.1450\gamma-1.7007)(\Delta\rho/\rho)(\Delta\alpha/\alpha)(\Delta\beta/\beta)$$
$$+(-0.078385-0.10846\gamma)(\Delta\rho/\rho)^2(\Delta\alpha/\alpha)$$
$$+(2.1887-5.2094\gamma)(\Delta\beta/\beta)^3$$
$$+(0.88819-1.6166\gamma)(\Delta\rho/\rho)(\Delta\beta/\beta)^2$$
$$+(-1.3353+3.7162\gamma)(\Delta\beta/\beta)^2$$
$$+(-0.0032124\gamma+0.0046488)(\Delta\rho/\rho)^2(\Delta\beta/\beta)$$
$$+(-0.42698+1.0271\gamma)(\Delta\rho/\rho)(\Delta\beta/\beta)$$
$$+(0.024367-0.047666\gamma)(\Delta\rho/\rho)^3$$
$$+(0.061323-0.013578\gamma)(\Delta\rho/\rho)^2 \qquad (9)$$

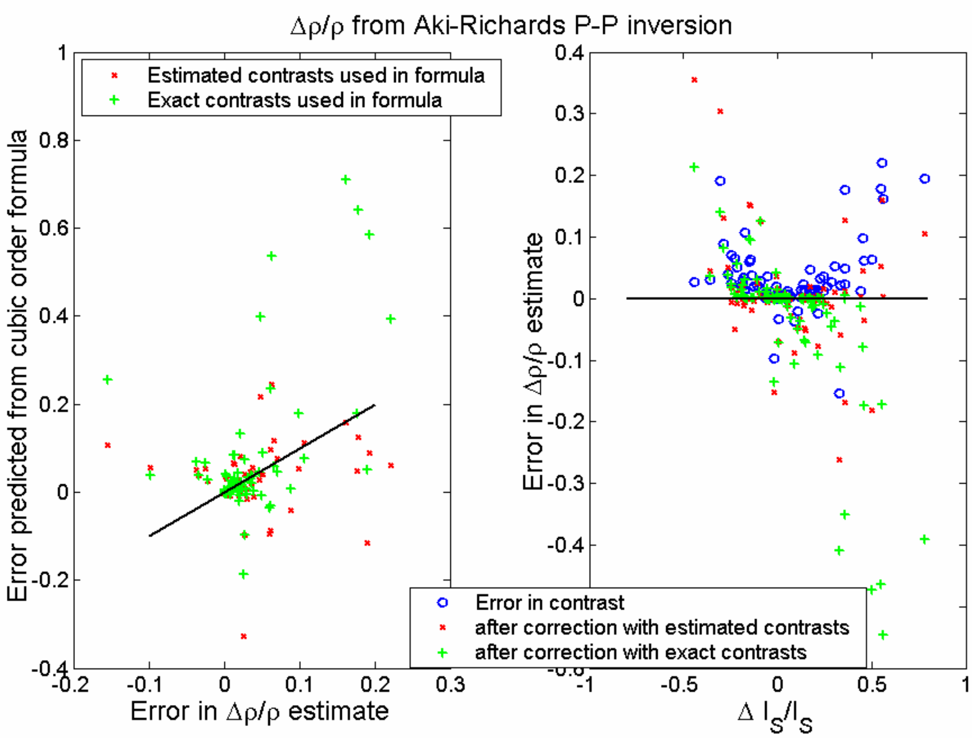


FIG. 9. This figure describes the errors of Δρ/ρ estimation by the Aki-Richards method from P-P data, and their comparison with Equation (9). (See caption of Figure 1 for detailed explanation.)

$\Delta\rho/\rho$ from P-S data by the Aki-Richards method

$$
\begin{aligned}
\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{AR}}^{\mathrm{PS}} - \left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{exact}}^{\mathrm{PS}} &= \left(-0.91190-0.097836\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha)^2 \\
&+\left(-1.1276+0.29550\gamma\right)(\Delta\rho/\rho)(\Delta\alpha/\alpha)^2 \\
&+\left(-0.24037+0.65761\gamma\right)(\Delta\beta/\beta)^2(\Delta\alpha/\alpha) \\
&+\left(-0.21531+1.5616\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha)(\Delta\rho/\rho) \\
&+\left(0.22179-1.6622\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha) \\
&+\left(-0.54678+0.81016\gamma\right)(\Delta\alpha/\alpha)(\Delta\rho/\rho)^2 \\
&+\left(-1.6062\gamma+1.1078\right)(\Delta\alpha/\alpha)(\Delta\rho/\rho) \\
&+\left(-1.6108\gamma+0.76113\right)(\Delta\beta/\beta)^3 \\
&+\left(-3.4160\gamma+1.3780\right)(\Delta\beta/\beta)^2(\Delta\rho/\rho) \\
&+\left(-1.9941+6.0649\gamma\right)(\Delta\beta/\beta)^2 \\
&+\left(-1.1220\gamma+0.62995\right)(\Delta\beta/\beta)(\Delta\rho/\rho)^2 \\
&+\left(2.3444\gamma-1.0605\right)(\Delta\beta/\beta)(\Delta\rho/\rho) \\
&+\left(0.039672\gamma-0.020204\right)(\Delta\rho/\rho)^3 \\
&+\left(0.21825-0.34404\gamma\right)(\Delta\rho/\rho)^2
\end{aligned} \tag{10}
$$

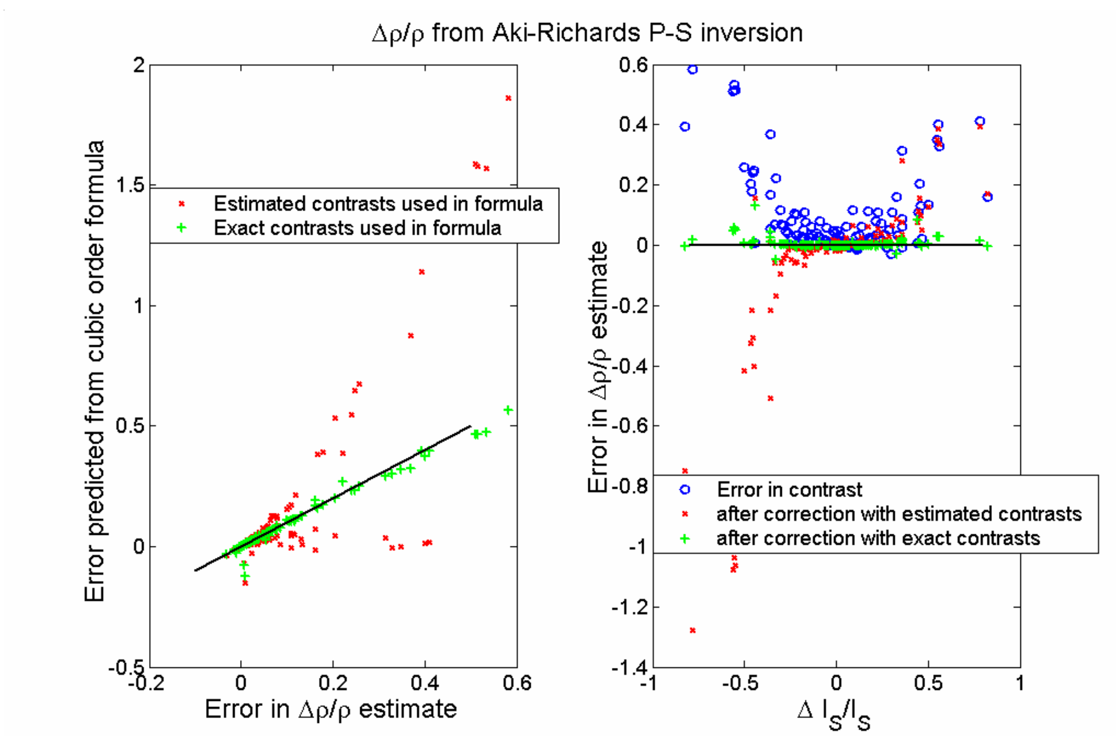


FIG. 10. This figure describes the errors of $\Delta\rho/\rho$ estimation by the Aki-Richards method from P-S data, and their comparison with Equation (10). (See caption of Figure 1 for detailed explanation.)

Δρ/ρ from P-P data by the Pseudo-Linear method

$$\begin{aligned}\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{PL}}^{\mathrm{PP}}-\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{exact}}^{\mathrm{PP}} &=\left(-3.2353+8.6936\gamma\right)(\Delta\alpha/\alpha)(\Delta\beta/\beta)^2\\
&+\left(3.1450\gamma-1.7007\right)(\Delta\rho/\rho)(\Delta\alpha/\alpha)(\Delta\beta/\beta)\\
&+\left(-0.078385-0.10846\gamma\right)(\Delta\rho/\rho)^2(\Delta\alpha/\alpha)\\
&+\left(2.1887-5.2094\gamma\right)(\Delta\beta/\beta)^3\\
&+\left(0.88819-1.6166\gamma\right)(\Delta\rho/\rho)(\Delta\beta/\beta)^2\\
&+\left(-1.3353+3.7162\gamma\right)(\Delta\beta/\beta)^2\\
&+\left(-0.0032124\gamma+0.0046488\right)(\Delta\rho/\rho)^2(\Delta\beta/\beta)\\
&+\left(-0.42698+1.0271\gamma\right)(\Delta\rho/\rho)(\Delta\beta/\beta)\\
&+\left(0.024367-0.047666\gamma\right)(\Delta\rho/\rho)^3\\
&+\left(0.061323-0.013578\gamma\right)(\Delta\rho/\rho)^2\end{aligned}\tag{11}$$

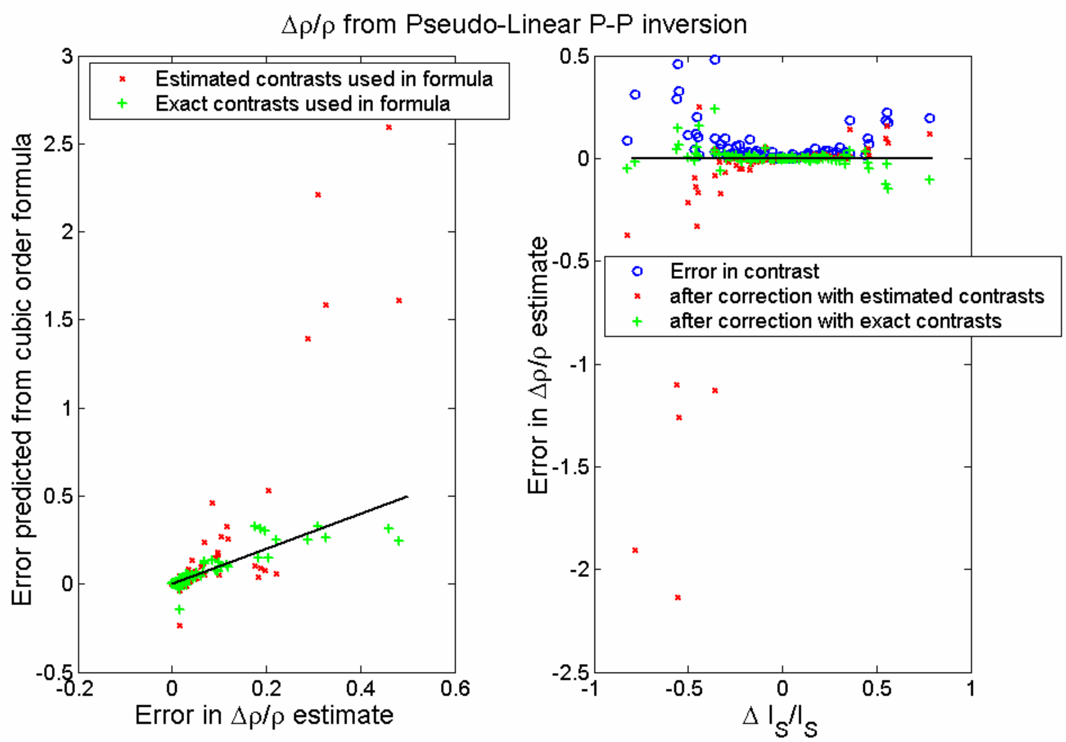


FIG. 11. This figure describes the errors of Δρ/ρ estimation by the Pseudo-linear method from P-P data, and their comparison with Equation (11). (See caption of Figure 1 for detailed explanation.)

Δρ/ρ from P-S data by the Pseudo-Linear method

$$\begin{aligned}\left(\frac{\Delta\rho}{\rho}\right)^{PS}_{PL}-\left(\frac{\Delta\rho}{\rho}\right)^{PS}_{exact} &= \left(1.2947-4.3078\gamma\right)(\Delta\beta/\beta)^2(\Delta\alpha/\alpha)\\ &+\left(0.92981-1.2352\gamma\right)(\Delta\beta/\beta)(\Delta\alpha/\alpha)(\Delta\rho/\rho)\\ &+\left(-0.60721+0.84688\gamma\right)(\Delta\alpha/\alpha)(\Delta\rho/\rho)^2\\ &+\left(-1.6175\gamma+0.76389\right)(\Delta\beta/\beta)^3\\ &+\left(-3.4027\gamma+1.3733\right)(\Delta\beta/\beta)^2(\Delta\rho/\rho)\\ &+\left(-1.9941+6.0649\gamma\right)(\Delta\beta/\beta)^2\\ &+\left(-1.1220\gamma+0.62995\right)(\Delta\beta/\beta)(\Delta\rho/\rho)^2\\ &+\left(2.3444\gamma-1.0605\right)(\Delta\beta/\beta)(\Delta\rho/\rho)\\ &+\left(0.039672\gamma-0.020204\right)(\Delta\rho/\rho)^3\\ &+\left(0.21825-0.34404\gamma\right)(\Delta\rho/\rho)^2\end{aligned} \tag{12}$$

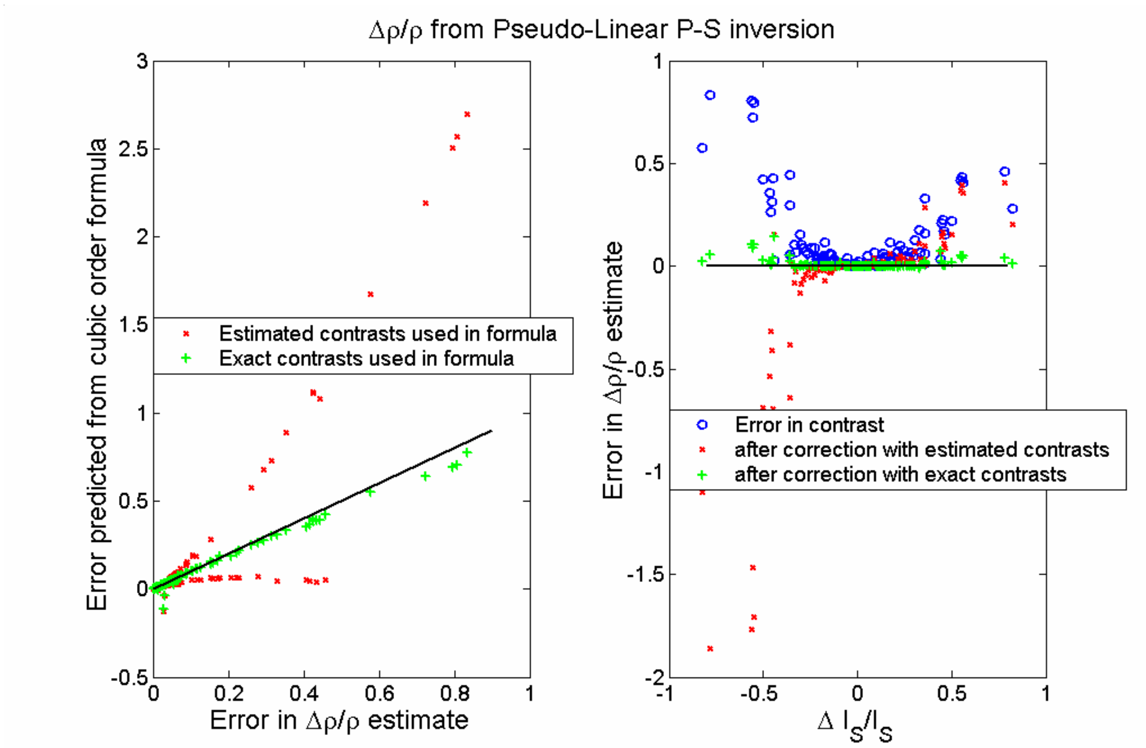
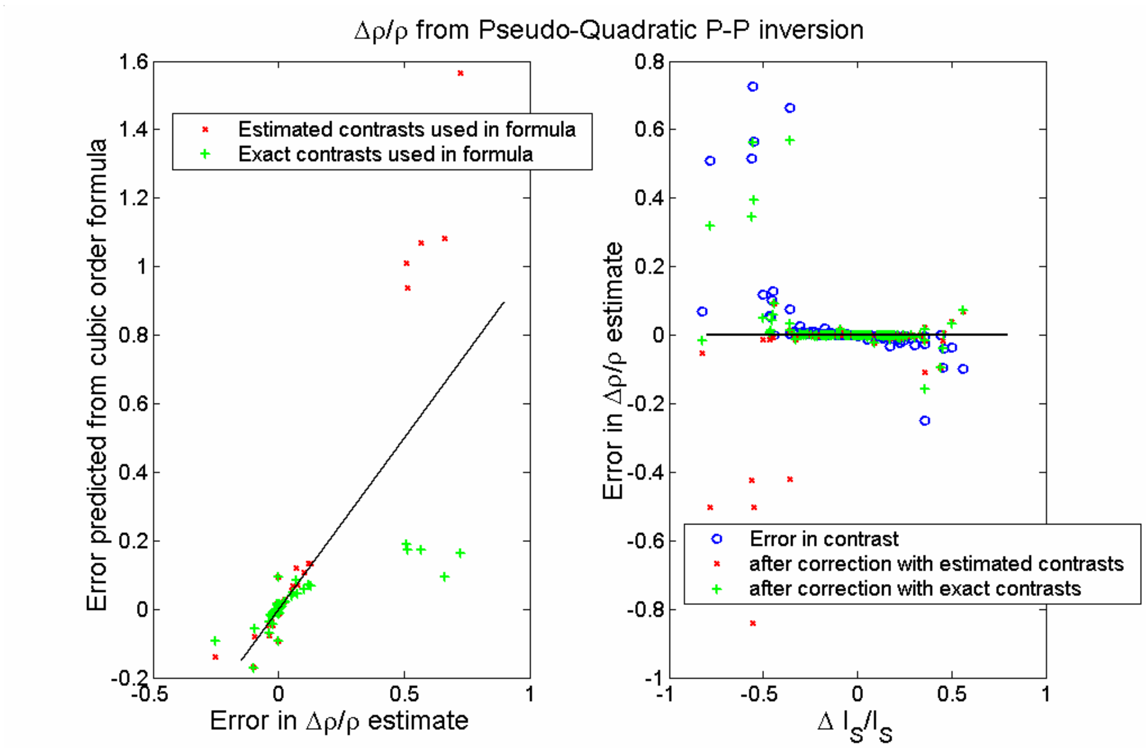


FIG. 12. This figure describes the errors of Δρ/ρ estimation by the Pseudo-linear method from P-S data, and their comparison with Equation (12). (See caption of Figure 1 for detailed explanation.)

Δρ/ρ from P-P data by the Pseudo-Quadratic method

$$\begin{aligned}\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{PQ}}^{\mathrm{PP}}-\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{exact}}^{\mathrm{PP}} &= \left(2.1887-5.2094\gamma\right)\left(\Delta\beta/\beta\right)^{3} \\ &+\left(0.88819-1.6166\gamma\right)\left(\Delta\rho/\rho\right)\left(\Delta\beta/\beta\right)^{2} \\ &+\left(-0.0032122\gamma+0.0046488\right)\left(\Delta\rho/\rho\right)^{2}\left(\Delta\beta/\beta\right) \\ &+\left(0.024367-0.047666\gamma\right)\left(\Delta\rho/\rho\right)^{3}\end{aligned} \tag{13}$$



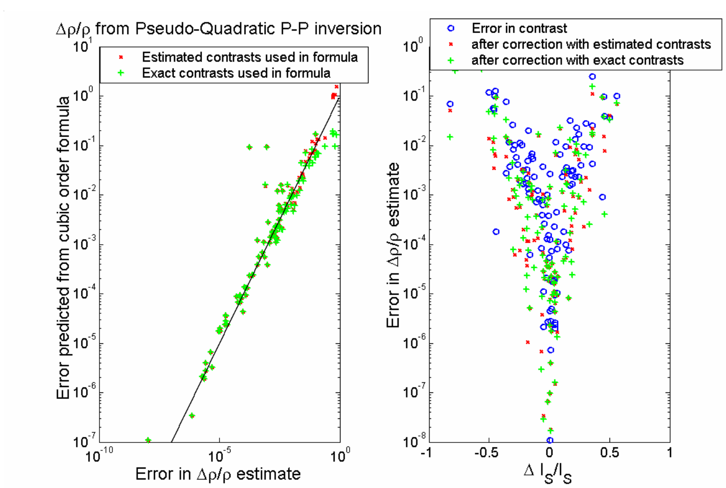


FIG. 13. This figure describes the errors of Δρ/ρ estimation by the Pseudo-quadratic method from P-P data, and their comparison with Equation (13). (See caption of Figure 1 for detailed explanation.) The lower panels display the same data but on log-log and semilog plots.

Δρ/ρ from P-S data by the Pseudo-Quadratic method

$$\begin{aligned}\left(\frac{\Delta\rho}{\rho}\right)^{\mathrm{PS}}_{\mathrm{PQ}} - \left(\frac{\Delta\rho}{\rho}\right)^{\mathrm{PS}}_{\mathrm{exact}} &= \left(-1.6175\gamma + 0.76389\right)(\Delta\beta/\beta)^3 \\ &+ \left(-3.4027\gamma + 1.3733\right)(\Delta\beta/\beta)^2(\Delta\rho/\rho) \\ &+ \left(-1.1220\gamma + 0.62995\right)(\Delta\beta/\beta)(\Delta\rho/\rho)^2 \\ &+ \left(0.039672\gamma - 0.020204\right)(\Delta\rho/\rho)^3\end{aligned} \tag{14}$$

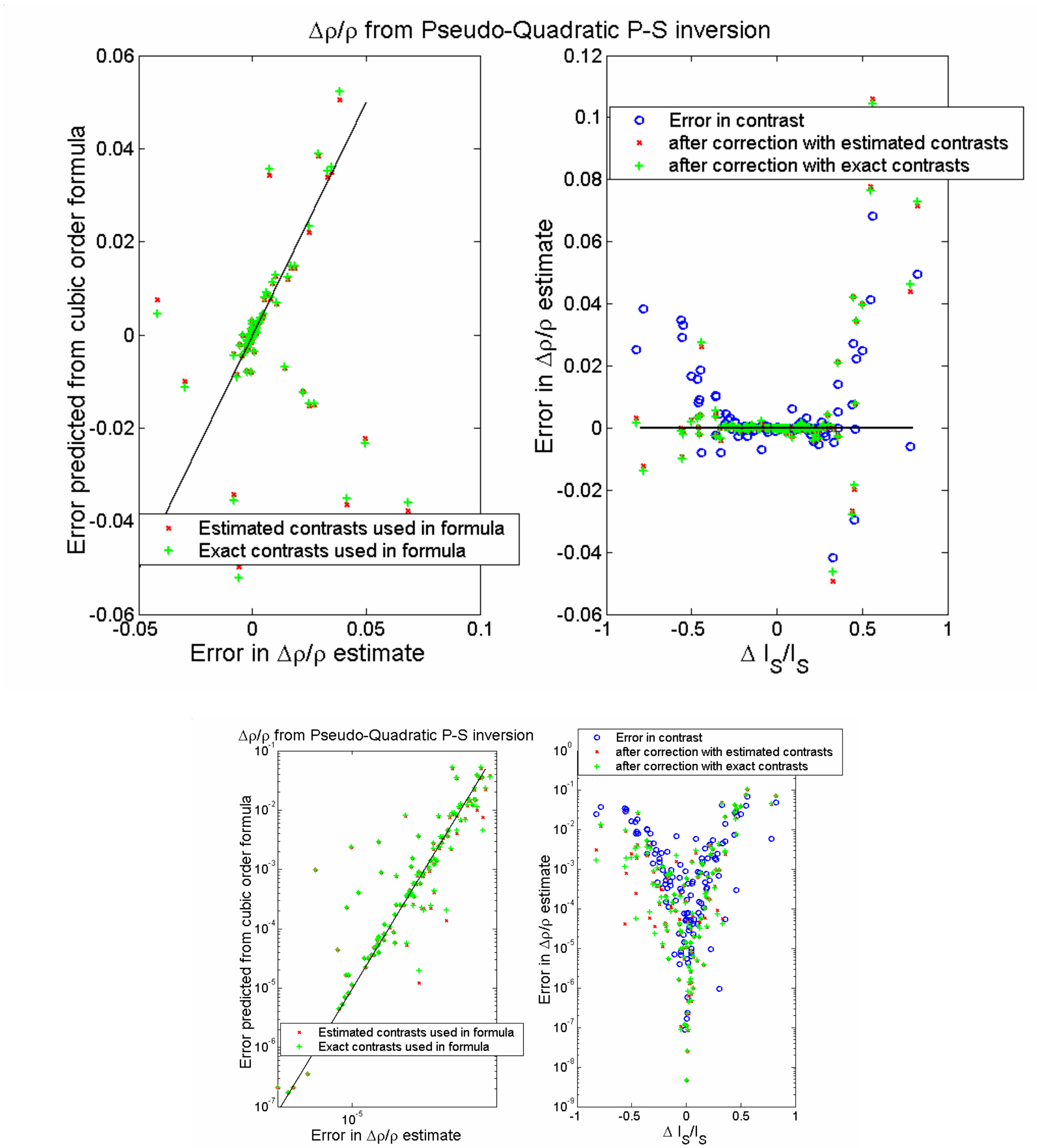


FIG. 14. This figure describes the errors of Δρ/ρ estimation by the Pseudo-quadratic method from P-S data, and their comparison with Equation (14). (See caption of Figure 1 for detailed explanation.) The lower panels display the same data but on log-log and semilog plots.

DISCUSSION

Do the derived error functions describe the actual errors realistically? Several of the error quantities seem well-described, such as $(\Delta I_{\mathrm{P}} / I_{\mathrm{P}})^{\mathrm{PP}}_{\mathrm{AR}}$, $(\Delta I_{\mathrm{S}} / I_{\mathrm{S}})^{\mathrm{PS}}_{\mathrm{AR}}$, $(\Delta I_{\mathrm{S}} / I_{\mathrm{S}})^{\mathrm{PS}}_{\mathrm{PL}}$, $(\Delta \rho / \rho)^{\mathrm{PS}}_{\mathrm{AR}}$, $(\Delta \rho / \rho)^{\mathrm{PP}}_{\mathrm{PL}}$, and $(\Delta \rho / \rho)^{\mathrm{PS}}_{\mathrm{PL}}$. The other quantities are usually well-described for small contrasts, but have noticeable outliers elsewhere. Of course higher-order terms in the error functions become more important for larger contrasts.

Are estimated contrasts more accurate after correction using the formulae? When exact contrasts are used in the formulae, the answer mirrors that of the previous question. **What if one employs estimated contrasts in the formulae?** For several of the quantities, the results seem similar to those obtained using exact contrasts. However the Aki-Richards and Pseudo-Linear P-S formulae, while quite accurate with the true contrasts, are considerably less accurate with estimated formulae. A more sophisticated application of the formulae could probably extract some of their inherent accuracy with only a knowledge of the estimated and background contrasts.

CONCLUSIONS

We have derived formal expressions for the error of contrasts estimated by three theoretical methods from both conventional and converted-wave data. They describe reasonably well the errors obtained by actual inversion.

The formulae show explicitly that errors in P-wave impedance contrasts are an order of magnitude smaller than other those of other contrasts.

One general observation is that errors are estimated quite accurately for extraction of contrasts by linear theories from converted-wave data. If practical use of the formulae is further developed, it will thus extend the accuracy of multicomponent AVO inversion.

REFERENCES

Ursenbach, C.P., 2003, Testing pseudo-linear Zoeppritz approximations: P-wave AVO inversion, CREWES 2003 Research Report.

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