Equivalent offset migration in anisotropic medium

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ABSTRACT

Equivalent offset migration (EOM) significantly improves the quality of time migrations, and is also very fast. It is shown in this paper that EOM is very robust in the presence of velocity perturbation (both due to inhomogeneities and anisotropy). A technique for extending EOM (depth) to anisotropic media is also proposed based on the shifted hyperbola NMO approach.

INTRODUCTION

Compensating for velocity anisotropy is a very important step in the process of imaging seismic data. Many workers have worked on the topic of imaging seismic data in anisotropic medium, including Alkhalifah (1997) and Grechka and Tsvankin (1999). Equivalent offset migration (EOM), which is a fast and easy prestack time migration technique, has been extended to imaging in an anisotropic medium (Bancroft and Vestrum, 1999). In this paper we show that the time formulation of EOM is very robust. We show that the effect of velocity anisotropy and other velocity perturbations on the accuracy of the EOM is negligible. This is not the case with depth migration. We propose an improved depth image formulation for inhomogeneous and anisotropic medium.

In this paper we assume that the non-hyperbolic nature of the moveout curve may include the effects of:

1. Intrinsic anisotropy of the medium;
2. Alternate high and low velocity layering;
3. Velocities varying with depth.

As stated above, the definition of anisotropy in this paper is not the same as used in literature. Therefore, it should be noted that Thomsen's anisotropy parameters ϵ and δ (Thomsen, 1986), which parameterize anisotropy, and the parameters used in this paper are not the same. In this paper, the effect of velocity perturbations on EOM in both the time domain and depth domain and a robust EO formulation will be presented which will handle the non-hyperbolic nature of the moveout curve.

EQUIVALENT OFFSET MIGRATION

Equivalent offset migration (EOM) has been developed by Bancroft et al. (1998). EOM can be defined as a prestack migration technique in which the EO gathers are formed at every common midpoint (CMP) and conventional velocity analysis is performed on these EO gathers. NMO correction is then applied to these gathers, and they are flattened and stacked. The advantages of EOM are a high signal-to-noise ratio (SNR) due to high fold coverage, and higher resolution velocity estimation.

EOM has been developed with the assumption of an isotropic constant velocity medium. This assumption of straight rays is only valid for small offsets in an inhomogeneous and anisotropic medium. In inhomogeneous and anisotropic medium this assumption of straight rays breaks down at higher offsets. In this paper we will try to analyze the effect of velocity perturbations (due to both inhomogeneity and anisotropy), which cause ray-bending, on the formulation of equivalent offset gathers in both the time and depth domains.

Equivalent Offset

Equivalent offset is defined by converting the double square-root equation, which defines the traveltime from a source to a scatterpoint and then back to the receiver on the surface, into a single square-root equation. Figure 1 shows a scatterpoint embedded in a medium and the ray paths to the scatterpoint from both shot and receiver. This conversion of the double square-root equation into a single square-root equation is accomplished by defining an equivalent collocated source and receiver position. The equivalent offset can be calculated using

$$h_e{}^2 = x^2 + h^2 - \frac{4x^2h^2}{Tv^2}, \tag{1}$$

where x is the distance between the midpoint and scatterpoint, h is the half shot-receiver offset, and v is the RMS velocity of the medium. Reiterating the fact that the straight-ray assumption is a RMS velocity assumption, we now investigate how the EO formulation behaves in the presence of velocity perturbations.

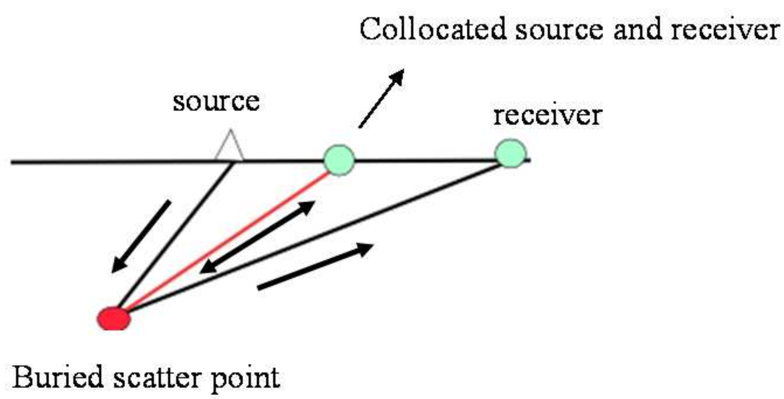

FIG. 1. Raypaths to the scatterpoint and collocated source and receiver.

Effect of velocity perturbations on equivalent offset

Equation 1 defines the calculation of the EO. The contribution due to velocity inhomogeneity and velocity anisotropy can be written as ($\frac{\partial h_e}{\partial v}$). Therefore, the new EO in inhomogeneous and anisotropic medium can be written as the EO in isotropic medium plus the perturbation term

$$h_{e_{aniso}} = h_e \pm \frac{\partial h_e}{\partial v}, \tag{2}$$

where $\frac{\partial h_e}{\partial v}$ is the contribution of the velocity perturbation due to anisotropy and inhomogeneity. It can be written as Equation 3. The derivative follows immediately by differentiating Equation 1 with respect to the velocity v:

$$\frac{\partial h_e}{\partial v} = \frac{4x^2h^2}{v^3 h_e T}. \tag{3}$$

It can be easily observed that the contribution of velocity perturbation to the EO calculated as given by Equation 3 is very small. The quantity $(\frac{\partial h_e}{\partial v})$ is very small when the velocity v is much greater than x. This condition is valid for most of the cases, associated with exploration seismology. When $x >> v$ the value of $(\frac{\partial h_e}{\partial v})$ is significant and cannot be ignored. This condition is usually associated with the shallow reflectors. This is not a big concern as we don't acquire with data large offsets for shallower times to begin with.

EQUIVALENT OFFSET MIGRATION IN DEPTH

We have shown that the EOM in the time domain is pretty stable in the presence of anisotropy. In time imaging, we are mostly concerned with focusing of energy. This objective is met quite well by the EOM. Depth imaging or migration of seismic data is a different matter all together. In depth migration, both the positioning and focusing of the events become important.

EOM (depth) anisotropic media

Chernis (1998) has shown that EOM can be easily extended to the depth domain. He showed that the traveltime tables generated using raytracing can be used to form the equivalent offsets. The time in an EO gather is given by

$$T = \sqrt{{T_0}^2 + \frac{4{h_e}^2}{{v_{ave}}^2}}, \tag{4}$$

where T_0 is the zero offset traveltime, v_{ave} is the average velocity and T is the traveltime calculated from the traveltime map. Using Equation 4, an expression for EO h_e can be written as

$$T = \sqrt{\frac{{z_0}^2 + 4{h_e}^2}{{v_{ave}}^2}}. \tag{5}$$

This equation can be rewritten as

$${h_e}^2 = {v_{ave}}^2 T^2 - {Z_0}^2, \tag{6}$$

where z_0 is the depth of the scatterpoint.

Some observations

The traveltime maps are generated without any assumptions, except for the high-frequency assumption, if ray theory was used for the computation. During this traveltime calculation, no assumption of constant velocity is being made, but Equation 4, which has been derived for constant velocity medium, is no longer valid in this medium as it is now both inhomogeneous and anisotropic. Figure 2 shows the ray-bending encountered in an anisotropic and inhomogeneous medium.

The ray-bending encountered in complex medium can be handled by using a higher order traveltime equation. The next section discusses these higher order formulations.

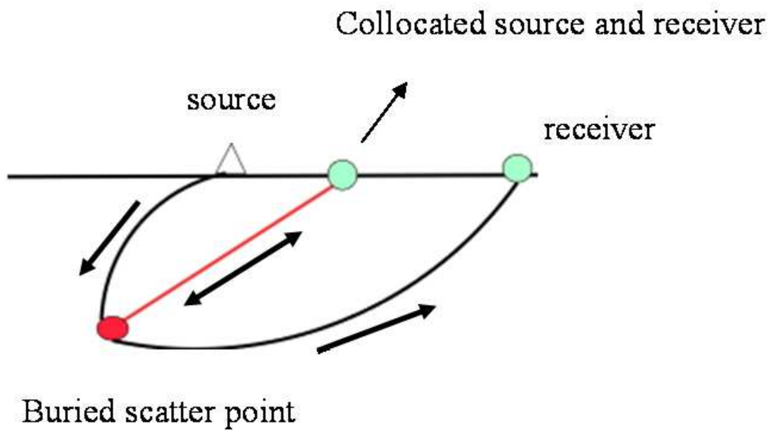

FIG. 2. Raypaths to the source and receiver in an inhomogeneous anisotropic medium.

Higher-order moveout equations

Initially, Taner and Koehler (1969) proposed a multi-term approximation for the traveltime vs. offset formulation for a multi-layered medium. They state that the two-term approximation by Dix (1955) is just a short-offset approximation. More terms are needed to fit the traveltime data at longer offsets. Many authors (Castle (1994); Alkhalifah and Tsvankin (1995)) have proposed different higher order traveltime vs. offset formulations.

We will be using the shifted hyperbola normal moveout approach proposed by Castle (1994) to extend the EO depth formulation to anisotropic medium.

SHIFTED HYPERBOLA

The shifted hyperbola equation (Equation 7) was proposed by Castle (1994). A comparison between shifted hyperbola and normal hyperbola is shown in Figure 3.

$$t^2 = \tau_s + \sqrt{{\tau_0}^2 + \frac{x^2}{{v_s}^2}} \tag{7}$$

Castle (1994) showed that the shifted hyperbola equation is accurate to the fourth order in offset. The advantages of shifted hyperbola formulation are as follows:

- Traveltimes calculated using shifted hyperbola NMO formulation are much more accurate than those obtained using Dix's equation.

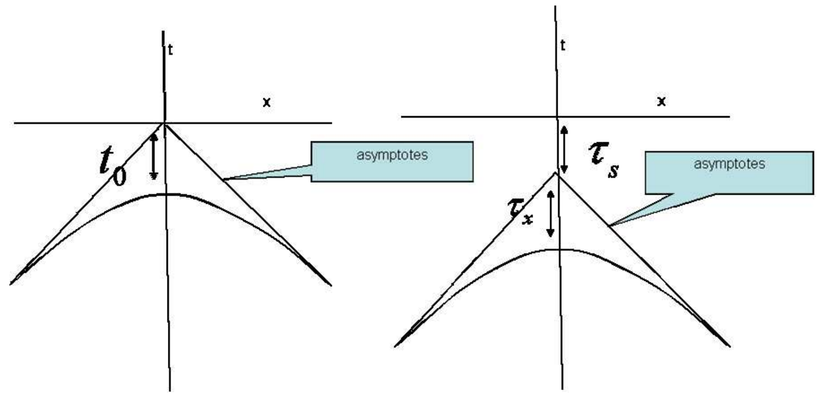

FIG. 3. Comparison of shifted hyperbola to normal hyperbola.

- Velocities estimated using this equation are much more accurate than those based on Dix's equation.

Equivalent offset migration in depth

Equation 5 can be applied to a complex medium, by extending this two-term equation into a three-term equation, which takes care of the ray-bending associated with the medium.

Equation 5 can therefore be written as a shifted hyperbola equation (Equation 7), which is a three-term equation. Since the shifted hyperbola equation is a three-term equation, it is a bit more complicated to apply to the traveltime data.

Therefore, this three-term equation is written as Equation 8, which can be fitted to the traveltime data using least-square fitting as shown by Castle (1994): i.e.,

$$at^2 + bt + c = x^2, \tag{8}$$

where

$$\tau_S = -\frac{b}{2a}, \tag{9}$$

$$\tau_0 = \frac{\sqrt{b^2 - 4ac}}{2a}, \tag{10}$$

$$v = \sqrt{a}, \tag{11}$$

$$t_0 = \tau_s + \tau_0, \tag{12}$$

$$S = 1 + \frac{\tau_s}{\tau_0}, \tag{13}$$

$$V = \frac{v}{\sqrt{S}}. \tag{14}$$

Thus, the equivalent offset in the depth domain for a complex medium can be written as a shifted hyperbola formulation, and easily applied to the medium. This is accomplished by writing the equivalent offset in depth (Equation 5) in the form of Equation 8. Equation 15 can now be used to calculate the equivalent offset in depth:

$$h_e{}^2 = at^2 + bt + c. \tag{15}$$

By knowing the values of v_{ave}, S, τ_s and τ_0, we can calculate h_e. The new equation for h_e, being a three-term equation, will be valid for the complex medium and will certainly improve the imaging of seismic data.

CONCLUSIONS

We have shown in this paper that equivalent offset migration technique is very robust in the presence of velocity perturbations in the time domain. In the depth domain, we propose a new equation to calculate equivalent offset to take care of velocity perturbations.

REFERENCES

Alkhalifah, T., 1997, Prestack time migration for anisotropic media, *in* 67th Ann. Internat. Mtg, Soc. of Expl. Geophys., 1583–1586.

Alkhalifah, T., and Tsvankin, I., 1995, Velocity analysis for transversely isotropic media: Geophysics, **60**, 1550–1566.

Bancroft, J., and Vestrum, R., 1999, Anisotropic depth migration using the equivalent offset method (EOM), *in* 69th Ann. Internat. Mtg, Soc. of Expl. Geophys., 1178–1181.

Bancroft, J. C., Geiger, H. D., and Margrave, G. F., 1998, The equivalent offset method of prestack time migration: Geophysics, **63**, 2041–2053.

Castle, R. J., 1994, Theory of normal moveout: Geophysics, **59**, 983–999.

Chernis, L., 1998, Depth migration by a generalized equivalent migration offset mapping, *in* 68th Ann. Internat. Mtg, Soc. of Expl. Geophys., 1550–1553.

Dix, C. H., 1955, Seismic velocities from surface measurements: Geophysics, **20**, 68–86.

Grechka, V., and Tsvankin, I., 1999, 3-D moveout inversion in azimuthally anisotropic media with lateral velocity variation: Theory and a case study: Geophysics, **64**, 1202–1218.

Taner, M. T., and Koehler, F., 1969, Velocity spectra - Digital computer derivation and applications of velocity functions: Geophysics, **34**, 859–881.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966.