Relative polarity of PP and PS events in the registration process and approximations to the PS reflection coefficient

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ABSTRACT

For most sedimentary interfaces, the PP and PS reflection coefficients, R_{PP} and R_{PS}, have opposite sign. In such cases, the corresponding PP and PS events have the same display polarity on their respective seismic sections. In processing and interpreting PS seismic sections, one normally relies on correlation of corresponding PS and PP events in the so-called registration process. Getting the polarity wrong on an event on the PS section can lead to a mistie of at least half a cycle between correlated events on the two sections and, consequently, over- or underestimated V_P/V_S ratios for the affected intervals. An unexpected polarity switch could conceivably even inhibit the correct correlation altogether and lead to a totally spurious correlation.

We examine under just what conditions an event will be recorded with opposite display polarities on PP and PS and show that the sign of R_{PS} depends on the sign of ΔU, where $U = \rho\beta^h$ and h is a function of P and S velocities as well as densities. In this analysis we derived an accurate small-angle R_{PS} approximation, one that we show is better than the Aki-Richards expression in many situations, not just for small angles.

INTRODUCTION

The SEG polarity standard (Thigpen et al., 1975) implies, when using for display a minimum-phase wavelet from a compressive source, that a PP reflection from an interface with a *positive* PP reflection coefficient ($R_{PP} > 0$) will begin with a downward (negative) deflection on the recorded seismogram (Sheriff, 2002). Recommended SEG standards for horizontal-component geophones (Landrum et al., 1994) and subsequent proposed standards (Brown et al., 2002) imply that a PS reflection from the same source, when the interface has a *negative* PS reflection coefficient ($R_{PS} < 0$), will also lead to a downward (negative) deflection on the inline geophone on positive-offset traces. Negative-offset traces, which are flipped in preprocessing, originally have the opposite polarity (see e.g. Tessmer & Behle, 1988; Brown et al., 2002). Therefore, because for most interfaces, R_{PP} and R_{PS} have opposite sign, an event usually has the same display polarity on PP and PS. In this paper, we study the 'unusual situation', where events on the two sections display opposite apparent polarities, i.e., where $R_{PS}/R_{PP} > 0$.

That sedimentary interfaces may fairly commonly exhibit opposite display polarities on PP versus PS, i.e., that R_{PP} and R_{PS} may have the same sign, has been shown abundantly by Vant (2003) for several lithologic-interface types. Figure 1 demonstrates this for two lithologic-interface examples of the many given by Vant (2003). In each example, a number of published values of the rock parameters, V_P, V_S and ρ, have been gathered from the literature, for both of the lithology types constituting the interface. These were then combined to simulate many possible examples of this type of lithologic interface. Of the pairs of reflection coefficients plotted (Figure 1), those points falling in quadrants 1 and 3 represent cases of the unusual situation, i.e., opposite PP-PS polarities.

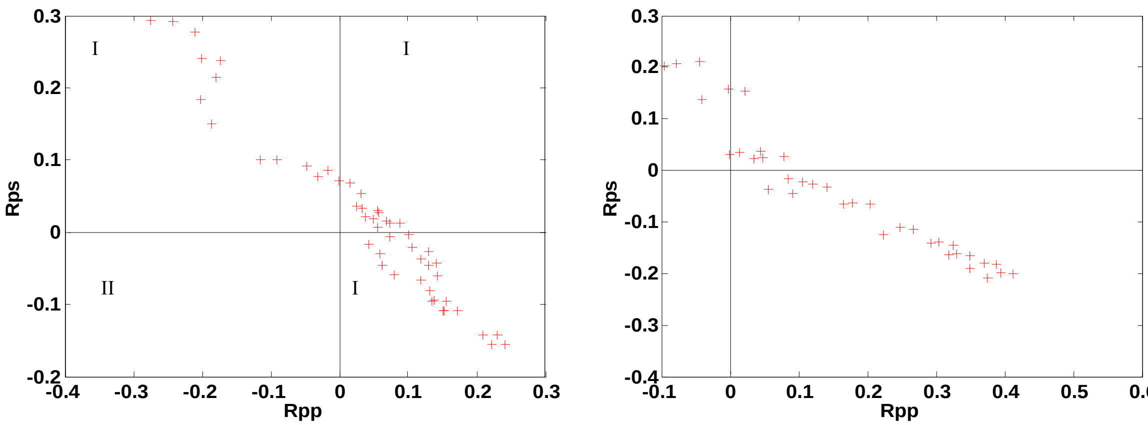


FIG. 1. Plots of R_{PP} versus R_{PS} for the following interfaces: wet sandstone over wet dolomite (left) and dry sandstone over dry limestone (right).

Preliminary (or first-order) estimates of V_P/V_S ratios, which are needed in the PS processing flow, can be obtained after the registration process in which PP and PS events are correlated. Typically, we generate synthetic seismograms using available log data for V_P, V_S and ρ from nearby wells – if available – then match PP and PS events on the field records with correlated PP and PS events on these synthetics. This correlation will usually have started with a zeroth-order V_P/V_S estimate, namely $V_P/V_S = 2$. First-order V_P/V_S ratios can then be calculated from traveltime ratios over selected intervals.

However, if nearby logs for V_S and/or ρ are not available, this procedure must be modified by estimating V_S and/or ρ , thereby introducing greater uncertainty. Without logs, ρ may be estimated from V_P using Gardner's empirical relationship (Gardner et al., 1974) or modified versions thereof for specific lithologies (Castagna et al., 1993). If V_S logs are unavailable, less reliable user-defined estimates of interval V_P/V_S ratios must be used to create PS synthetic stacks (Lawton & Howell, 1992), which could then be fine-tuned for optimal correlation with the stacked field data (Miller, 1996). But, as shown in Figures 2 and 3, without all the logs one could easily arrive at the wrong relative PP-versus-PS polarity for a particular event.

APPROXIMATIONS TO THE ZOEPPRITZ EQUATIONS

In adopting approximations for reflection amplitudes, we do not wish to restrict ourselves to low contrasts, or small changes in rock parameters, so we assume small angles of incidence. Such approximations should also be valid for high-contrast interfaces. We also assume the polarity relationship at small angles to be representative of the polarity relationship of the stacked events in all but the rarest of cases. Figure 4 shows a rather extreme case in which there is a polarity change in R_{PP}. However, it occurs at sufficiently large offset that the stacked-trace polarity is the same as the small-offset polarity, even though this stack includes some rather long-offset opposite-polarity traces.

Many approximations to the Knott-Zoeppritz equations governing P-SV waves at a welded interface have been derived (mainly for R_{PP} and R_{PS}), some of the earliest being

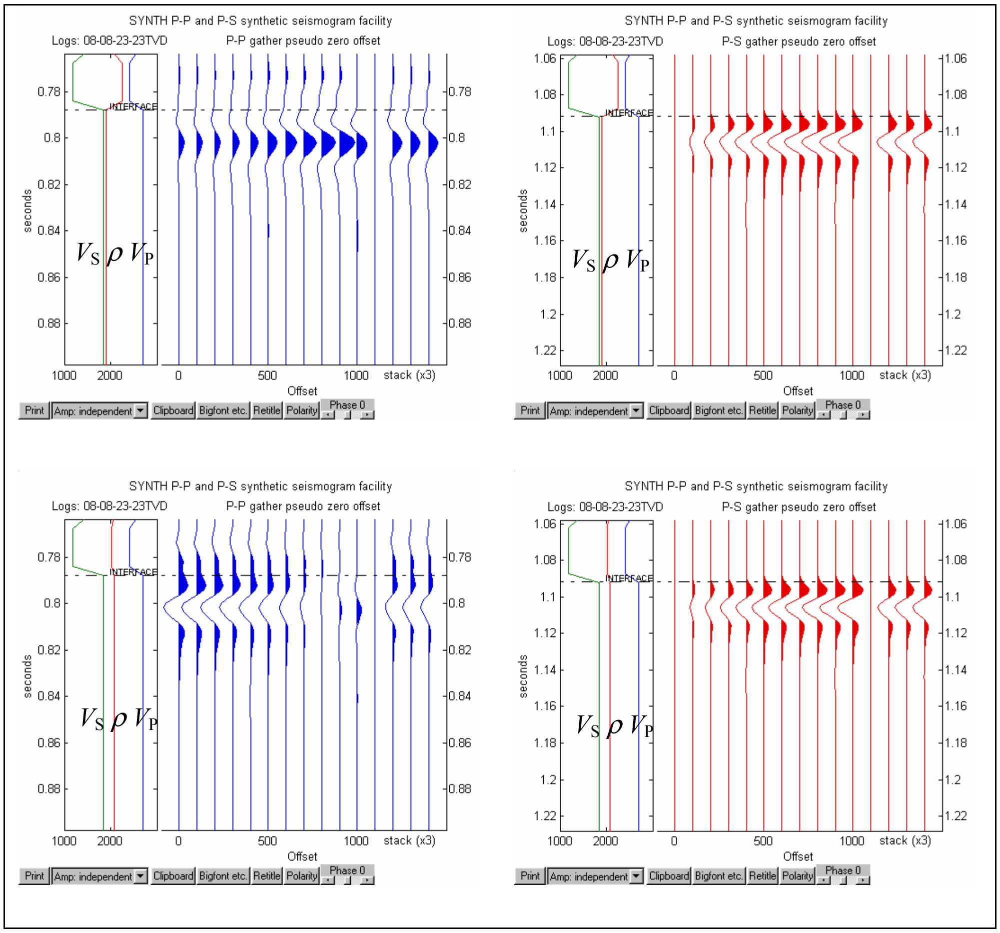


FIG. 2. AVO responses and synthetic stacks [P-P (blue) and PS (red)] for a model of a wet sand over a coal, an example of opposite PP-PS display polarities. In the upper figure, all three logs were used to model the interface. In the lower figure, for the scenario of a missing density log, Gardner's equation was used to supply a substitute for the density log.

those of Bortfeld (1961), Richards & Frasier (1976) and Aki & Richards (1980), whose notation we essentially follow. We use the symbol r to represent rock parameters generally (e.g. α, β, ρ, σ or μ) and we express any rock parameter and its change over an interface in terms of r (the average of r_1 and r_2) and Δr (the difference $r_2 - r_1$), where 1 and 2 denote media 1 and 2, respectively. Contrary to what has been implied by some authors, the definitions of r and Δr are exact and do not require any assumption of small parameter changes. However, caution is required in expressions like:

$$\Delta Z = \Delta(\rho\alpha) = \rho\Delta\alpha + \alpha\Delta\rho \qquad \text{and} \qquad \Delta\mu = \Delta\left(\rho\beta^2\right) = \rho\Delta\beta^2 + \beta^2\Delta\rho \tag{1}$$

(where μ is rigidity). Both are exact if one defines notation like β^2 or $\rho\alpha$ to be the average of the squares $\beta_2{}^2$ and $\beta_1{}^2$ or the average of the products $\rho_2\alpha_2$ and $\rho_1\alpha_1$, not the square of β (the average of β_1 and β_2) or the product of the averages ρ and α. The difference between these two is second-order in $\Delta\beta$ or in $\Delta\rho\Delta\alpha$, so no such caution is needed in first-order low-contrast theory.

For R_{PP}, many approximations have been published (e.g. Bortfeld, 1961; Richards & Frasier, 1976; Aki & Richards, 1980; Shuey, 1985; Zheng, 1991; Wang, 1999;

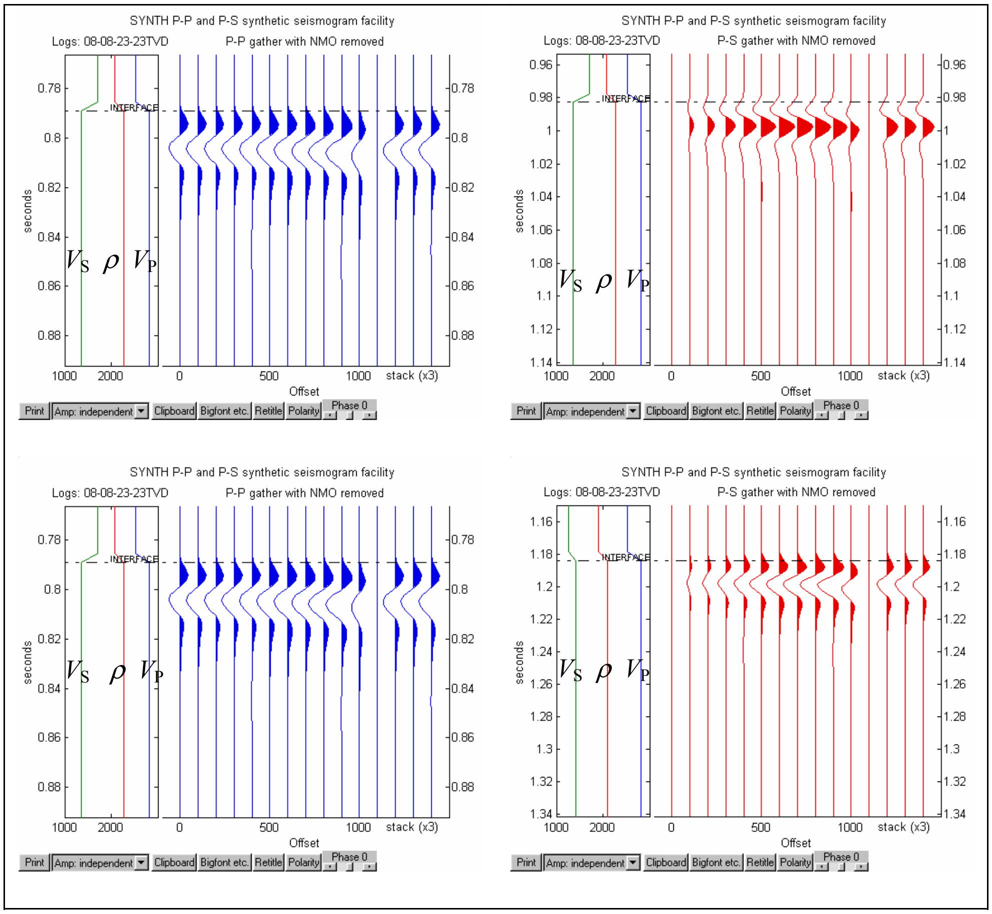


FIG. 3. AVO responses and synthetic stacks [P-P (blue) and PS (red)] for a model of a gas sand over a limestone, an example of opposite PP-PS display polarities. In the upper figure, all three logs were used to model the interface. In the lower figure, for the scenario of a missing shear-wave log, a V_P/V_S ratio of 2.0 was used to supply a substitute for the shear-wave log.

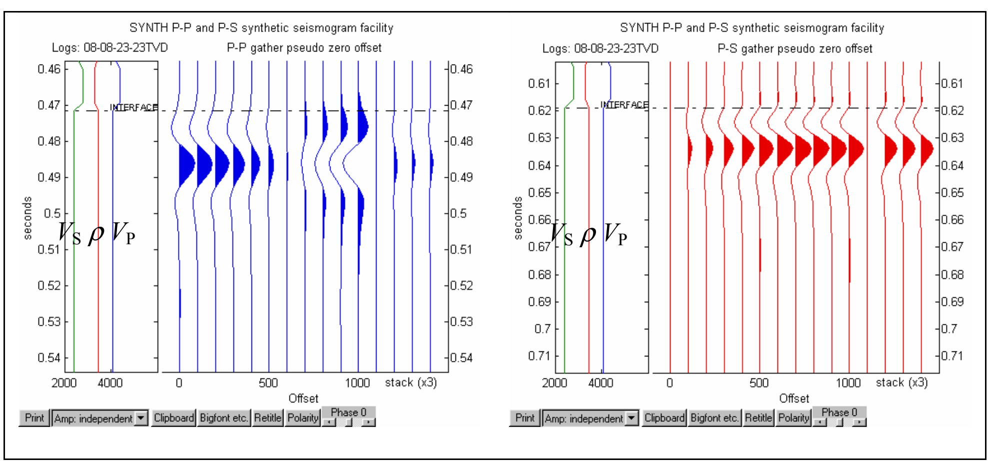


FIG. 4. AVO responses and synthetic stacks [P-P (blue) and PS (red)] for an interface model of water-saturated sandstone over chalk at a depth of 1000 m. The polarity of PP changes at a moderate offset but the stack retains the small-offset polarity.

Ursenbach, 2002) However, for R_{PP}, we use the zero-offset expression as sufficient for characterizing polarity. Some of the published R_{PS} approximations assume small parameter changes (Aki & Richards, 1980; Zheng, 1991; Xu & Bancroft, 1997; Gulati & Stewart, 1997; Donati & Martin, 1998; Ursenbach, 2002), some, like ours (below) assume small angles (Bortfeld, 1961; Richards & Frasier, 1976; Zaengle & Frasier, 1993; Wang, 1999; Ramos & Castagna, 2001; Carcuz, 2001; Geldart & Sheriff, 2004). It turns out that the differences between the two are not as great as one might think because in the Taylor expansions the terms of higher order in sin*i* tend also to be the terms of higher order in $\Delta r/r$.

OUR APPROXIMATION FOR R_{PS}

In deriving our own approximation to R_{PS} (Figure 5), we started with the exact formula given by Aki & Richards (1980, p. 150):

$$R_{\mathrm{PS}} = -2\left[\frac{\cos i_1}{\alpha_1}\left(ab + cd\,\frac{\cos i_2}{\alpha_2}\frac{\cos j_2}{\beta_2}\right)p\alpha_1\right]\Big/\left(\beta_1 D\right) \tag{2}$$

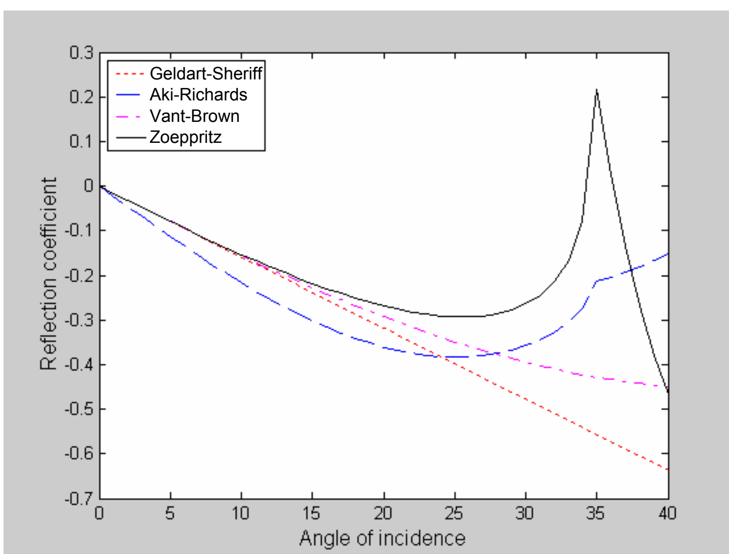
where p is horizontal slowness. The definitions of a, b, c, d and D (Aki & Richards, 1980) were reprinted by Xu & Bancroft (1997), Ramos & Castagna (2001) and Vant (2003), though Ramos & Castagna err in their expression for d. It should be:

$$d = 2\left(\rho_2\beta_2^2 - \rho_1\beta_2^2\right) \quad \text{and not} \quad d = 2\left(\rho_2\beta_2^2 p^2 - \rho_1\beta_2^2\right). \tag{3}$$

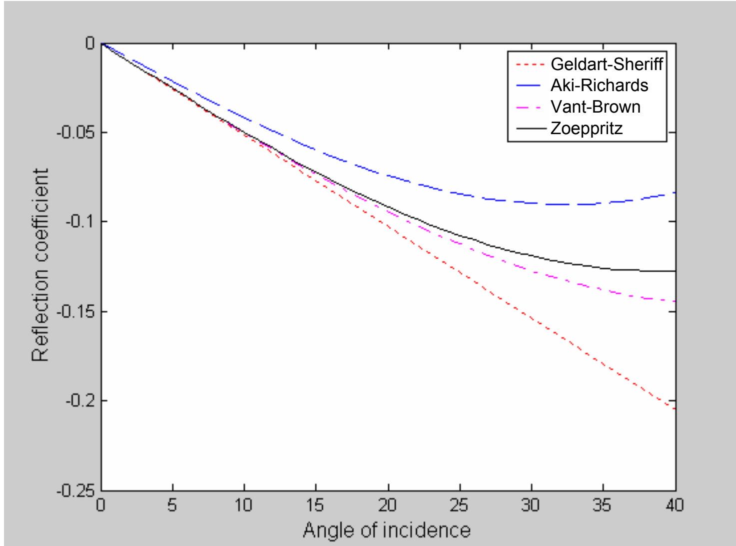
To get our R_{PS} approximation we first rewrite (2) as:

$$R_{\mathrm{PS}} = -\left[\sin 2i_1\left(ab + cd\,\frac{\cos i_2}{\alpha_2}\frac{\cos j_2}{\beta_2}\right)\right]\Big/\left(\alpha_1\beta_1 D\right) \tag{4}$$

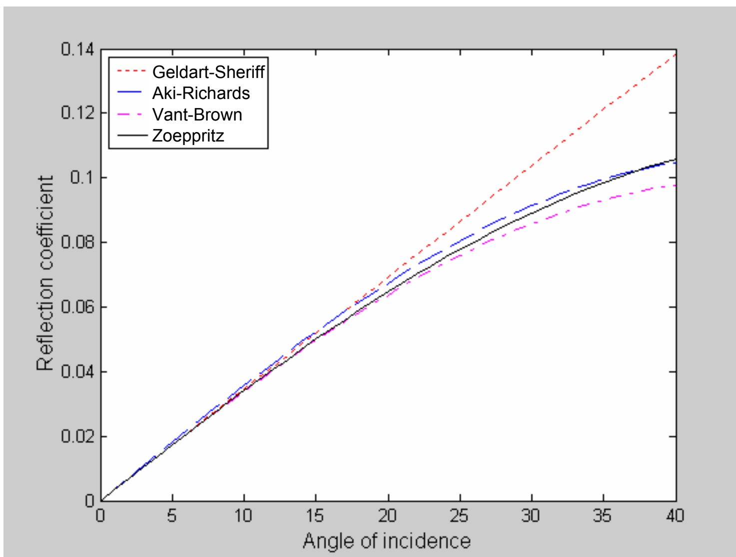
then apply the small-angle approximation by setting $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ for sines and cosines of all incidence angles, i_1, i_2, j_1 and j_2, in the expressions for a, b, c, d and D, giving:



(a)



(b)



(c)

FIG. 5. Comparison of the exact R_{PS} curve (solid black) with three approximations: our equation (5) (dashed magenta); Geldart & Sheriff (2004) (dotted red) [equivalent to (5) with $\sin 2i_1 \rightarrow 2i_1$], and Aki & Richards (1980) (coarsely dashed blue). The interface models are (a) young shale over old shale, (b) shale over gas-sand and (c) sandstone over salt (Brown et al., 2002).

$$R_{\mathrm{PS}} = \frac{-\sin 2i_1 \left(\alpha_2 \beta_2 \rho_2 \Delta\rho + 2\rho_1 \Delta\mu\right)}{\left(\rho_1 \alpha_1 + \rho_2 \alpha_2\right)\left(\rho_1 \beta_1 + \rho_2 \beta_2\right)}. \tag{5}$$

If we also approximate the explicit sine factor, it simply reduces from $\sin 2i_1$ to $2i_1$ and we have an expression equivalent to one given by Geldart & Sheriff (2004, p. 70). However this is only a slight simplification and it costs significantly in accuracy at moderate-to-large angles (Figure 5); so we usually choose to retain the explicit sine factor.

Wang (1999) started with the exact formulae for R_{PP} and R_{PS} (Aki & Richards, 1980) and developed Taylor-series expansions in powers of p, and therefore in powers of sin θ, or θ, i.e., small-angle approximations. He presents one R_{PS} approximation [his equation (C-3)] that is correct up to terms in p^5. However, to obtain a second simplified approximation [his equation (C-5)], Wang introduces two assumptions, one of which is quite unjustified [coming from his equation (A-10)]. For R_{PS} this amounts to assuming that $\Delta\rho = 0$, which eliminates one of the two first-order terms in his expressions, even though he retains other terms up to fifth order. In comparing the accuracy of Wang's and other R_{PS} approximations for reasonable interfaces, Vant (2003) found Wang's first fifth-order approximation [his (C-3)] to be extremely accurate but the second [his (C-5)] to be quite inaccurate. Truncation of Wang's two R_{PS} approximations after first order gives, for the first, an expression whose accuracy is about the same as that of our equation (5) but which is much more complicated; and for the second, an expression that is much less accurate than (5).

CONDITIONS FOR OPPOSITE PP AND PS POLARITIES

We shall use the two small-angle approximations to R_{PP} and R_{PS} to determine under what conditions we get opposite display polarities on the same event on PP versus PS data, that is, when $R_{PP}/R_{PS} > 0$. Thus, we start with our own approximation for R_{PS} [equation (5)] and the normal-incidence expression for R_{PP}:

$$R_{PP} = \frac{\rho_2\alpha_2 - \rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1}. \tag{6}$$

With respect to equation (6):

$$\operatorname{sgn} R_{PP} = \operatorname{sgn}\left(\rho_2\alpha_2 - \rho_1\alpha_1\right) = \operatorname{sgn}\left(\rho\Delta\alpha + \alpha\Delta\rho\right) = \operatorname{sgn}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right). \tag{7}$$

And with respect to equation (5):

$$\begin{aligned} \operatorname{sgn} R_{PS} &= -\operatorname{sgn}\left\{\left[\alpha_2\beta_2\rho_2 + \rho_1\left(\beta_2^2 + \beta_1^2\right)\right]\Delta\rho + 2\rho_1\left(\rho_2\beta_2 + \rho_1\beta_1\right)\Delta\beta\right\} \\ &= -\operatorname{sgn}\left[f\frac{\Delta\rho}{\rho} + g\frac{\Delta\beta}{\beta}\right] \end{aligned} \tag{8}$$

where $$f = \rho\left[\alpha_2\beta_2\rho_2 + \left(\beta_2^2 + \beta_1^2\right)\right] \quad \text{and} \quad g = 2\rho_1\beta\left(\rho_2\beta_2 + \rho_1\beta_1\right). \tag{9}$$

So, for opposite display polarities, or $R_{PP}/R_{PS} > 0$, we need either:

$$\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho} > 0 \text{ and } f\frac{\Delta\rho}{\rho} + g\frac{\Delta\beta}{\beta} < 0\text{; i.e. } -\frac{\Delta\alpha}{\alpha} < \frac{\Delta\rho}{\rho} < -\frac{g\Delta\beta}{f\beta} \text{ for } R_{PP} > 0 \text{ and } R_{PS} > 0 \tag{10}$$

or:

$$\frac{\Delta\alpha}{\alpha}+\frac{\Delta\rho}{\rho}<0 \text{ and } f\frac{\Delta\rho}{\rho}+g\frac{\Delta\beta}{\beta}>0\text{; i.e. } -\frac{g\Delta\beta}{f\beta}<\frac{\Delta\rho}{\rho}<-\frac{\Delta\alpha}{\alpha} \text{ for } R_{PP}<0 \text{ and } R_{PS}<0. \quad (11)$$

Equations (9) to (11) give conditions for occurrence of the unusual situation, that is, reversed display polarity of a reflection event on PS versus PP. We can formulate these conditions from (10) and (11) in another way by first noticing that:

$$\frac{\Delta\rho}{\rho}+\frac{g\Delta\beta}{f\beta}=\frac{\Delta\rho}{\rho}+\Delta\ln\beta^h=\frac{\Delta U}{U}, \quad \text{where} \quad h=\frac{g}{f} \quad \text{and} \quad U=\rho\beta^h. \quad (12)$$

Then we will get the unusual polarity situation, $R_{PP}/R_{PS} > 0$, if either:

$$\frac{\Delta U}{U}<0 \quad (\text{i.e. } R_{PS}>0) \quad \text{when} \quad \frac{\Delta Z}{Z}>0 \quad (\text{i.e. } R_{PP}>0) \quad (13)$$

or:

$$\frac{\Delta U}{U}>0 \quad (\text{i.e. } R_{PS}<0) \quad \text{when} \quad \frac{\Delta Z}{Z}<0 \quad (\text{i.e. } R_{PP}<0). \quad (14)$$

It seems to be ‘conventional wisdom’ for many that the sign of R_{PS} is determined by the sign of ΔY, where Y (= $\rho\beta$) is shear impedance. However, we have shown that it actually depends on the sign of ΔU, where $U = \rho\beta^h$. The exponent h, given by (12) and (9), involves not only β_1 and β_2, but also ρ_1, ρ_2 and α_2 – but not α_1 explicitly. The quantity U could be termed the converted-wave impedance, or PS impedance. We believe U to be a more fundamental and diagnostic parameter in converted-wave analysis than Y or Z. It should actually not be surprising that the amplitude of a reflection involving both P and S waves should not depend only on shear-wave velocities.

CONCLUSIONS

We have reaffirmed the possibility of opposite polarities on corresponding PP and PS reflections and some of the related pitfalls in the registration process. To quantify when this might occur, we have derived mathematical expressions that give the conditions for opposite PP and PS polarities in terms of the interface rock parameters. In the course of this work, we required an approximation to R_{PS} for small angles of incidence. The R_{PS} approximation we thus derived turns out to be more accurate than the Aki-Richards approximation, at least for three interfaces tested, and not just for small angles: beyond 25° incidence in two cases and beyond 40° in the third case.

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