Higher-order statistics: improved event resolution?

David C. Henley and Arnim B. Haase

ABSTRACT

In any area where near-surface earth structure causes one or more small time delays in seismic energy transmission, a seismic trace can be represented as a bandlimited reflectivity signal convolved with a short sequence of scaled copy spikes, each spike representing the delay due to a particular travel path variation. If the pattern (distribution) of spikes or static time shifts can be determined, its effects can be removed from the seismic trace by an explicit deconvolution operation. Accurate estimation of the spike pattern (static distribution function), however, is a key difficulty.

The cross-correlation between a multi-shifted seismic trace and a "pilot" reflectivity trace can be used to estimate the embedded static shift distribution function. For a trace consisting of the simple convolution of a reflectivity series and a short sequence of scaled static delay spikes, with no additive noise, the cross-correlation can perfectly extract the spike sequence. The cross-correlation is bandlimited by the mutual spectra of the input trace and pilot trace, however, and any noise present in the input time series degrades the resolution of the underlying spike sequence, the amount of degradation being inversely proportional to the bandwidth and to the S/N ratio. Hence, we seek an improved alternative to the cross-correlation as an event detector.

The cross-bicorrelation function is a so-called "higher-order statistic" which decomposes the correlation between events on two time series into a two-dimensional correlation function, of which selected individual profiles may show better resolution between closely spaced or weak events than the conventional cross-correlation. Furthermore, the cross-bicorrelation function can be spectrally "normalized", which increases its resolution further.

Preliminary work shows that individual slices of the cross-bicorrelation function can, indeed have greater resolution than a cross-correlation. Selecting the appropriate slice of the function requires extra knowledge or interpretation, however, and the presence of any substantial amount of noise degrades the cross-bicorrelation function, reducing its advantage over the cross-correlation. The frequency-normalized cross-bicorrelation, however, appears to provide improved resolution; and there are tricks which may help to preserve this resolution in the presence of noise.

INTRODUCTION

Motivation—statics deconvolution

Because of the nature of the transmission of seismic energy through the irregular near-surface sediments, it is known that the conventional statics model whereby a seismic trace can be corrected for near-surface effects by application of a single static time shift is only approximately correct for most seismic data and quite inappropriate for some data sets. Under the conventional statics model assumptions, an uncorrected seismic trace can be regarded as having been convolved with a single time-shifted unit spike which describes the single transmission delay of all seismic events through the surface weathered zone. In some near-surface settings, however, near-surface scattering can lead to an irregular pattern of energy arrivals along various neighbouring raypaths for any particular reflection. This motivates the extension of the single shift statics model to that of a seismic trace convolved with a small group, or distribution, of positive spikes, each with a slightly different shift. Each spike in this model represents the delay and signal strength along a particular raypath. This is the so-called "static distribution function" model. Such a model can simultaneously accommodate various raypath phenomena like multi-path transmission and interbed multiple reflection as well—anything that leads to the delayed reception of the basic reflection series at the geophone (see Fig. 1). Since there is no way to explicitly remove more than one time shift simultaneously from a seismic trace, we must consider the "distribution" of static shifts to be a wavelet to be removed from the seismic trace (Henley, 2004). Given a sufficiently accurate representation of the wavelet (or static distribution function), its effects can be removed from the seismic data via a match filter designed to convert the static distribution function to a single unshifted (but bandlimited) spike (see Fig. 2). This principle has been demonstrated on both model data and seismic field data (Henley, 2004). The key limitation of the method seems to be the accurate determination of the embedded static distribution function for each seismic trace. Methods tested thus far all rely on the cross-correlation function computed between a given trace and either a neighbouring trace or a "pilot" or model trace of some kind. Although it can be modified by various ad hoc algorithms to more closely resemble a "distribution function", the low inherent bandwidth of the cross-correlation limits the resolution of closely spaced (or low amplitude) "static spikes" convolved with a trace. This led us to explore alternative statistical methods for event detection.

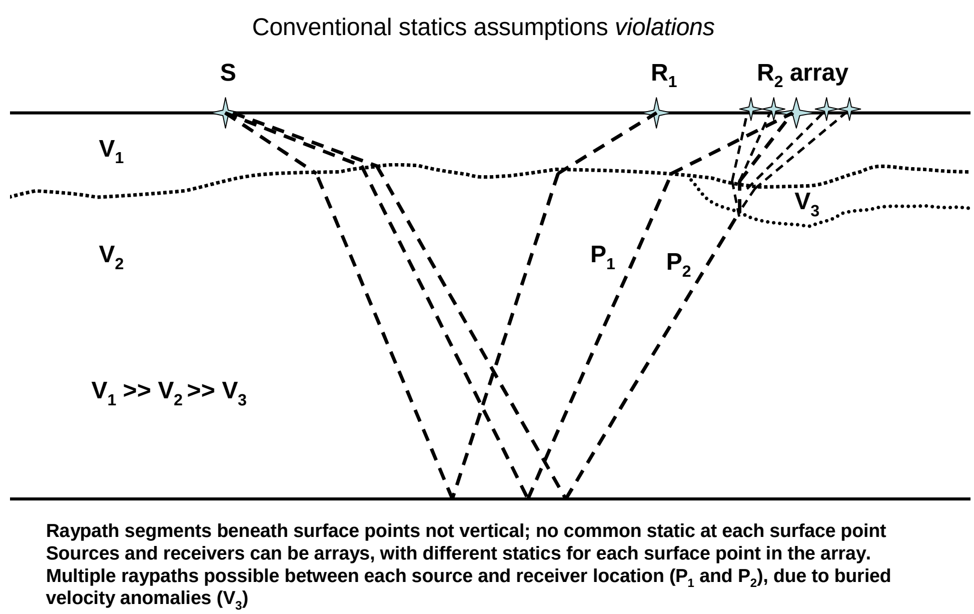
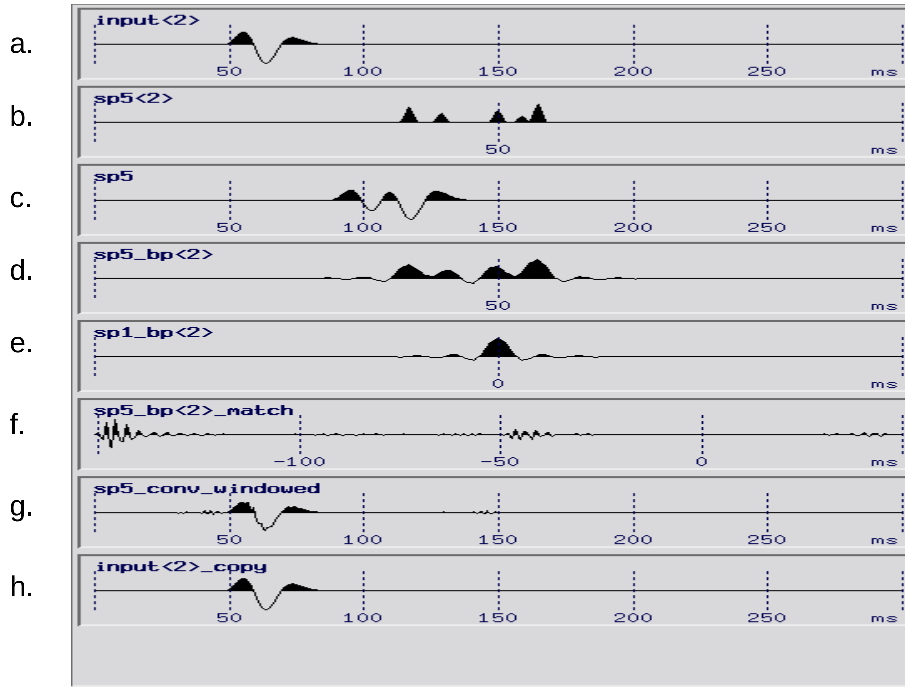


FIG. 1. Schematic showing some circumstances that can lead to more than one static on a seismic trace.



The static deconvolution principle

FIG. 2. The static deconvolution principle demonstrated. A waveform, or short reflectivity sequence in a) is convolved with a series of five scaled positive spikes, or “statics”, in b) to yield the multi-shifted and smeared waveform in c), the input seismic trace. A bandlimited representation of the static distribution in d), as might be obtained from the trace c) using an unspecified correlation technique, is used with a bandlimited spike at zero time, in e), to derive a match filter in f). The match filter is applied to the seismic trace in c) to obtain the unshifted reflectivity sequence in g), which can be compared with the original input in h). If even an imperfect representation of the static distribution function b) can be obtained, for instance, the severely bandlimited version in d), the effects of the statics can be removed from the seismic trace.

Higher-order statistics—the cross-bicorrelation function

The standard cross-correlation function is one of a class of “second-order statistics”, so-called because it involves multiplication of input samples to only the second power. As stated by Lu and Ikelle (2001), second-order statistics define the limit of resolution for time series only if the time series are Gaussian processes. Seismic traces are, for the most part non-Gaussian, however, and their details may fruitfully be resolved using suitable statistics higher than second order. One such function is the cross-bicorrelation, which basically constitutes a simultaneous two-way correlation of one time series with another time series and itself. We can define the conventional cross-correlation of two input time series, $x(t)$ and $y(t)$, as

$$r_{xy}(\tau) = E[x(t)y(t+\tau)], \tag{1}$$

where $E[\ ]$ is the expectation operator (in the discrete case, a normalized summation over the range of t). The cross-bicorrelation between the same two time series is then defined by

$$r_{xyx}(\tau_1,\tau_2) = E[x(t)y(t+\tau_1)x(t+\tau_2)]. \tag{2}$$

Each function has its corresponding Fourier transform, which in the case of the cross-correlation function is the cross-spectrum, $P_{xy}(\omega)$, defined as

$$P_{xy}(\omega) = E[X(\omega)Y^*(\omega)], \tag{3}$$

where $X(\omega) = F[x(t)]$ is the Fourier transform of $x(t)$ and Y^* is the complex conjugate of Y. For the cross-bicorrelation function, its Fourier transform is the cross-bispectrum, $B_{xyx}(\omega_1,\omega_2)$, given by

$$B_{xyx}(\omega_1,\omega_2) = E[Y(\omega_1)X(\omega_2)X(\omega_1+\omega_2)]. \tag{4}$$

Following the usual conventions, auto-correlation, auto-bicorrelation, and various auto-spectra can be defined simply by substituting the appropriate input time series and their corresponding indices.

The cross-bicorrelation (Eq. 2) is a function of both τ_1 and τ_2, independent time shift variables. This means that the function itself forms a two-dimensional surface with various correlation peaks located at positions whose coordinates are discrete values of τ_1 and τ_2. These can be used to locate the particular events in the two input time series that contribute to the particular observed correlation peak. As well, the function can be summed (or *projected*) along either axis to yield a single one-dimensional output function. Summation over the variable τ_2 yields the autocorrelation of the input series $x(t)$, while summation over the variable τ_1 yields the cross-correlation of the two input time series. It should also be noted that there is a second, complementary cross-bicorrelation function, $r_{yxy}(\tau_2,\tau_1)$, in which the roles of the input time series $x(t)$ and $y(t)$ are reversed. In this case, summation over τ_1 gives the autocorrelation of the input series $y(t)$, while summation over τ_2 gives the cross-correlation of $y(t)$ and $x(t)$. Hence, for example,

$$r_{xx}(\tau_2) = \sum_{\tau_1} r_{xyx}(\tau_1,\tau_2), \text{ and} \tag{5}$$

$$r_{xy}(\tau_1) = \sum_{\tau_2} r_{xyx}(\tau_1,\tau_2). \tag{6}$$

Spectral normalization, or "whitening", can be used to extend the bandwidth of the correlation functions and hence potentially increase their resolution. To generate the normalized cross-correlation, $\gamma_{xy}(\tau)$, we start with the cross-spectrum $P_{xy}(\omega)$, divide it by the square root of the product of the individual auto-spectra, and take the inverse Fourier transform:

$$\gamma_{xy}(\tau) = F^{-1}\left[P_{xy}(\omega) \middle/ \sqrt{\{P_{xx}(\omega)P_{yy}(\omega)\}}\right]. \tag{7}$$

The normalized cross-bicorrelation, $\gamma_{xyx}(\tau_1,\tau_2)$, is created in a similar way by dividing the cross-bispectrum by the auto-bispectrum, then inverse Fourier transforming:

$$\gamma_{xyx}(\tau_1,\tau_2) = F^{-1}\left[B_{xyx}(\omega_1,\omega_2) / B_{xxx}(\omega_1,\omega_2)\right]. \tag{8}$$

The normalized cross-bicorrelation function can, like its unnormalized parent, be summed over either of its time indices to yield a whitened version of either the autocorrelation or the cross-correlation. The latter is similar to the better-known coherence function.

IMPLEMENTATION

The cross-bicorrelation function (Eq. 3) was coded in Fortran by A. Haase and tested in a rudimentary version on model data. Subsequently, the code was enhanced for use in the ProMAX package by D. Henley (2005), and the spectral normalization feature added. As a practical matter, since spectral division always carries the risk of instability due to the possible presence of very small amplitude values, a stability factor was inserted in the formula in equation 8:

$$\gamma_{xyx}(\tau_1,\tau_2) = F^{-1}\left[B_{xyx}(\omega_1,\omega_2) / \{B_{xxx}(\omega_1,\omega_2) + \lambda\}\right], \tag{9}$$

where λ is a fractional additive constant scaled by the peak value of the auto-bispectrum, $B_{xxx}(\omega_1,\omega_2)$.

As implemented, the cross-bicorrelation function creates an output trace gather for each pair of input traces selected. The number of traces in the gather equals the number of time lags, *n*, specified by the parameters. The number of live samples in each trace of the output gather is also *n*, since the cross-bicorrelation function is a two dimensional function by definition.

Figure 3 shows a bandlimited spike test model in which the first trace contains a single spike and all subsequent traces contain two spikes separated by increasing time delays. This model was used to demonstrate the utility of the cross-bicorrelation function for resolving closely spaced bandlimited events on seismic traces.

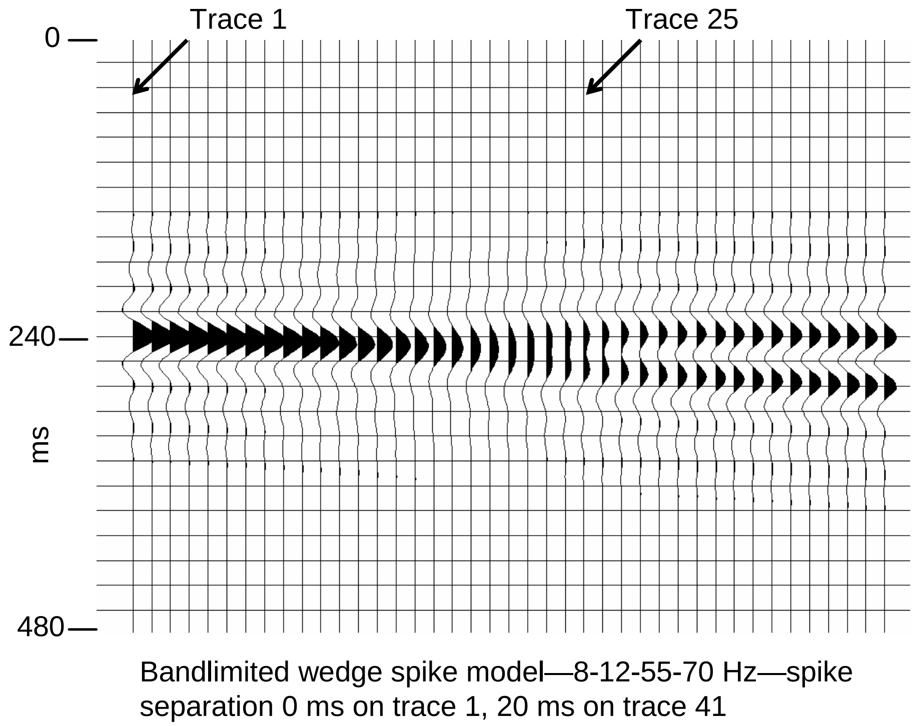
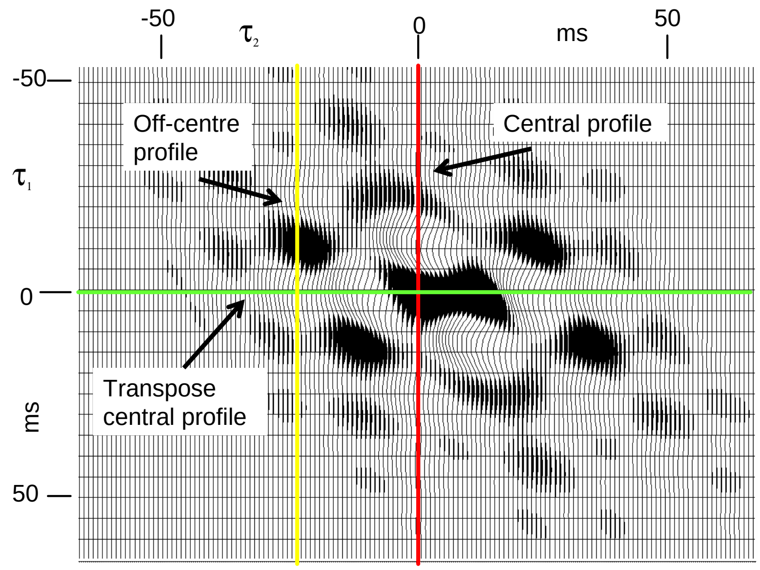


FIG. 3. A model used for testing the resolution capabilities of the cross-bicorrelation function. The event separation on trace 25 is just visible for the bandwidth used.

First, however, we illustrate some features of the function. The cross-bicorrelation function displayed in Figure 4 has been computed using traces 1 and 25 of the input model. While this display is in the usual wiggle/var mode, the function can also be usefully displayed in grey-shade or colour contour mode, as in Figures 5 and 6, respectively. In each of the figures, the symmetric pattern of peaks and valleys constitutes a decomposition of the relationship between the two input traces in terms of the relative shifts between their individual events. By careful study of the two input traces, it is possible to determine exactly which events on each trace are responsible for each of the correlation peaks of their cross-bicorrelation function.

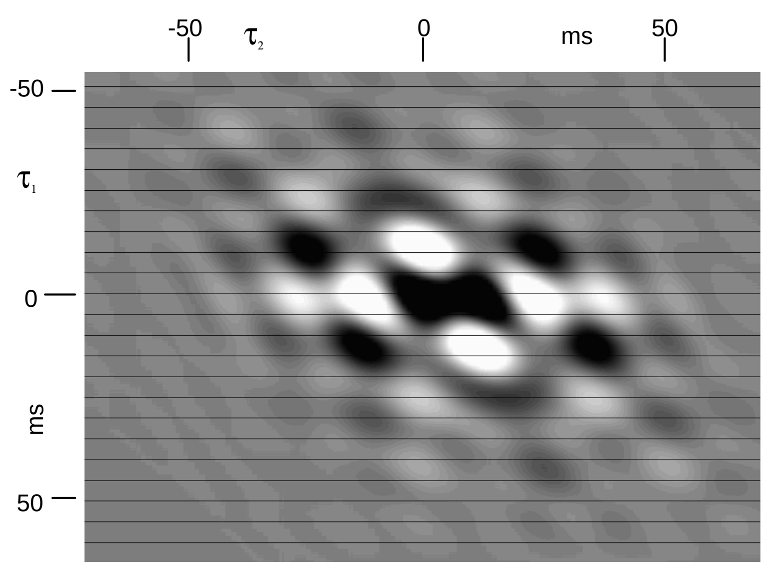
A note on terminology:

In what follows, the term "transpose" simply indicates that the cross-bicorrelation function displayed is the same as the "regular" function, except that the values have been reflected about the diagonal, and the τ_2 axis is now the left axis of the plot, while the τ_1 axis is the top axis, rather than vice versa.



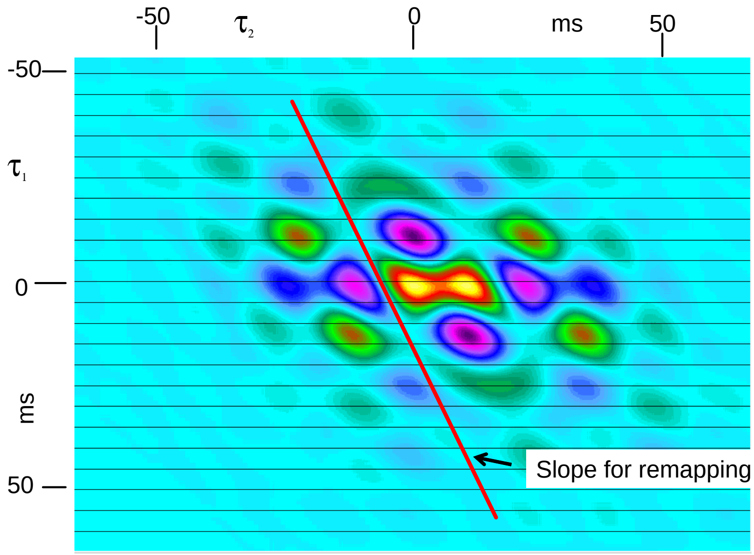
Cross-bicorrelation function of traces 1 and 25 of Figure 3

FIG. 4. The cross-bicorrelation function computed for traces 1 and 25 of the wedge model. The time shift variable τ_1 denotes the relative shift between the two copies of trace 1, while the time shift variable τ_2 denotes the relative shift between trace 1 and trace 25. The red line indicates the position at which the central autocorrelation profile is located; the green line marks the central cross-correlation profile, and the yellow line shows a possible choice for a profile through a secondary correlation peak.



Cross-bicorrelation function of traces 1 and 25 of Figure 3

FIG. 5. The cross-bicorrelation function for traces 1 and 25 of the wedge model displayed with a greyshade representation.



Cross-bicorrelation function of traces 1 and 25 of Figure 3

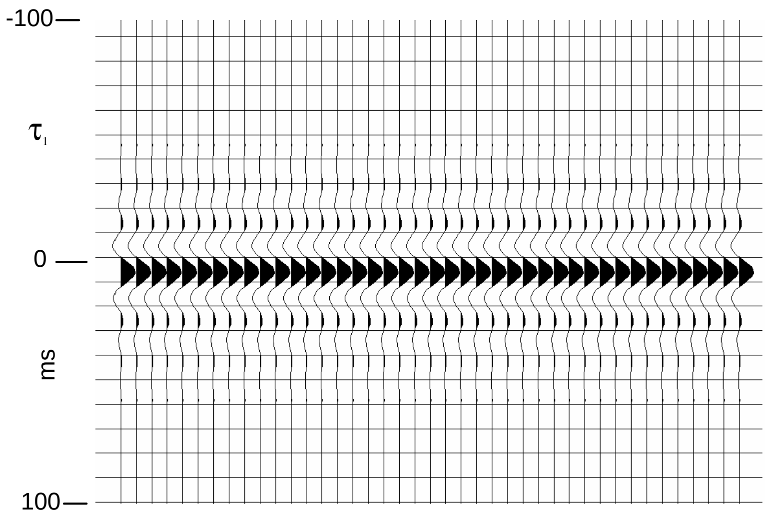
FIG. 6. The cross-bicorrelation function for traces 1 and 25 of the wedge model displayed in colour contour format. This representation best shows the relative heights of the various correlation peaks.

INTERPRETATION METHODS

“Slice” methods

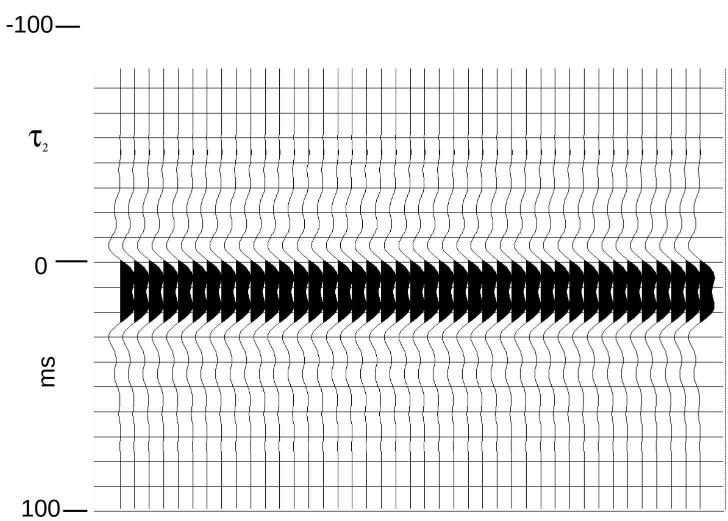
While the individual peaks in the cross-bicorrelation function can help identify the location and relative timing of contributory events, there are other ways in which to use the function for interpretation. The ones to be considered here can be classed according to two broad categories; “slice” methods, and “summation” methods. In the former, a single correlation profile, or “slice” is extracted from the 2-D cross-bicorrelation function along a trajectory defined in terms of the time lag coordinate axes. The “summation” methods, however, are characterized by a summation, along a specified direction, of some or all of the cross-bicorrelation function, both the summation trajectory and length being defined relative to the independent time lag coordinates of the function.

As an obvious example of profile extraction, consider the cross-bicorrelation function shown in Figures 4-6. If we extract and replicate the curve parallel to the τ_1 axis but passing through the centre (0,0) of the time lag axes, the result is shown in Figure 7; whereas the curve parallel to the τ_2 axis passing through (0,0) is as shown in Figure 8. In the first case, the curve resembles the autocorrelation of trace 1 of the bandlimited spike model of Figure 3; while in the second, the curve resembles the cross-correlation of traces 1 and 25. In fact, both of these curves are more sharply peaked (more highly resolved) than the actual cross-correlation or autocorrelation functions, and on these noiseless model data, might actually be preferable for analysis purposes. In a similar fashion, we can select curves from the cross-bicorrelation function parallel to either of the time lag axes, but passing through a specific correlation peak, instead of the time lag origin. Such curves can be used in an interpretive sense to highlight the correlation of relatively minor events in the input traces while de-emphasizing the more major correlations. Such a curve, replicated, is shown in Figure 9. Note here the relatively low correlation side lobes around the zero lag origin of this curve.



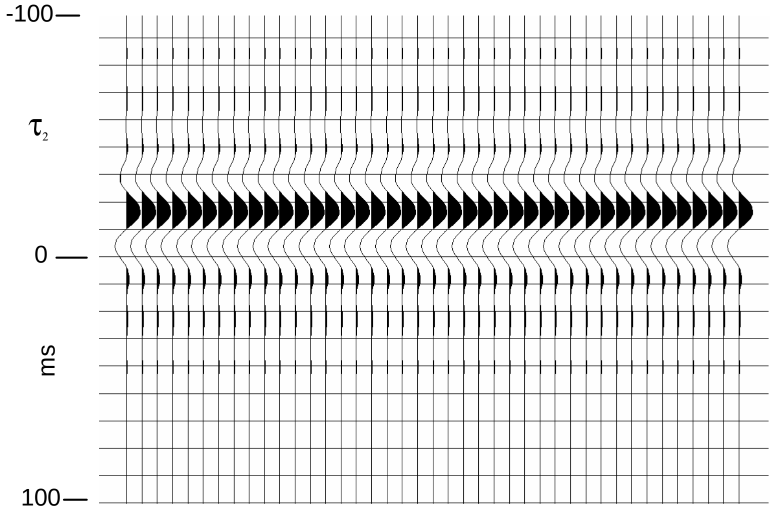
Central profile of the cross-bicorrelation function in Figure 4

FIG. 7. This is the central profile (or trace) of the cross-bicorrelation function in Figure 4, repeated. Since it runs parallel to the τ_1 direction, it is similar to the autocorrelation function; and it might properly be termed a "marginal" autocorrelation function.



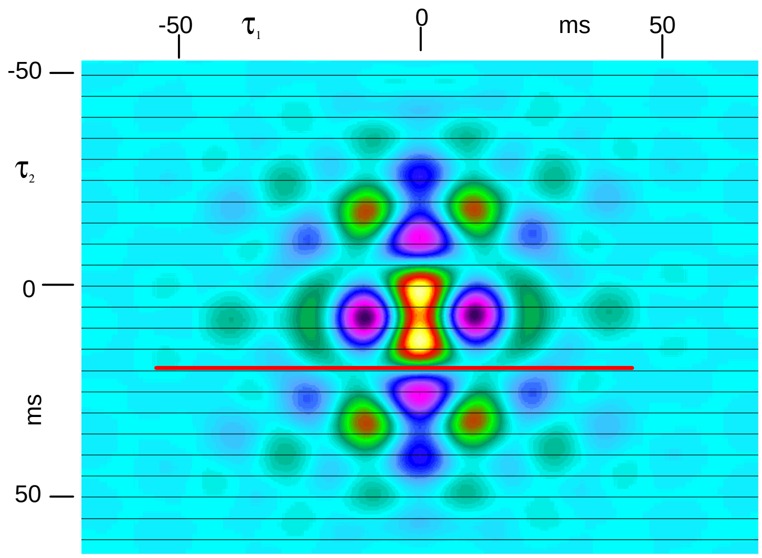
Central profile of the transpose of the cross-bicorrelation function in Figure 4

FIG. 8. This is the central profile of the transpose of the cross-bicorrelation function in Figure 4, repeated. Since it runs parallel to the τ_2 direction, it is similar to the cross-correlation function.



Off-centre profile of the transpose of the cross-bicorrelation function in Figure 4

FIG. 9. This repeated profile corresponds to the off-centre position indicated in Figure 4 and is a subsidiary correlation within the overall cross-bicorrelation function.



Transpose Cross-bicorrelation function of traces 1 and 25 of Figure 3 after remapping along constant slope in figure 6.

FIG. 10. The transpose of the cross-bicorrelation function in Figure 6 (note the interchange of τ_1 and τ_2 on the axes) has been remapped along trajectories of constant slope to change the alignment of the various correlation peaks.

The correlation peaks of the cross-bicorrelation function in Figure 6 may not be uniform in cross-section. This means that a profile extracted from the cross-bicorrelation function perpendicular to the long axis of a given correlation peak may have a narrower peak profile than one extracted parallel to this axis, and may thus show greater resolution of individual correlations. To assist in extracting a profile along a sloping trajectory, the samples of the cross-bicorrelation function can be "remapped" parallel to the desired trajectory. Figure 10 shows the example cross-bicorrelation function with its samples re-mapped parallel to the slanted trajectory shown in Figure 6. Extraction and replication of the profile parallel to the new axis and through the origin results in Figure 11. Comparing with Figure 7, however, it can be seen that the event resolution is unchanged with this re-oriented function, relative to the original. This means that in this particular case, contrary to appearances in Figure 6, the correlation peaks are apparently uniform in cross-section.

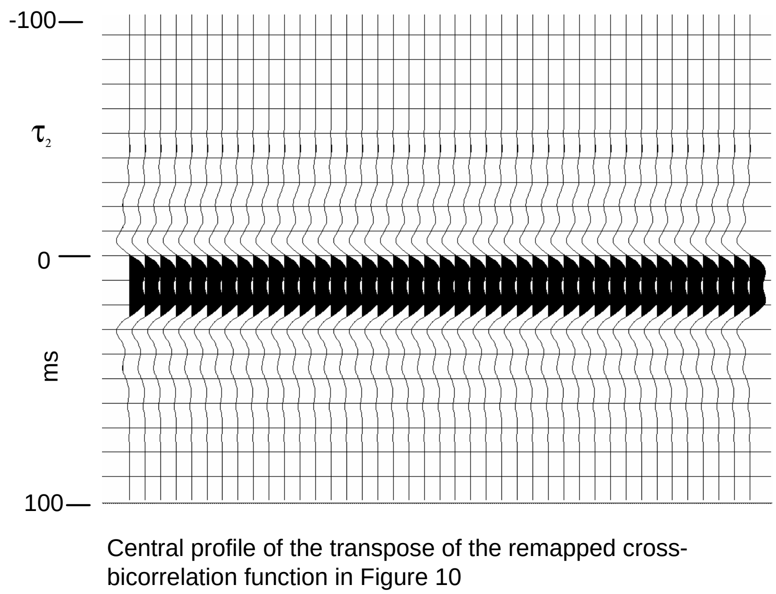


FIG. 11. The repeated central profile of the remapped transpose cross-bicorrelation function in Figure 10. Re-orienting the function has had little effect on the central profile.

"Summation" methods

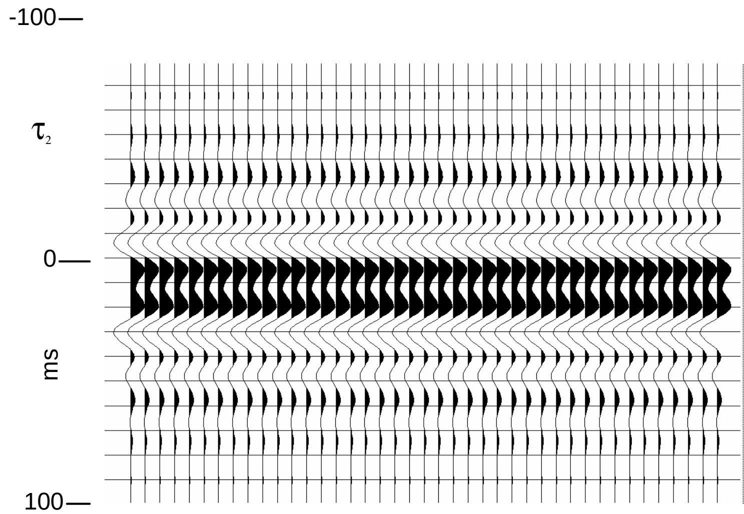
Profile extraction can be used to analyze the details of correlation between the events on two input traces. However, there is no way to determine, a priori, which events on an input trace are legitimate signal and which may be additive random noise, except possibly to arbitrarily ignore correlation peaks in the cross-bicorrelation function smaller than a particular threshold. For this reason, summation methods can be useful in extracting the signal information from the cross-bicorrelation function. The simplest of these methods is simply to "project", or sum the entire cross-bicorrelation function parallel to one of its time lag axes. When the summation is parallel to the τ_1 axis, the result is the cross-correlation of the input traces; and when the summation is parallel to the τ_2 axis, the autocorrelation of input trace 1 results. Figures 12 and 13 show the replicated summations along each of these two axes, respectively, for the cross-bicorrelation function in Figures 4-6. Comparing with Figures 7 and 8, we can see that the "projections" have different side lobes, from the summation of minor correlations, than the single extracted profiles, but similar resolution. Surprisingly, the re-mapping of the cross-bicorrelation function as in Figure 10, and its projection, as in Figure 14, yields a function which exhibits better resolution than the projection of the original function in Figure 12. However, the side lobes are higher, reflecting the effects of constructive interference of the remapped subsidiary correlation peaks on the summation.

Another summation technique which will be demonstrated later is that of partial summation, or weighted mixing, which is particularly useful for noisy data. While a projection collapses the cross-bicorrelation function into a single profile, another summation operator, the weighted partial sum, “running sum”, or “mixing” operation preserves larger features while smoothing out or cancelling smaller features which may be manifestations of random noise correlations. In this method, a mixing or smoothing step is applied along the axis perpendicular to the desired profile trajectory in order to smooth out fluctuations in the cross-bicorrelation function. Extraction of the desired single profile then preserves most of the correlation resolution while being less affected by noise correlation.

Operationally, the mixing operation can be applied using various smoothing lengths and trace weighting functions; and it can be applied along any axis through the cross-bicorrelation function. The single curve required for interpretation can then be extracted along a particular trajectory as described above.

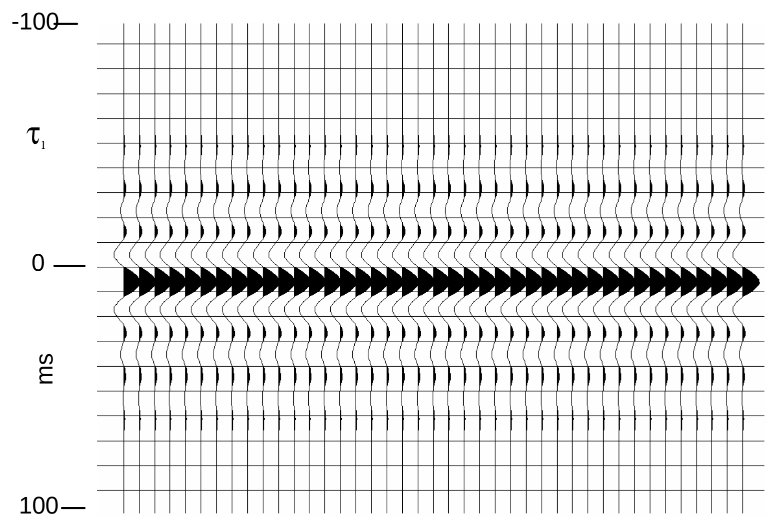
Spectral normalization

Spectral normalization, as defined by Eq. 9, can lead to sharper and better defined correlation peaks in the cross-bicorrelation function, but at the risk of creating more spurious side lobes unrelated to legitimate time series events. Figure 15 shows the normalized cross-bicorrelation function for model traces 1 and 25, and Figure 16 displays its “cross-correlation” projection, which exhibits slightly greater resolution than the projection of the un-normalized function in Figure 14. However, if we remap the function along a slope aligned with the major axes of its major correlation peaks before projecting it, we get a much better resolved “coherence” function in Figure 17. On the other hand, the subsidiary correlation peaks also sum constructively to give increased background side lobes. In this case, if we extract the central profile of the remapped function instead of projecting the function, we get the result in Figure 18, with much reduced side lobes, and with better resolution than the projected normalized cross-correlation in Figure 16. Figures 17 and 18 both indicate that remapping the normalized cross-bicorrelation function is beneficial, due to the apparent elongation of the cross-section of the correlation peaks in Figure 15. Hence, while a cross-bicorrelation function in its unnormalized form may have nearly circular peak cross-sections, the normalized form may exhibit elongation of those cross-sections due, probably, to a different amount of whitening with respect to each of the function axes.



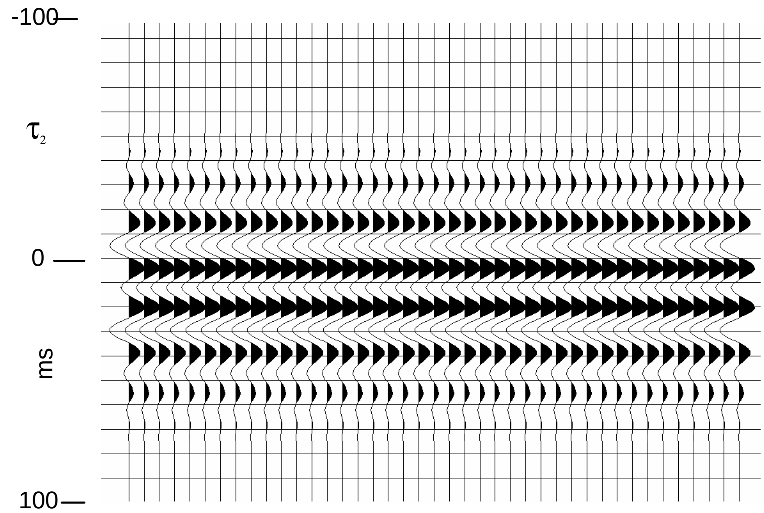
Projection of the transpose of the cross-bicorrelation function in Figure 4—cross-correlation

FIG. 12. Projection or summation of the cross-bicorrelation function in Figure 4 parallel to the τ_1 axis. This is the full cross-correlation function.



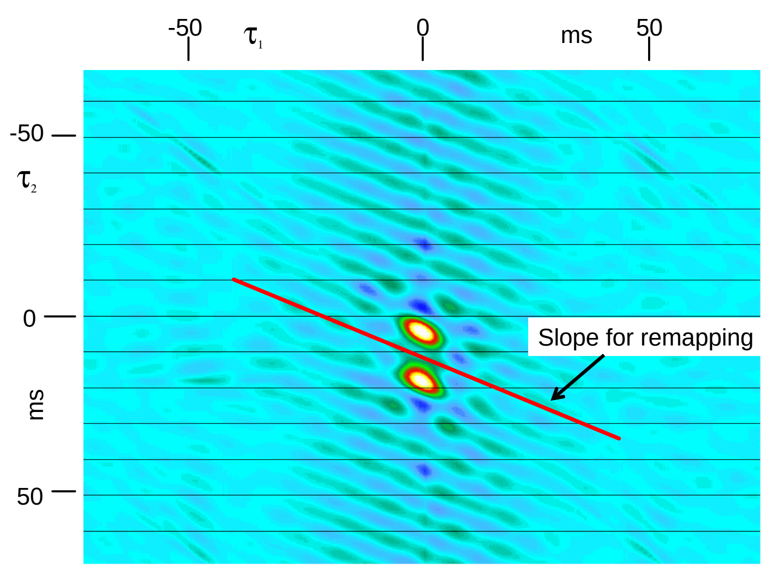
Projection of the cross-bicorrelation function in Figure 4—auto-correlation

FIG. 13. Projection or summation of the cross-bicorrelation function in Figure 4 parallel to the τ_2 axis. This is the full autocorrelation function.



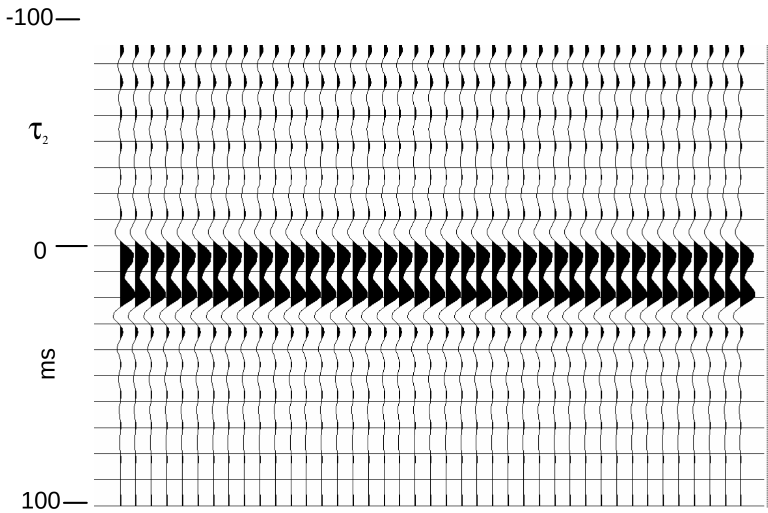
Projection of the transpose of the remapped cross-bicorrelation function in Figure 10—cross-correlation

FIG. 14. The cross-correlation function obtained by projecting the remapped cross-bicorrelation function in Figure 10.



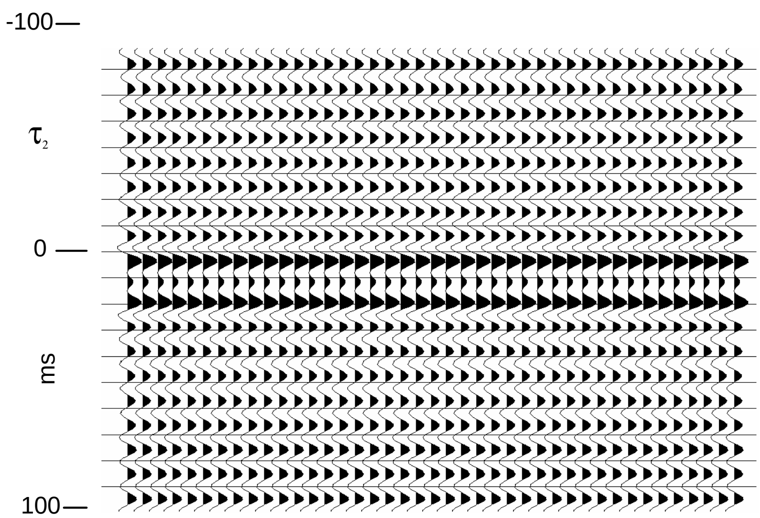
Transpose Cross-bicorrelation function of traces 1 and 25 of Figure 3 with spectral normalization applied.

FIG. 15. This transpose cross-bicorrelation function has been spectrally normalized to increase its resolution. Note that the energy in the function has been greatly concentrated at the two central peaks.



Projection of the normalized transpose cross-bicorrelation function in Figure 15—cross-correlation

FIG. 16. Cross-correlation function projected from the normalized cross-bicorrelation function in Figure 15. The summation has reduced the apparently greater resolution.



Projection of the remapped normalized transpose cross-bicorrelation function

FIG. 17. Cross-correlation function projected from the cross-bicorrelation function remapped along a slope indicated in Figure 15. The resolution of the event correlations is greater, but so is the constructive interference of the subsidiary correlation peaks.

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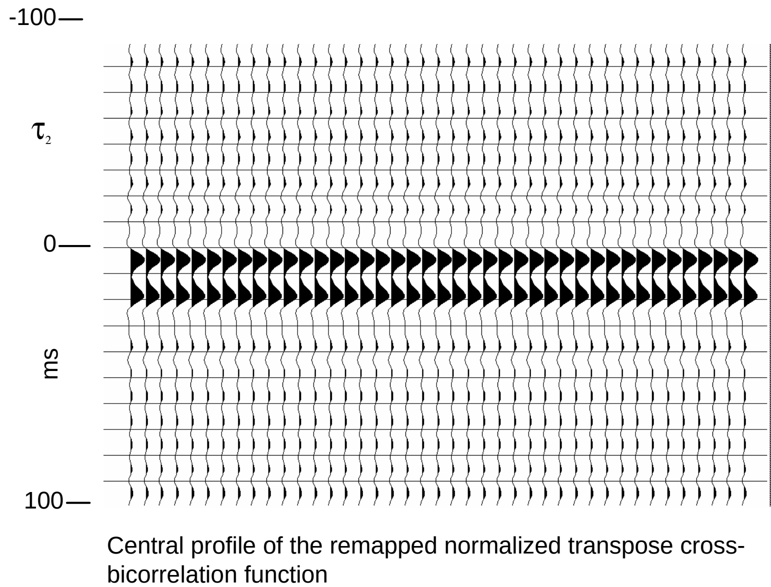


FIG. 18. Choosing the central profile of the remapped normalized cross-bicorrelation function leads to higer resolution as well as reduced background interference.

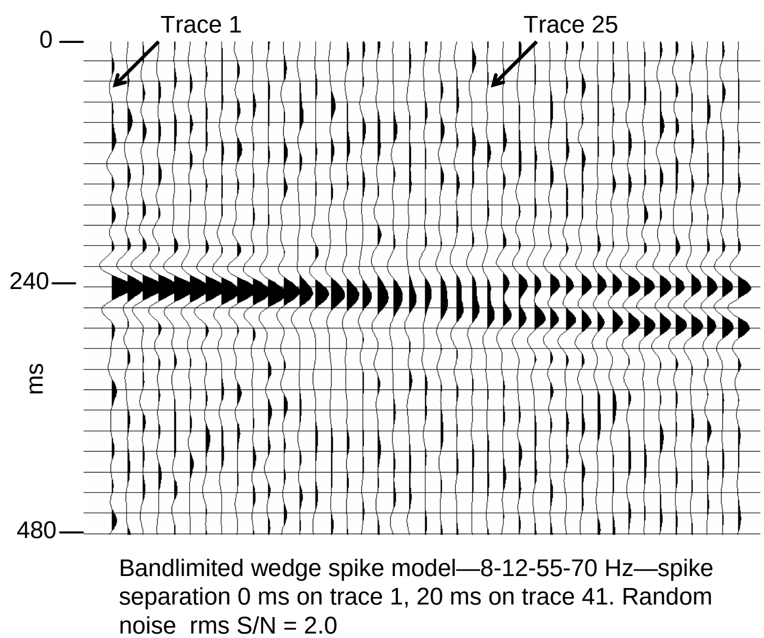


FIG. 19. Bandlimited spike wedge model with random noise added prior to bandlimiting—the rms S/N = 2.0.

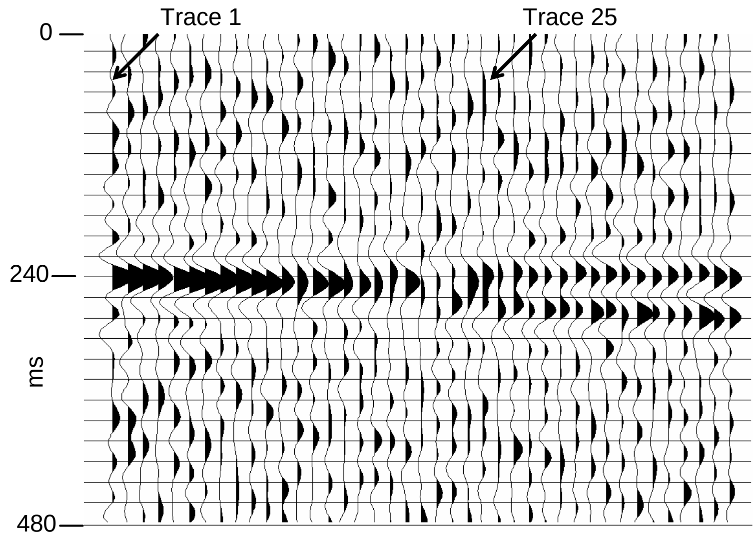


FIG. 20. Bandlimited spike wedge model with added random noise—rms S/N = 1.0.

MODELS WITH NOISE

In order to further explore the resolving potential of the cross-bicorrelation function, the function was generated for the same trace pair previously chosen from the synthetic wedge model. Here, however, bandlimited additive noise was introduced to the model at two different S/N levels to investigate the effects of noise on the various correlation function techniques described previously. Figure 19 shows the bandlimited spike wedge model of Figure 3, except that white random noise with rms S/N of 2.0 has been added to the traces before application of the bandpass filter. The same model with rms S/N of 1.0 is shown in Figure 20. As in our previous results with noiseless data, traces 1 and 25 were used to illustrate the differences in resolution for the different coherent measures for the chosen bandlimits. In Figures 19 and 20 it can be seen that the additive noise significantly impacts the amplitudes of the model spikes, and hence is expected to degrade the resolution of coherence measures.

S/N level 2.0

Figures 21 and 22 show the regular and normalized cross-bicorrelation functions for the spike model with S/N = 2.0. The presence of noise is immediately apparent in the many new correlation peaks in both functions, although those in the normalized function appear to have lower amplitude relative to the main signal correlation peaks than those in the regular function. Because the amplitudes of the model spikes have themselves been altered by the additive noise, it becomes a little less clear which peaks in the cross-bicorrelation function are the signal-related ones.

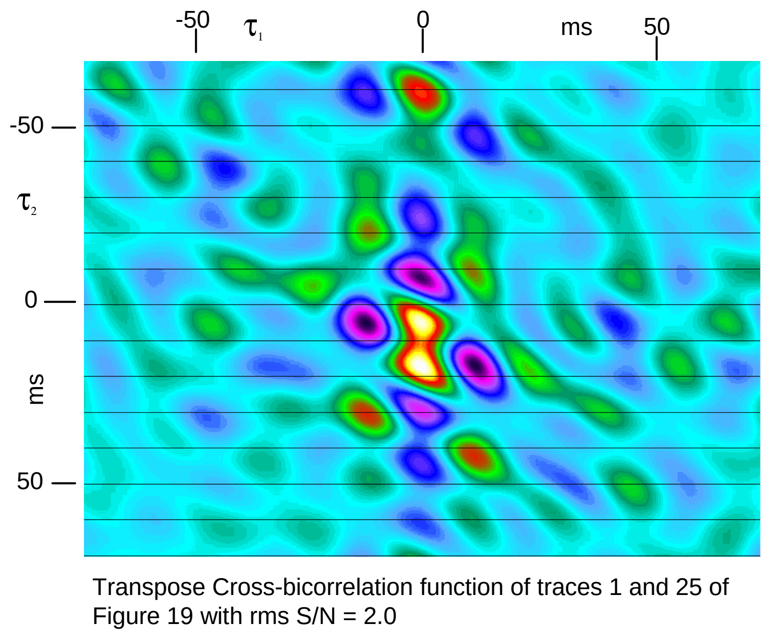


FIG. 21. Cross-bicorrelation function of traces 1 and 25 of the model shown in Figure 19.

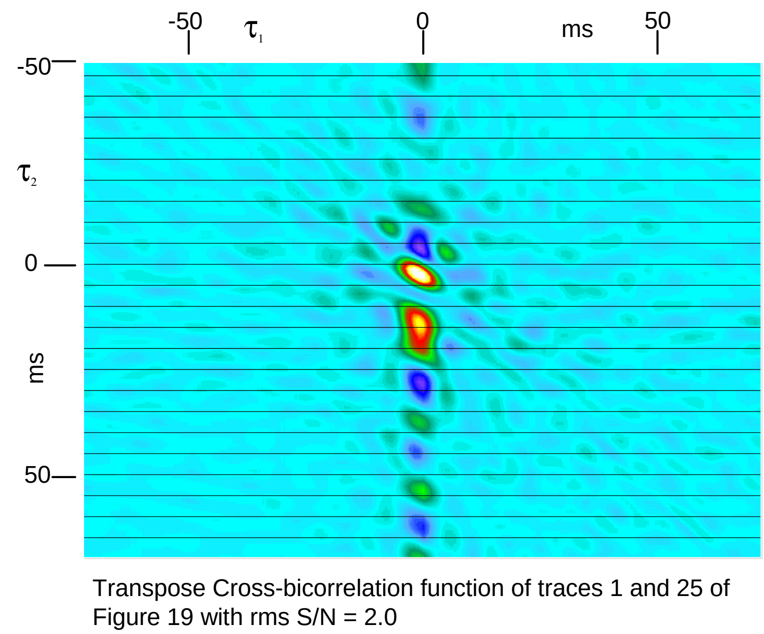


FIG. 22. Normalized cross-bicorrelation function of traces 1 and 25 of the model in Figure 19.

Figure 23 shows the central correlation curve extracted from the bicorrelation function, while Figure 24 shows the cross-correlation obtained from the projection of the function along the τ_1 axis, for comparison. Figures 25 and 26 show the equivalent correlation curves for the normalized cross-bicorrelation function in Figure 22. In this case, similar to the results for the noiseless case, the extracted central profile is better resolved than the projection, or cross-correlation. Remapping along a slope, however, contrary to the noiseless results, seems to give no improvement in resolution, as shown in Figure 27.

We hypothesized that partial summation methods would be most useful on noisy data, providing more resolution than a full projection and more noise cancellation than single profile extraction. We applied a weighted trace mix to the cross-bicorrelation function in Figure 21. Figure 28 shows the cross-bicorrelation function of figure 21 after a weighted trace mix of 21 traces with triangular weights. The central profile of this smoothed correlation is shown in Figure 29. A similar display for the remapped cross-bicorrelation function appears in Figure 30. Here, the remapping appears to have improved the function resolution, probably because of a favorable reorientation of smaller correlation peaks prior to the mixing operation. Figure 31 shows the smoothed normalized cross-bicorrelation, and Figure 32 the central profile of this smoothed function. Normalization and partial summation methods both seem to provide modest improvements to the resolution of the cross-bicorrelation function when used on these mildly noisy data. Applying both in tandem does not necessarily provide incremental improvements, however.

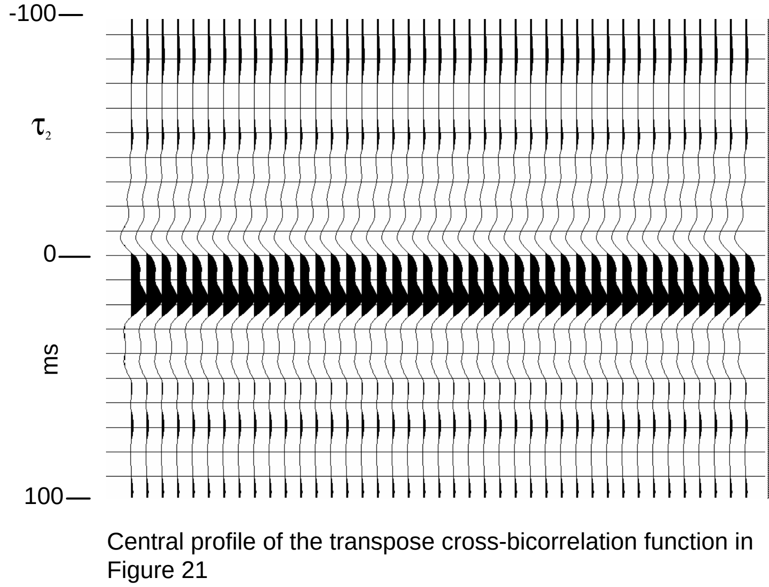
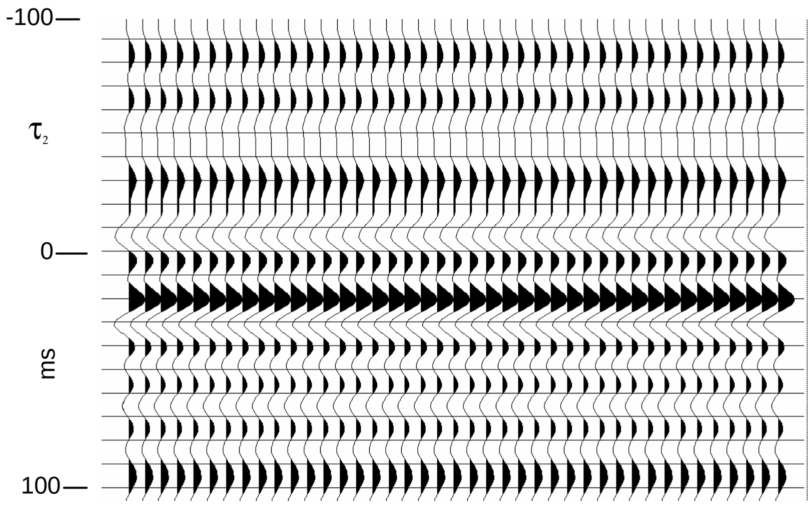
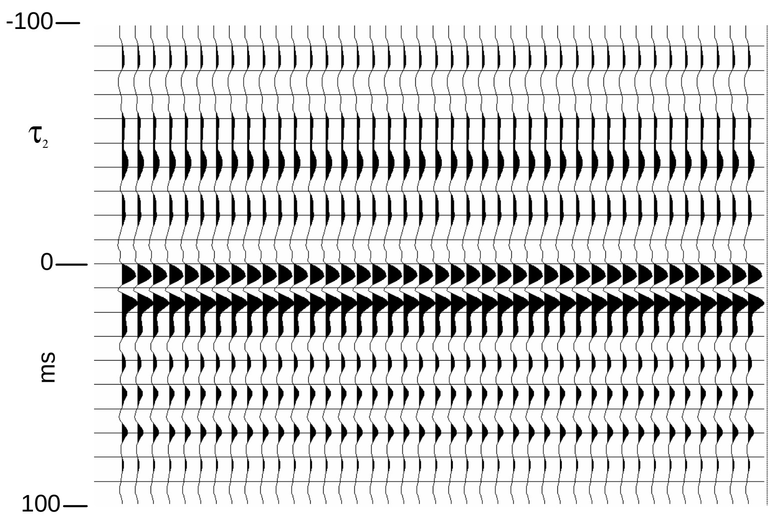


FIG. 23. Central profile of the cross-bicorrelation function in Figure 21



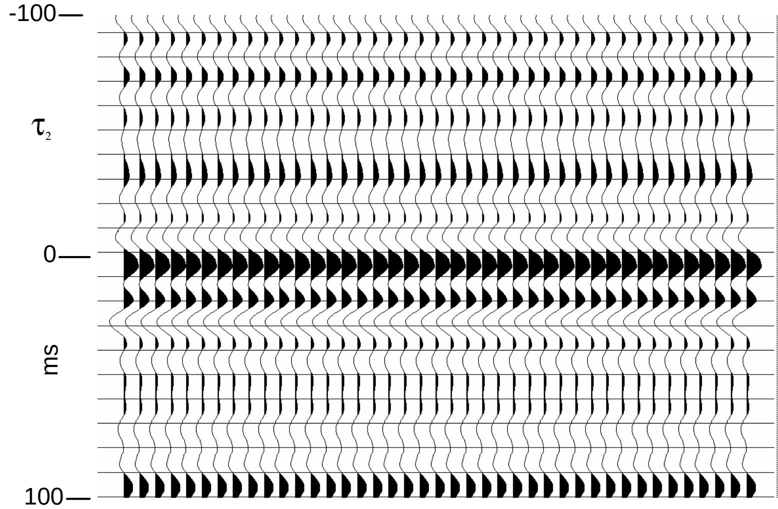
Projection of the transpose cross-bicorrelation function in Figure 21

FIG. 24. Projection of the cross-bicorrelation function in Figure 21—the cross-correlation function.



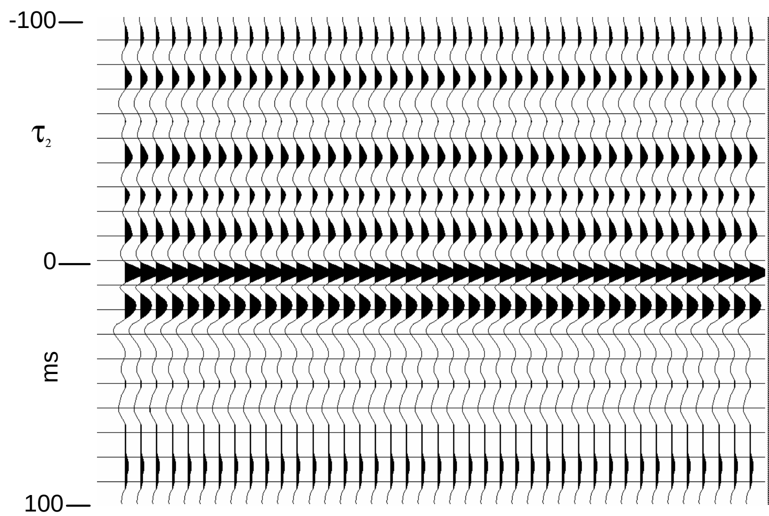
Central profile of the transpose normalized cross-bicorrelation function in Figure 22

FIG. 25. Central profile of the normalized cross-bicorrelation function in Figure 22.



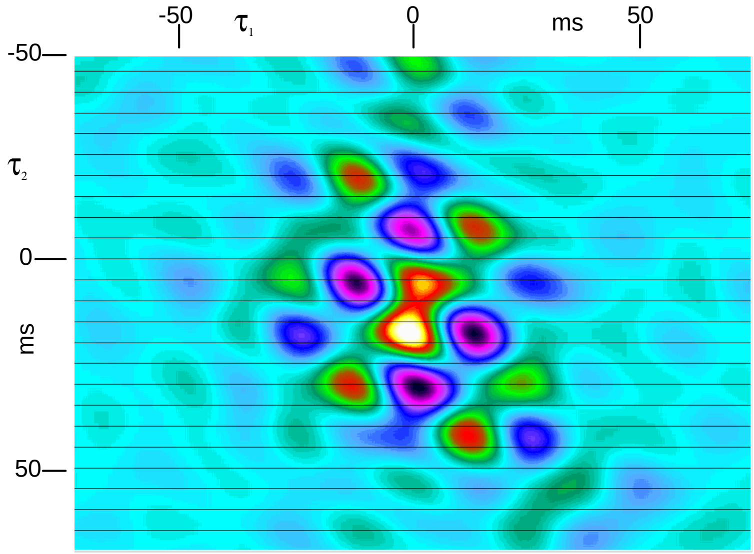
Projection of the normalized transpose cross-bicorrelation function in Figure 22

FIG. 26. Projection of the normalized cross-bicorrelation function in Figure 22—the cross-correlation function. The function appears less well resolved than the central profile in Figure 25.



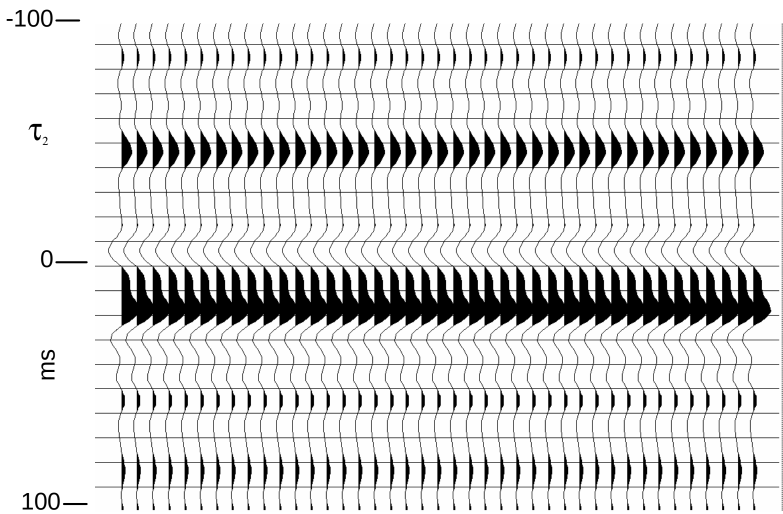
Central profile of the remapped normalized transpose cross-bicorrelation function in Figure 22

FIG. 27. In this case, remapping the normalized cross-bicorrelation function does not improve resolution.



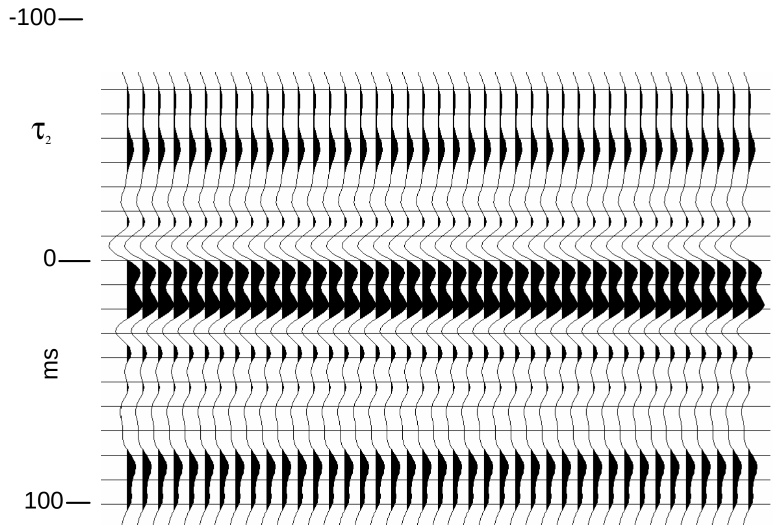
Transpose cross-bicorrelation function of traces 1 and 25 of Figure 19 with 21 point weighted lateral smoothing.

FIG. 28. Cross-bicorrelation function smoothed laterally with 21 point triangular smoothing.



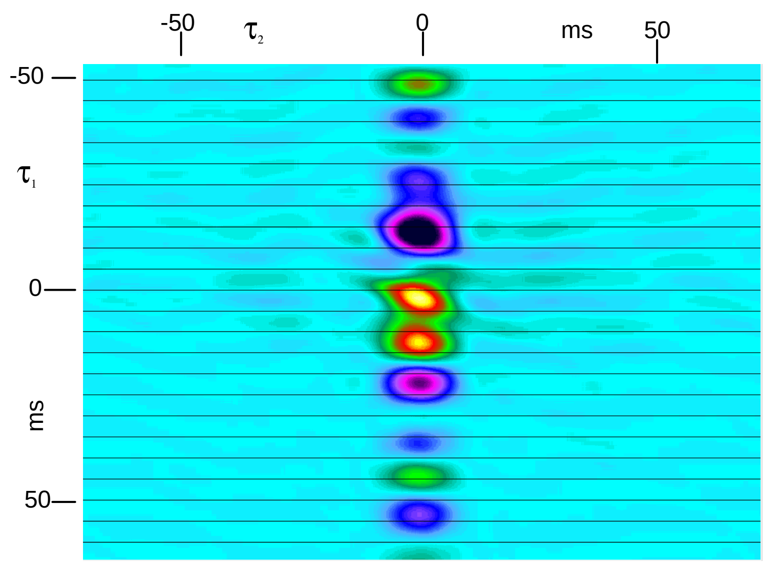
Central profile of the smoothed transpose cross-bicorrelation function in Figure 28

FIG. 29. Central profile of the smoothed cross-bicorrelation function in Figure 28. Compare with Figure 23.



Central profile of the remapped smoothed transpose cross-bicorrelation function in Figure 28

FIG. 30. Remapping before smoothing appears to improve the resolution of the cross-bicorrelation function, probably due to reorientation of subsidiary peaks.



Normalized transpose cross-bicorrelation function of traces 1 and 25 of Figure 19 with 21 point weighted lateral smoothing.

FIG. 31. Normalized cross-bicorrelation function for noisy wedge model in Figure 19 after 21 point weighted lateral smoothing.

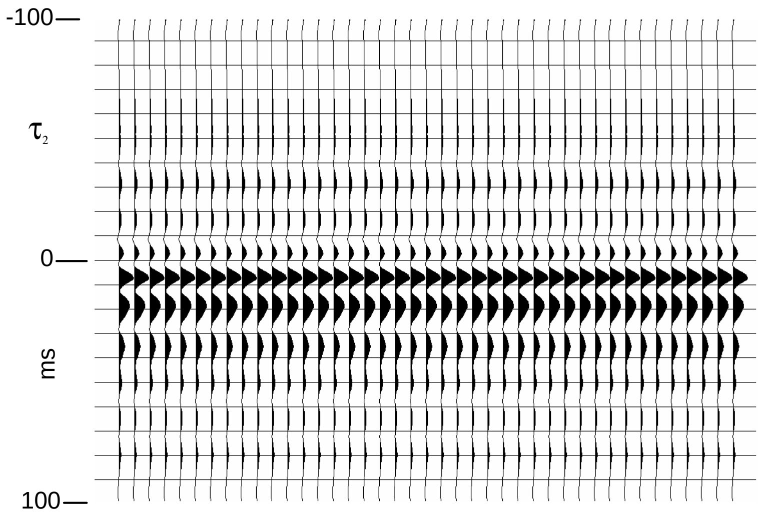
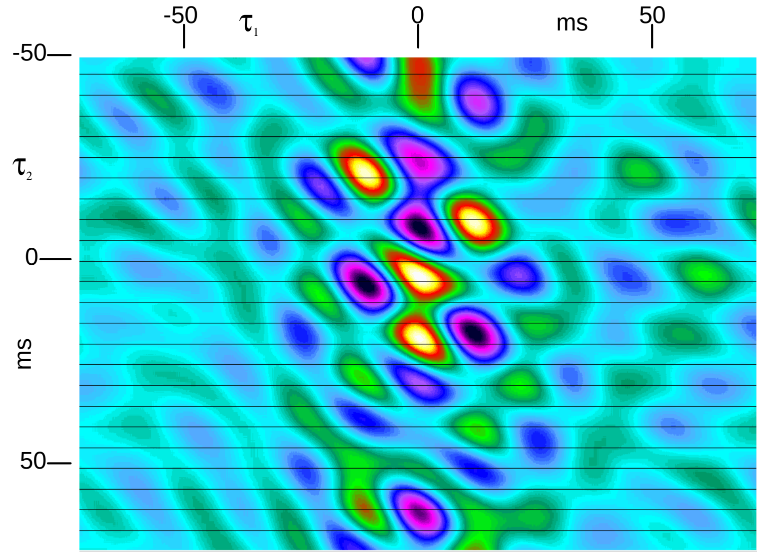


FIG. 32. Central profile of the smoothed normalized cross-bicorrelation function in Figure 31. In this case, smoothing yields little improvement. Compare with Figure 25.

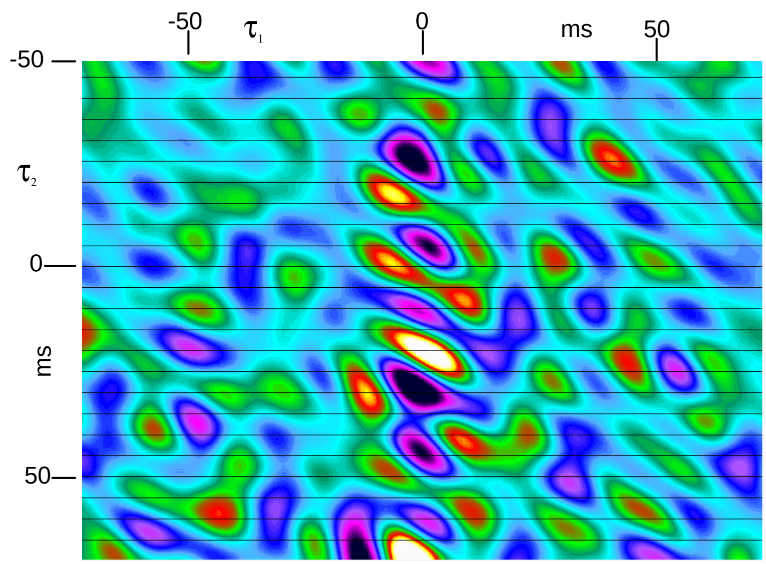
S/N level 1.0

Although signal is still easily visible on most of the model traces in Figure 20 when the S/N is unity, we find that attempts to increase resolution by selecting appropriate correlation curves from the cross-bicorrelation function shown in Figure 33 are defeated by the noise. In this case, the projection of the function, or full cross-correlation is probably the best that can be done. The normalized cross-bicorrelation function and corresponding projected cross-correlation function are shown in Figures 34 and 35, respectively. In this case, the whitening of the result, as well as the summation, appear to improve the resolution marginally. Nearby interference peaks are higher than the principal correlations, however, so interpretation of the results becomes ambiguous.



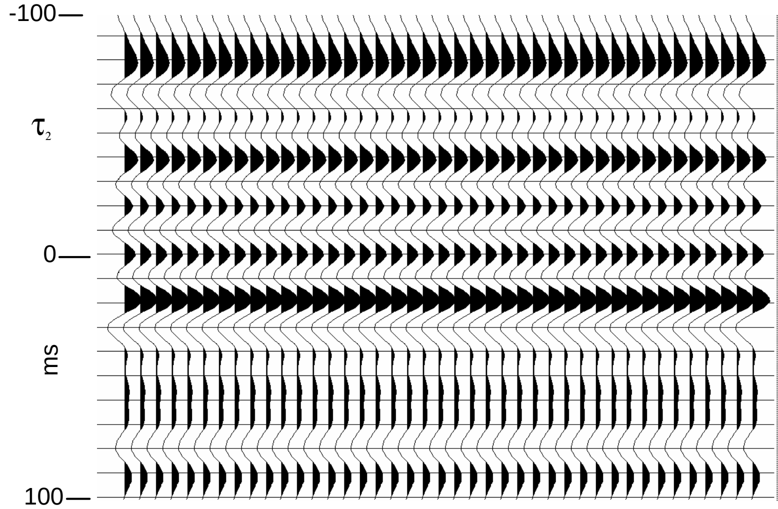
Transpose cross-bicorrelation function of traces 1 and 25 of Figure 20 with rms S/N = 1.0

FIG. 33. Cross-bicorrelation function for noisy wedge model in Figure 20.



Normalized transpose cross-bicorrelation function of traces 1 and 25 of Figure 20 with rms S/N = 1.0

FIG. 34. Normalized cross-bicorrelation function for noisy wedge model in Figure 20.



Projection of the normalized transpose cross-bicorrelation function in Figure 34.

FIG. 35. Projection of the normalized cross-bicorrelation function in Figure 34. Subsidiary peaks are comparable to or larger than primary correlation peaks in this cross-correlation.

SUMMARY OF RESULTS

While it may be useful in detailed analysis of two input time series, the cross-bicorrelation function, one of a family of so-called "higher order statistics" seems to be limited in its utility for increasing the detection resolution of individual events in seismic traces because of its sensitivity to noise. We have two sets of observations on the performance of the function, depending upon whether the input traces are contaminated with noise or not:

Noise-free data

- On noise-free data, the individual correlation peaks can be used to analyze the specific correlations between isolated spikes on the input traces

- Individual correlation profiles selected from the cross-bicorrelation function may show better resolution than the regular cross-correlation created by the projection of the cross-bicorrelation function parallel to one of the time lag axes.

- The normalized cross-bicorrelation function has greatly enhanced resolution of individual correlations within the bicorrelation, depending upon the stability factor used in the computation.

- The spectrally normalized cross-correlation function projected from the cross-bicorrelation function will usually show enhanced resolution with respect to the cross-correlation.

- Re-mapping the cross-bicorrelation function along a slope defined by the alignment of the major axes of correlation peaks can result in enhanced resolution of the coherence function, due to fortuitous cancellation of aligned correlations and anti-correlations.

Noisy data

The presence of even a modestly realistic level of noise consistent with the type of noise present on seismic data (additive bandlimited random noise) seems to degrade the performance of the cross-bicorrelation with respect to resolution improvement. However, for modestly noisy data:

- Partial summation or weighted mixing followed by single profile extraction provides modestly improved event resolution compared to the standard cross-correlation

- The spectrally normalized cross-bicorrelation function provides improved event resolution as long as the required stability factor is relatively small (the larger the stability factor, the less the whitening).

- Remapping of the cross-bicorrelation function can still improve resolution results, as long as subsidiary peak realignment does not lead to increased constructive interference.

For very noisy data, the only resort may be collapsing the cross-bicorrelation function to the standard cross-correlation function. Even in this case, however, the options exist to remap the function or to weight the summation as a way to improve resolution or reduce side lobe energy; options that are not available when computing the cross-correlation from the usual formula (Equation 1).

CONCLUSIONS

This report has demonstrated the use of a so-called “higher order statistic” to analyze event correlation between two time series. This statistic, the cross-bicorrelation function, has been tested on a synthetic model, both with and without added noise. Preliminary results indicate that there are several techniques which may be used with the cross-bicorrelation function to improve the resolution of individual events on input time series traces. It is not clear at this early stage, however, whether there is a universal technique, using the cross-bicorrelation function, which can consistently lead to significantly improved extraction of a “static distribution function”, the ultimate goal of this investigation. Further work is required, particularly with real data rather than models. One advantage of computing the cross-bicorrelation function is that the resulting decomposed correlation has components which can be weighted or otherwise modified prior to projection of the function to form the cross-correlation.

ACKNOWLEDGEMENTS

The authors thank the CREWES project and its sponsors for support.

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