Applying residual phase shifts to wavefield extrapolation

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ABSTRACT

In the generalized phase shift plus interpolation (GPSPI) method, the phase shift extrapolation is used to produce a reference wavefield for each output point using its velocity, and the process is repeated for all output points. This assumes a locally homogenous medium for each extrapolation, and that velocity differences between input and output points are small.

Applying residual phase shifts that account for differences in velocities between input and output points in wavefield extrapolation methods can be helpful in areas that have complicated subsurface structures. In this report, we show how this type of correction can be applied to wavefield extrapolation methods in the space-frequency domain. We use the Marmousi dataset to show the effect of adding this correction. Although no dramatic changes resulted from applying these shifts, some faults and dipping events are better focused and delineated because of them.

INTRODUCTION

The generalized phase shift plus interpolation (GPSPI) algorithm (Margrave and Ferguson, 1999) is the limiting form of the well known phase shift plus interpolation (PSPI) algorithm (Gazdag and Squazerro, 1984). The space-frequency wavefield extrapolation methods are an approximation to the GPSPI where for each output point a different operator is used so that strong lateral velocity variations can be handled properly. Explicit space-frequency extrapolation methods are a good approximation to GPSPI (Margrave et al., 2005). These methods are only an approximation because each extrapolator is a dip-limited approximation to the inverse Fourier transform of the phase shift operator (Gazdag, 1978; Hale, 1991). In other words, for an extrapolator to migrate a 90 degree event, it would need to have an infinite spatial extent, which is practically not possible.

The split Fourier methods (Stoffa et al., 1990) use a thin lens term or static shift and a focusing shift term to perform the wavefield extrapolation. In its simplest form, the wavefield is extrapolated to the next depth level using the Gazdag (1978) phase shift migration with a reference velocity. Then residual phase shifts are applied to the extrapolated data to account for differences in velocities between the input and output points. This method does not work very well in the presence of complicated subsurface structures. However, the same idea can be extended to GPSPI to make it even more powerful.

In this report, we show how the residual phase shifts can be applied to space-frequency wavefield extrapolation methods. We will use the Marmousi dataset to show the effect of adding these shifts to the extrapolated data and discuss the results.

THEORY

We start with the 2D variable wave speed Helmholtz equation (Thomson, 2005)

$$\partial_z^2\psi + \partial_x^2\psi + \frac{\omega^2}{v^2(x)}\psi = 0, \tag{1}$$

where z is the direction along which the wavefield is extrapolated, x is the lateral position, $\psi = \psi(x,z,\omega)$ is the Fourier transform of pressure wavefield over the temporal coordinate, ω is the temporal frequency, and $v(x)$ is the velocity field. Equation (1) can rewritten as

$$(\partial_z + iB)(\partial_z - iB)\psi - B^2\psi + M^2\psi + i(\partial_z B)\psi = 0, \tag{2}$$

where

$$M^2\psi = \left(\frac{\omega^2}{v^2} + \partial_x^2\right)\psi \tag{3}$$

and

$$(\partial_z B)\psi = \partial_z B\psi - B(\partial_z\psi), \tag{4}$$

where $B(x,\partial_x,z,\omega)$ is a pseudodifferential operator (PSDO) acting on the lateral coordinate *x*. The form of equation (2) suggests that if operator B can be found such that

$$B^2 - i(\partial_z B) = M^2, \tag{5}$$

then

$$(\partial_z - iB)\psi = 0 \tag{6}$$

is an exact one-way wave equation. If the medium is independent of *z*, or range independent, then forward $(+z)$ and backward $(-z)$ propagating waves are uncoupled. These spatial Fourier conventions of the wavefield will be used

$$\hat{\psi}(k_x,\omega,z) = \int \psi(x,\omega,z)e^{ik_x x}dx \tag{7}$$

and

$$\psi(x,\omega,z) = \frac{1}{2\pi}\int \hat{\psi}(k_x,\omega,z)e^{-ik_x x}dk_x. \tag{8}$$

The symbol of the partial differential operator M^2 is $\left(\omega^2/v^2 - k_x^2\right)$. That is,

$$\left(\frac{\omega^2}{v^2} + \partial_x^2\right)\psi\left(x,\omega,z\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\omega^2}{v^2} - k_x^2\right)\hat{\psi}\left(k_x,\omega,z\right)e^{-ik_x x}dk_x. \tag{9}$$

Also, the symbol of the operator, *B*, will be denoted by $\hat{B}\left(x,k_x,z,\omega\right)$ as in

$$B\left(x,\partial_x,\omega,z\right)\psi\left(x,\omega,z\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{B}\left(x,k_x,\omega,z\right)\hat{\psi}\left(k_x,\omega,z\right)dk_x. \tag{10}$$

The theory of PSDOs provides the following exact composition rule for symbols of the standard-ordering type (Thomson, 1999)

$$symbol\left(\hat{B}^2\right)\left(x,k_x,z,\omega\right) = \exp\left(-\frac{i}{\omega}\partial_y\partial_\eta\right)B\left(x,\eta,z,\omega\right)B\left(y,k_x,z,\omega\right)\Big|_{\substack{y\to x\\ \eta\to k_x}}. \tag{11}$$

The asymptotic expansion of the symbol can be given in the form

$$\hat{B}\left(x,k_x,z,\omega\right) = \hat{B}_0\left(x,k_x,z,\omega\right) + \frac{1}{\omega}\hat{B}_1\left(x,k_x,z,\omega\right) + \frac{1}{\omega^2}\hat{B}_2\left(x,k_x,z,\omega\right) + \ldots. \tag{12}$$

After taking the first term of the Taylor series expansion of equation (11), it yields

$$\hat{B}_0\left(x,k_x,z,\omega\right) = \left(\frac{\omega^2}{v^2\left(x\right)} - k_x^2\right)^{\frac{1}{2}}. \tag{13}$$

While the leading term of equation (12) is non-singular, taking more terms of the series expansion of equation (11) will lead to a non-uniform singular series (Fishman and McCoy, 1985)). If more terms are needed, a uniform asymptotic expansion has to be used. By using $\hat{B}_0$ a solution to the one way wave equation in equation (6) can be found. By using this solution, the Fourier-integral wavefield extrapolation expression can be expressed as

$$\psi_{v(x)}\left(x,z+\Delta z,\omega\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{\psi}\left(k_x,z,\omega\right)\hat{W}\left(x,k_x,\omega,\Delta z\right)e^{-ik_x x}dk_x, \tag{14}$$

where

$$\hat{W}\left(x,k_x,\omega,\Delta z\right) = e^{i\hat{B}_0\Delta z}. \tag{15}$$

The subscript $v(x)$ was used in equation (14) to indicate that the wavefield is range independent but dependent in the transverse (*x*-) direction. Equation (14) is known as the generalized phase shift plus interpolation (GPSPI) (Margrave and Ferguson, 1999). Moreover, Equation (14) is the limiting form of the phase shift plus interpolation (PSPI) (Gazdag and Squazerro, 1984). Further, when equation (14) is transversely independent then it reduces to Gazdag phase shift extrapolation (1978).

In the split Fourier methods (Stoffa et al., 1990) $\hat{W}$ can be written as

$$\hat{W}\left(x,k_x,\omega,\Delta z\right)=e^{i\Delta z\left(\phi_s+\phi_f\right)}, \tag{16}$$

where

$$\phi_s=\frac{\omega}{v(x)} \tag{17}$$

and

$$\phi_f=\frac{\omega}{v(x)}\left(\left(1-\frac{k_x^2 v(x)^2}{\omega^2}\right)^{\frac{1}{2}}-1\right), \tag{18}$$

where ϕ_s is also known as the static shift "or" thin lens term and ϕ_f is know as the focusing phase shift. Equation (14) can be rewritten as

$$\psi_{v(x)}\left(x,z+\Delta z,\omega\right)=\frac{1}{2\pi}W_s\left(x,\omega,\Delta z\right)\int_{-\infty}^{\infty}\hat{\psi}\left(k_x,z,\omega\right)\hat{W}_f\left(x,k_x,\omega,\Delta z\right)e^{-ik_x x}dk_x, \tag{19}$$

where

$$\hat{W}_f\left(x,k_x,\omega,\Delta z\right)=e^{\frac{i\omega\Delta z}{v(x)}\left(\left(1-\frac{k_x^2 v(x)^2}{\omega^2}\right)^{\frac{1}{2}}-1\right)}, \tag{20}$$

$W_s\left(x,\omega,\Delta z\right)=e^{i\Delta z\phi_s}$, and $\hat{W}_f\left(x,k_x,\omega,\Delta z\right)=e^{i\Delta z\phi_f}$. Note that W_s is not a function of k_x so it can go outside the integral (equation (19)). For each output point, the Gazdag (1978) phase shift extrapolation is used to produce a reference wavefield

$$\psi_{v_j}\left(x_j, z+\Delta z, \omega\right) = \frac{1}{2\pi} W_s\left(x_j, x, \omega, \Delta z\right) \int_{-\infty}^{\infty} \hat{\psi}\left(k_x, z, \omega\right) \hat{W}_f\left(x_j, k_x, \omega, \Delta z\right) e^{-ik_x x} dk_x, \quad (21)$$

and the process is repeated for all output points (Margrave and Ferguson, 1999) under the constraint that

$$\psi_{v(x)}\left(x_j, z+\Delta z, \omega\right) = \psi_{v_j}\left(x_j, z+\Delta z, \omega\right), \text{ if } v\left(x_j\right) = v_j . \quad (22)$$

The focusing term in Equation (20) can be decomposed into (where only the velocity of the output is used)

$$\hat{W}_f\left(x_j, k_x, \omega, \Delta z\right) = e^{i\Delta z\left(\frac{\omega^2}{v_j^2} - k_x^2\right)^{\frac{1}{2}}} e^{\frac{-i\Delta z \omega}{v_j}} \quad (23)$$

Inserting equation (23) into equation (19) gives

$$\psi_{v_j}\left(x_j, z+\Delta z, \omega\right) = \frac{1}{2\pi} \Gamma\left(x_j, x, \omega, \Delta z\right) \int_{-\infty}^{\infty} \hat{\psi}\left(k_x, z, \omega\right) \hat{W}\left(x_j, k_x, \omega, \Delta z\right) e^{-ik_x x} dk_x, \quad (24)$$

where

$$\Gamma\left(x_j, x, \omega, \Delta z\right) = e^{i\omega\Delta z\left(\frac{1}{v(x)} - \frac{1}{v_j}\right)} \quad (25)$$

and

$$\hat{W}\left(x_j, k_x, \omega, \Delta z\right) = e^{i\omega\Delta z\left(\frac{\omega^2}{v_j^2} - k_x^2\right)^{\frac{1}{2}}} . \quad (26)$$

The inside integral in equation (24) assumes a locally homogenous medium and Γ accounts for differences in velocities between output and input points. Further, equation (23) is an enhanced version of GPSPI since it applies some correction to the resulting error from the local homogeneity assumption. Wavefield extrapolation can be done in the $\omega - x$ domain as a nonstationary convolution according to

$$\psi_{v_j}\left(x_j, z+\Delta z, \omega\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma\left(x_j, x', \omega, \Delta z\right) \psi\left(x', z, \omega\right) W\left(x_j, x_j - x', \omega, \Delta z\right) dx' , \quad (27)$$

where

$$W\left(x_j, x' - x, \omega, \Delta z\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{W}\left(x_j, k_x, \omega, \Delta z\right) e^{-ik_x\left(x'-x\right)} dk_x, \qquad (28)$$

x' is the transverse coordinate at input, and x is the transverse coordinate at output. However, W is not compactly supported, i.e. it is infinitely long because it is the inverse Fourier transform of the symbol $\hat{W}$. There are different methods that can be used to design stable operators that can be used in a similar fashion to equation (27). In this report, we will use the enhanced forward operator and conjugate inverse (FOCI) algorithm to design wavefield extrapolators that approximate W (Margrave et al., 2005; Al-Saleh and Margrave, 2005).

DISCUSSION

There will be an error due to neglecting the difference in velocity between the output and input points in wavefield extrapolation methods. This error depends on the size of the depth step and complexity of the subsurface. Generally, the local homogeneity assumption does not induce a large error because the size of the depth step is usually relatively small.

The Marmousi dataset is used here to illustrate the effect of adding the residual phase shifts to the wavefield extrapolators. The 2-D acoustic Marmousi dataset was created at the Instut Francais du Petrole (IFP) (Bourgeois et al., 1991). With the presence of complex reflectors, steep dips and strong velocity gradients, it is widely recognized as an ideal synthetic dataset for testing migration algorithms. The dataset consists of 240 individual shot records of 96 traces each in a marine towed streamer configuration. The source and receiver intervals are 25 m and the highest coherent frequencies in the data are about 50 Hz. Prior to migration, we applied a wavelet shaping filter designed to whiten the signal spectrum and to remove an approximately 60 ms delay due to ghosting and water-bottom multiples. We also interpolated each shot to a receiver spacing of 12.5 m. The spatial extent of the operator that is used in the wavefield extrapolation is 25 points.

There was no dramatic difference after adding these shifts due to small depth steps in the extrapolation process. However, we can still discern little improvements in areas where there are dipping events and faults. Figure 1.a shows a portion of the shallow section of the Marmousi image where no phase shifts were applied, and Figure 1.b shows the same section but with the shifts applied. The dipping events were better focused as were the faulted areas (indicated by circles). Figure 2.a also shows some faulted areas of Marmousi, and Figure 2.b shows the result after applying the phase shifts. Again, there are some improvements, and we can see a better focusing of the faulted areas. Applying these shifts is computationally more expensive than wavefield extrapolation without them.

Adding these corrections to wavefield extrapolation methods doubles the computation cost. So using such corrections can be only justified in areas where there is a complicated subsurface.

CONCLUSIONS

Enhancing wavefield extrapolation methods with the residual phase shifts to account for difference in velocity between output and input points yielded slightly better results. Generally, the local homogeneity assumption by which each output point is computed does not introduce a large phase error as long as small depth steps are taken. However, we think that in highly faulted areas, applying these residual shifts might help interpreters to identify and delineate faults more accurately.

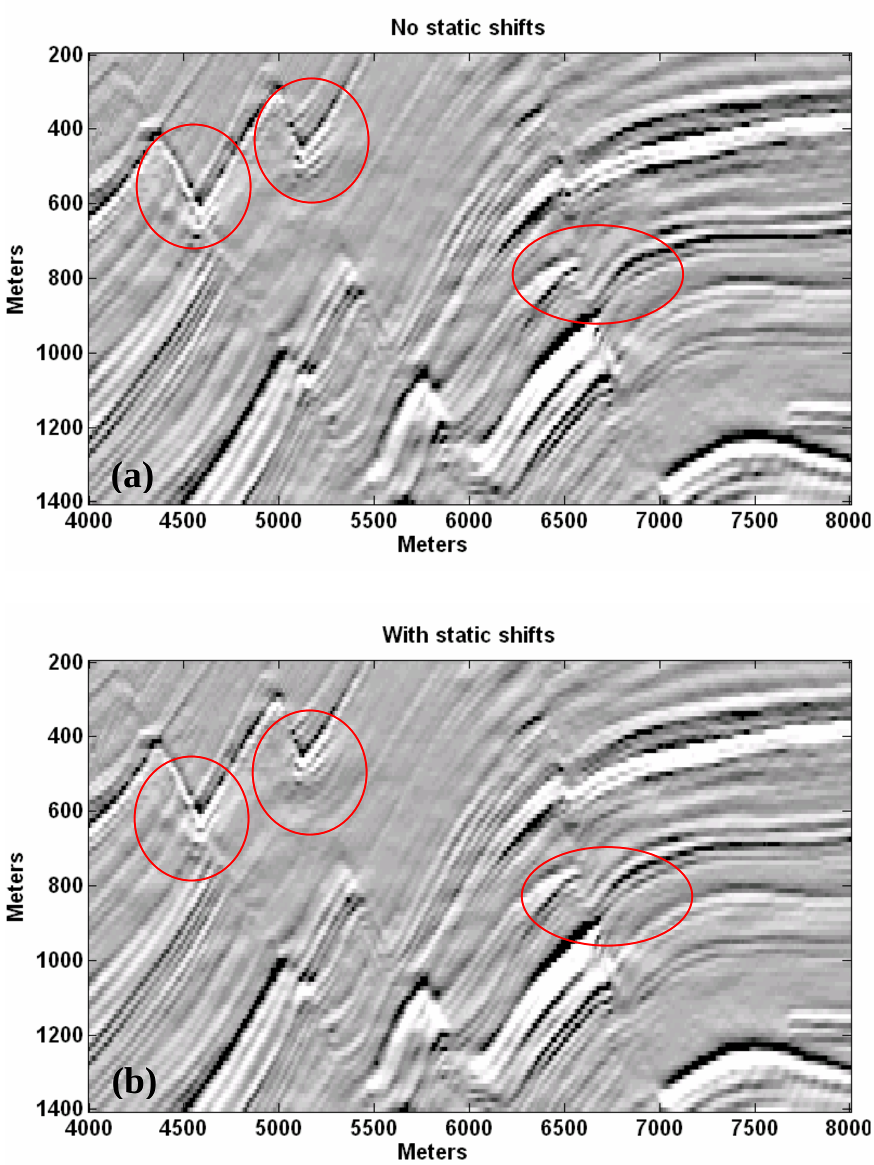

FIG. 1. A close-up of the shallow section of Marmousi where (a) shows images before applying the shifts and (b) shows the same images after. The circles show areas of improvement.

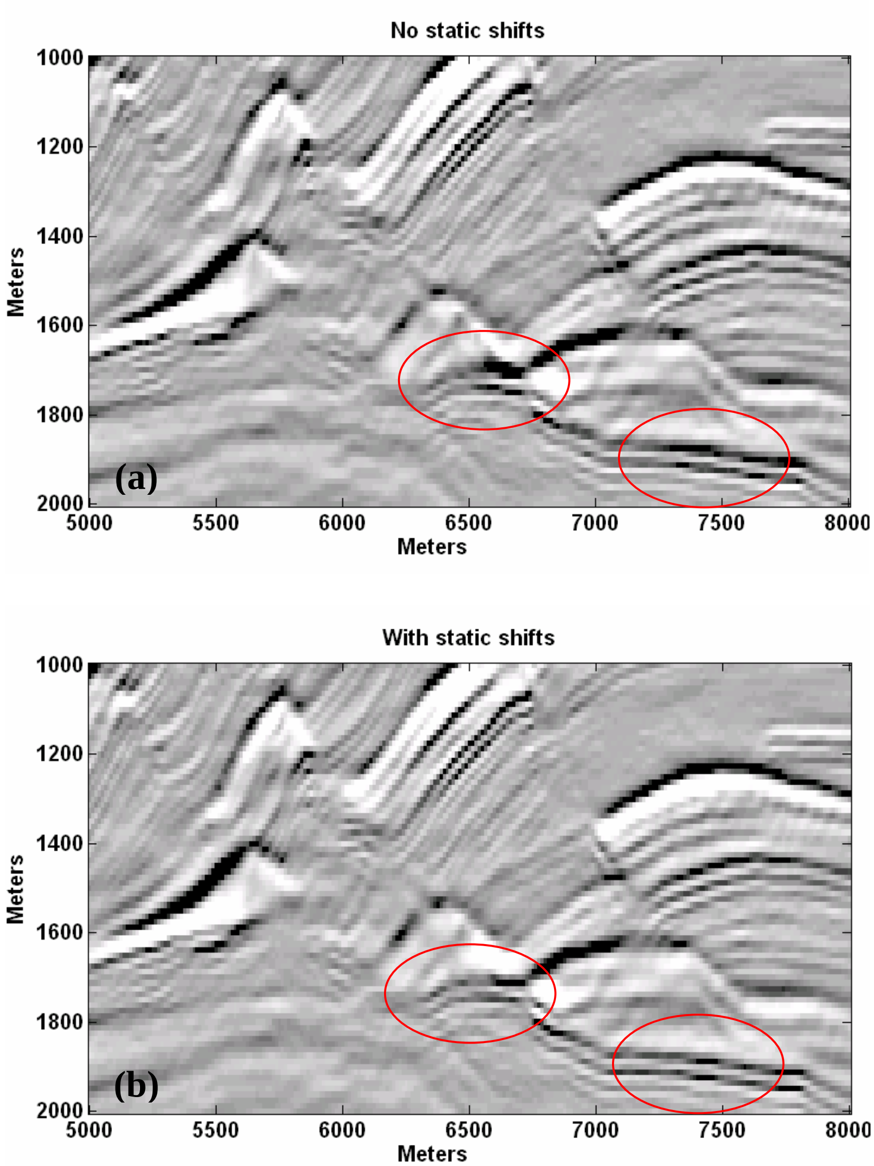

FIG. 2. A close-up of a faulted section of Marmousi where (a) shows an image before applying the shifts and (b) shows the same image after. The circles show areas of improvement.

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