A new adaptive windowing algorithm for Gabor depth imaging

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ABSTRACT

Adaptive windowing algorithms are critical for improving imaging speed with the Gabor wavefield extrapolator. These algorithms help to reduce redundancy of computation while keeping a certain level of imaging accuracy. We propose and test a new adaptive windowing algorithm for the Gabor depth imaging. The new algorithm uses a phase error criterion in the Gabor extrapolation algorithm to determine a set of spatially variable windows that sum to unity. The new windowing algorithm incorporates an important physical constraint because the phase is one of the basic quantities used to describe propagating wavefields. The phase error windowing algorithm is indirectly dependent upon both velocity and wavenumber since the phase is a function of both variables. Comparisons between the new algorithm and a previously published method using velocity alone as a criterion are presented.

INTRODUCTION

The Gabor wavefield extrapolation uses the Gabor transform, also known as the windowed Fourier transform, to deal with lateral velocity variations (Margrave and Lamoureux, 2001). This method is related to the 'phase-screen propagator' method (Wu and Huang, 1992; Roberts et al., 1997; Jin et al., 2002) imaging strong lateral velocity variations. Windows are chosen to localize wavefields in regions (spatially) and correspondingly the wavefield extrapolation problem is also localized. In each window, we calculate a local mean velocity from the migration velocity model and refer to these localized velocities as reference velocities and use them to calculate the Gabor extrapolators. The Gabor extrapolator is an approximation to the locally homogeneous extrapolator such as the generalized phase shift plus interpolation (GPSPI) extrapolator (Margrave and Ferguson, 1999).

The Gabor transform usually uses a set of uniform windows, where each window is a spatially translated (shifted) copy of a mother window, which in this paper is a Gaussian. We choose our set of translated Gaussian windows such that they satisfy a partition of unity (POU) (Margrave and Lamoureux, 2001), meaning that their sum is precisely unity over the real line. In the Gabor wavefield extrapolation, we use these localize velocities to calculate the locally homogeneous extrapolators and use them to extrapolate wavefields. If the width of Gaussian windows is very narrow, then it can be shown that the Gabor wavefield extrapolation algorithm approaches the same result as GPSPI. In practice, this is not useful because computation time will be overwhelming. For a practical algorithm, the window width must relate to the velocity variation, and since this is a function of position, it follows that we need to vary the window width with position. For example, in a laterally homogeneous velocity profile, there is no velocity variation in the lateral dimensions. In this case, only one reference velocity and one Fourier transform are required. For slowly varying velocity structures, more windows are required to achieve an accurate extrapolation. In abrupt velocity structures, we use many windows to get the approximate wavefield extrapolators accurate enough according to a certain accuracy criterion. Thus, we need an adaptive windowing algorithm to determine the minimum number of windows such that the Gabor extrapolation operator satisfies an accuracy criterion in wavefield extrapolations. Grossman et al. (2002) gave an adaptive windowing algorithm utilizing velocity gradients in the lateral dimensions to decide where windows are needed. The algorithm has been used in a Marmousi imaging with the Gabor wavefield extrapolator (Ma and Margrave, 2005) and has shown good imaging results. We introduce the new phase-error adaptive windowing algorithm, which considers velocity variations indirectly, by directly using phase error as the windowing criterion. The new algorithm gives a better adaptive windowing scheme than the Grossman et al. (2002) algorithm [1] does in some aspects, resulting in more physically controlled windowing in the Gabor imaging.

THE PHASE-ERROR ADAPTIVE WINDOWING ALGORITHM

The Gabor extrapolator in our depth imaging applications is used as an approximation of the GPSPI extrapolator[2]. A GPSPI extrapolation is an 'locally homogeneous' wavefield extrapolation method, which can be written as (after Margrave and Ferguson, 1999; Margrave et al., 2004).

$$\psi_P(x_T, z+\Delta z) = \int_{\mathbb{R}} \hat{\psi}(k_T, z)\hat{W}(k_T, x_T, \Delta z)\exp\left(-ik_T x_T\right)\mathrm{d}k_T, \tag{1}$$

where

$$\hat{W}(k_T, x_T, \Delta z) = \exp\left(ik_z\left(x_T\right)\Delta z\right), \tag{2}$$

and

$$k_z(x_T) = \begin{cases} \sqrt{\frac{\omega^2}{v^2(x_T)} - k_T^2}, & \frac{\omega^2}{v^2(x_T)} > k_T^2 \\ i\sqrt{k_T^2 - \frac{\omega^2}{v^2(x_T)}}, & \frac{\omega^2}{v^2(x_T)} < k_T^2, \end{cases} \tag{3}$$

where Δz is the step size in z (vertical) direction for the wavefield extrapolation, ω [3] is temporal frequency and $v(x_T)$ denotes velocity as a function of lateral position along a thin layer with thickness of Δz. Equation (1) extrapolates wavefields at depth z to depth $z+\Delta z$ in the frequency-wavenumber domain.

In the Gabor wavefield extrapolation, we use the Gabor extrapolator in windows to replace the exact extrapolators in equation (1) (defined by (2) and (3)). i.e., we use

[1] We will call it the velocity gradient adaptive windowing (VGAW) algorithm in the following sections.
[2] We treat it as a so-called 'exact' extrapolator.
[3] All the wavefields that we mentioned and will talk about are in the temporal Fourier domain. i.e., they have a temporal frequency (ω) dependency, which we haven't expressed explicitly.

$$\hat{W}(k_T, x_T, \Delta z) \approx \sum_{j \in \mathbb{Z}} \Omega_j(x_T) S_j(x_T) \hat{W}_j(k_T, \Delta z) \tag{4}$$

to calculate the approximation of the exact wavefield extrapolator $\hat{W}$ described in equations (2) and (3), where Ω_j is a family of windows composing a partition of unity (POU) (Margrave and Lamoureux, 2001), $S_j(x_T)$ is a split-step Fourier correction, $\hat{W}_j(k_T, \Delta z)$ is an approximate wavefield extrapolator calculated with a k_z defined by a similar formula to (3) using reference velocity v_j instead of v_T in the formula (see also equation (6)). The symbols in (4) are defined as

$$S_j(x_T) = \exp\left(i\omega\Delta z\left(\frac{1}{v(x_T)} - \frac{1}{v_j}\right)\right), \tag{5}$$

$$\hat{W}_j(k_T, \Delta z) = \exp\left(ik_z \Delta z\right) \tag{6}$$

and

$$v_j = \frac{\int_{\mathbb{R}} \Omega_j(x_T) v(x_T) \mathrm{d}x_T}{\int_{\mathbb{R}} \Omega_j(x_T) \mathrm{d}x_T}. \tag{7}$$

The $\hat{W}_j(k_T, \Delta z)$ in equation (6) has a dependency on the reference velocity v_j inside window $\Omega_j(x_T)$, which is defined in (7), and makes $\hat{W}_j(k_T, \Delta z)$ implicitly depend on x_T within window $\Omega_j(x_T)$.

Using (4) in (1) and rearranging gives

$$\psi_P(x_T, z + \Delta z) \approx \sum_{j \in \mathbb{Z}} S_j(x_T) \Omega_j(x_T) \int_{\mathbb{R}} \hat{\psi}(k_T, z) \hat{W}_j(k_T, \Delta z) \exp\left(-ik_T x_T\right) \mathrm{d}k_T, \tag{8}$$

which is the wavefield extrapolation formula used in the Gabor wavefield extrapolation.

An appropriate interpretation of equation (8) should be the Gabor wavefield extrapolation with a split-step Fourier correction; in (8), a local wavefield $\hat{\psi}(k_T, z)$ has been extrapolated with a local wavefield extrapolator [4] $W_j(k_T, \Delta z)$ related to a constant velocity only locally valid, spatially transformed with the inverse Fourier transform, applied with a split-step correction S_j and localized by a window $\Omega_j(x_T)$. After all these processes, we get the extrapolated wavefield $\psi_P(x_T, z + \Delta z)$ at depth $z + \Delta z$ from the one at depth z.

[4] We call it the Gabor extrapolator.

Since wavefield extrapolator is directly related to the accuracy of wavefield extrapolation, the error between the locally homogeneous wavefield extrapolator $\hat{W}(k_T, x_T, \Delta z)$ and its approximation $\sum_{j\in\mathbb{Z}}\Omega_j(x_T)S_j(x_T)\hat{W}_j(k_T, \Delta z)$ can be set as a criterion to control accuracy in the Gabor wavefield extrapolation. That is,

$$\epsilon = \left\| \arg\left(\hat{W}(k_T, x_T, \Delta z)\right) - \arg\left(\sum_{j\in\mathbb{Z}} S_j(x_T)\Omega_j(x_T)\hat{W}_j(k_T, \Delta z)\right)\right\|, \tag{9}$$

where ϵ is the phase error in terms of arguments of the wavefield extrapolators, $\arg$ denotes arguments of complex numbers defined in a form of

$$\arg(x + iy) = \tan^{-1}(\frac{y}{x}), \tag{10}$$

where x, y are real numbers, $\|\|$ is an appropriate norm. Notice that ϵ depends on the temporal frequency ω, the transverse wavenumber k_T and the transverse coordinate x_T.

In this paper, we select L-1 norm for the phase error estimation (9), which is

$$\begin{aligned} \epsilon &= \left\| \arg\left(\hat{W}(k_T, x_T, \Delta z)\right) - \arg\left(\sum_{j\in\mathbb{Z}} S_j(x_T)\Omega_j(x_T)\hat{W}_j(k_T, \Delta z)\right)\right\|_1 \\ &= \int_{\mathbb{R}} \left| \arg\left(\hat{W}(k_T, x_T, \Delta z)\right) - \arg\left(\sum_{j\in\mathbb{Z}} S_j(x_T)\Omega_j(x_T)\hat{W}_j(k_T, \Delta z)\right)\right| \mathrm{d}x_T. \end{aligned} \tag{11}$$

For convenience, we will discuss the phase error related to one of these windows. From equation (11), we can write a similar formula to estimate the phase error corresponding to a specific window Ω_j. We have

$$\epsilon_j = \left\| \arg\left(\Omega_j(x_T)\hat{W}(k_T, x_T, \Delta z)\right) - \arg\left(\Omega_j(x_T)\sum_{k\in\mathbb{Z}} S_k(x_T)\Omega_k(x_T)\hat{W}_j(k_T, \Delta z)\right)\right\|_1 \quad j, k \in \mathbb{Z}, \tag{12}$$

where ϵ_j is the total phase error within the j^{th} window Ω_j, $\hat{W}(k_T, x_T, \Delta z)$ and its approximation have been windowed by Ω_j, the reference velocity used to calculate the Gabor extrapolator equals to the j^{th} reference velocity v_j. There are methods for phase error estimations (Ferguson and Margrave, 2005). We select a fractional phase error estimation for this paper, which is

$$\epsilon_{jr} = \frac{\left\| \arg\left(\Omega_j(x_T)\hat{W}(k_T, x_T, \Delta z)\right) - \arg\left(\Omega_j(x_T)\sum_{k\in\mathbb{Z}}\Omega_k(x_T)S_k(x_T)\hat{W}_k(k_T, \Delta z)\right)\right\|_1}{\left\|\arg\left(\Omega_j(x_T)\hat{W}(k_T, x_T, \Delta z)\right)\right\|_1}, \quad j, k \in \mathbb{Z} \tag{13}$$

where ϵ_{jr} is defined as a fractional or relative phase error with respect to the phase of the exact extrapolator in the j^{th} window.

Notice that in equation (13), the calculation of the exact and approximate wavefield extrapolators is involved in three domains, i.e., they are functions of three variables, x_T, k_T and ω. For k_T in phase error estimation formula (13), we use the first part of equation (3) to calculate k_z in the propagation regime. k_z used in the calculation of the exact extrapolator is a function of x_T via $v(x_T)$. Using (13) and a threshold for phase errors, we can control phase errors for the Gabor wavefield extrapolator, thus the accuracy of propagating seismic wavefields. Since phase errors are related to the temporal frequency, we expect that the PEAW windowing results vary with wavefields at different frequencies, and its dependency on the lateral wavenumber k_T indicates that wavefields propagating in all directions will be counted in the PEAW windowing algorithm. The VGAW algorithm and the PEAW algorithm both use the lateral velocity variations to determine the number of windows. However, the PEAW use more physical parameters such as frequency and wavenumber. Therefore, the PEAW algorithm is a better windowing algorithm than the VGAW algorithm, in which wavefields with various frequencies and propagation directions are treated in the same way as each other.

Equation (13) is the formula used for the phase error estimation in a single window. To get the total phase error of the Gabor extrapolators from all the windows, we can sum up all the phase errors given by equation (13).

We give a brief description on how the phase error adaptive windowing (PEAW) algorithm works. In a 2D wavefield extrapolation, for example, given a velocity profile (1D), we start with one single window across the whole velocity profile. Using equation (13) and a relative phase error limit, say 10%, the algorithm can tell if the current window needs splitting or not. If $\epsilon_{jr} > 10\%$, the current window will be split in the middle; otherwise, leave it alone. If $\epsilon_{jr} < 10\%$ and this is the first and the only window and it has not been split before, we know that this is the case of an approximately laterally homogeneous velocity model, which does not cause the phase error ϵ_{jr} to exceed the limit 10%. If it is not the first and the only one, we will move into the next window and repeat the process until the last one. This process is called 'sweeping'. The algorithm starts 'sweeping' over and over until the phase error criterion is satisfied in all windows. Following this the algorithm begins another process called 'combining'.

Though all individual windows are not splittable by themselves, there is the possibility that some neighbouring windows may have the same (or very close) reference velocity as each other now, which means that the combined window with them may also satisfies the phase error criterion. This was not the case before because they were combined with other windows containing fairly different velocities and violated the phase error criterion. The algorithm goes back and inspects if this is true for each pair of neighbouring windows. The 'combining' process will merge them depending on the phase error in the newly combined window. The 'combining' process is also recursively executed until the phase error criterion does not hold for a combined window from any pair of neighbouring windows. In this way, we may reduce the number of windows and eliminate some redundancy in wavefield extrapolations.

APPLICATION OF THE ADAPTIVE WINDOWING ALGORITHMS

We design some velocity profiles such as a homogeneous model, a step velocity model and a combined velocity model recreated from Grossman et al. (2002).

Windowing with simple velocity profiles

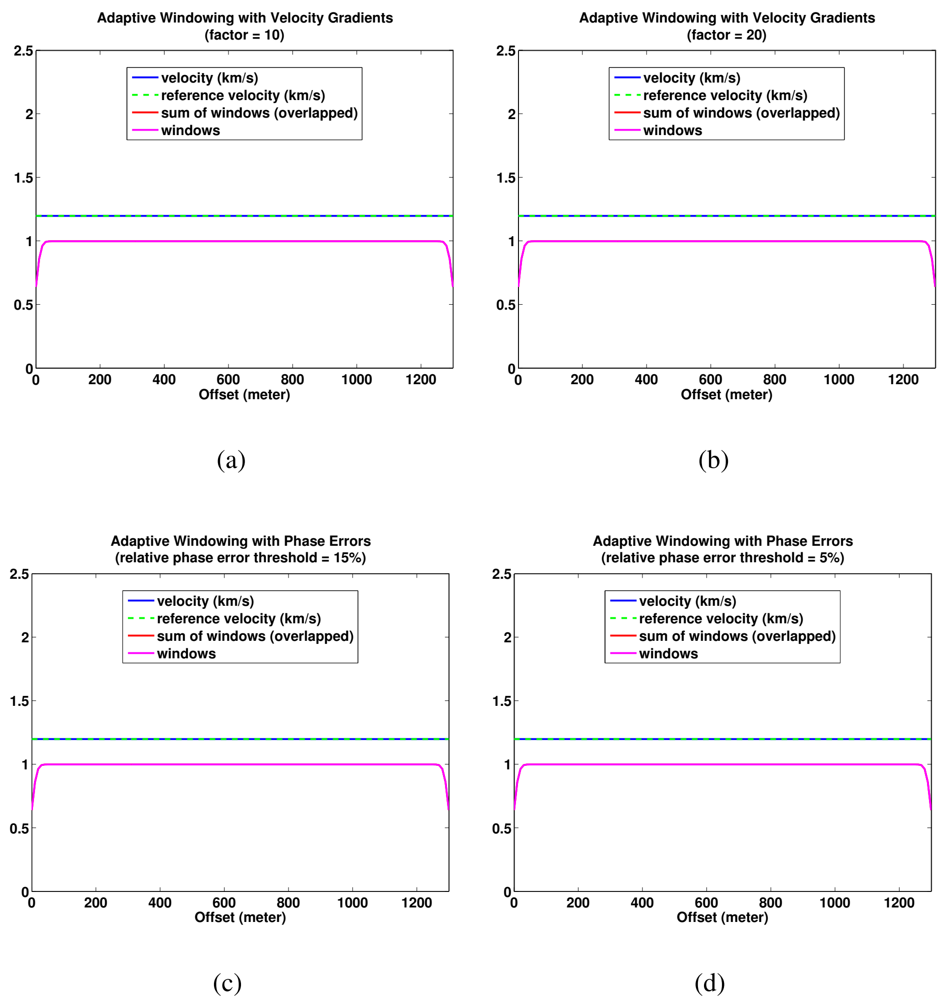

FIG. 1. Windowing with a Homogeneous Velocity Model

In the velocity gradient adaptive windowing (VGAW) algorithm, we use a parameter 'factor' to control the number of windows according to the lateral velocity variations. The larger the 'factor' is, the more windows are used; the smaller the 'factor' is, the fewer windows are used. For more discussion on parameter 'factor', see Ma and Margrave (2005). In the phase error adaptive windowing (PEAW) algorithm, we set a threshold as the percentage of the phase of the exact extrapolator, the relative phase errors of the Gabor extrapolators will be compared to this threshold to determine the number of windows. If the threshold is set small, the PEAW algorithm gives more windows. Otherwise, fewer windows will be created. For constant velocity profiles, we can predict that either of the adaptive algorithms should give one uniform window across a given velocity profile no matter what thresholds are used (see in Figure 1).

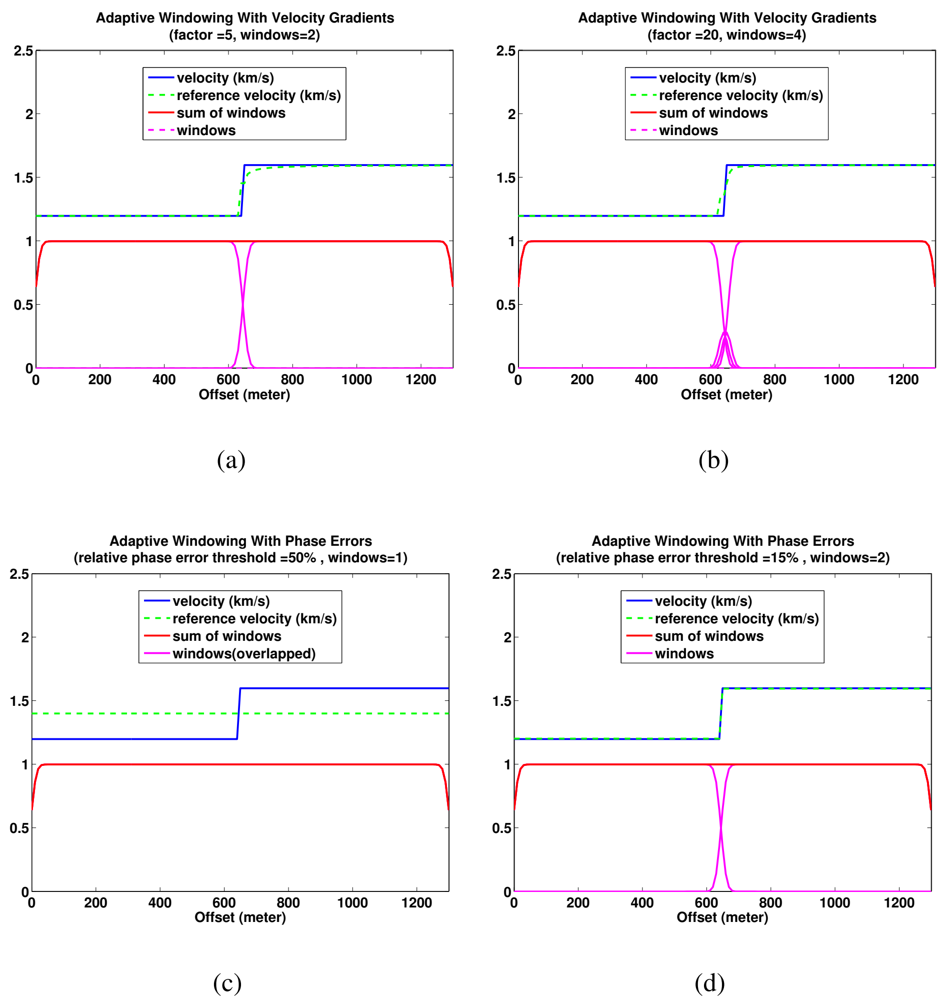

FIG. 2. Windowing with a Step Velocity Model

The adaptive windowing results with both algorithms on a step velocity profile are shown in Figure 2. We can see that the VGAW algorithm gives different number of windows with different thresholds; for factor=5, we have 2 windows, and for factor=20, we have 4 windows. For the PEAW algorithm, if the phase error threshold is set large (50%), then fewer windows are used; if the threshold is small, more windows are used (compare Figure 2 (c) and (d)). We also can see that when we change the controlling parameters in the both adaptive windowing algorithms, the number of windows varies in different ways. For the VGAW algorithm, windows are added in regions with velocity gradients. For the PEAW algorithm, windows are split according to the phase errors due to velocity variations.

Windowing with complicated velocity profiles

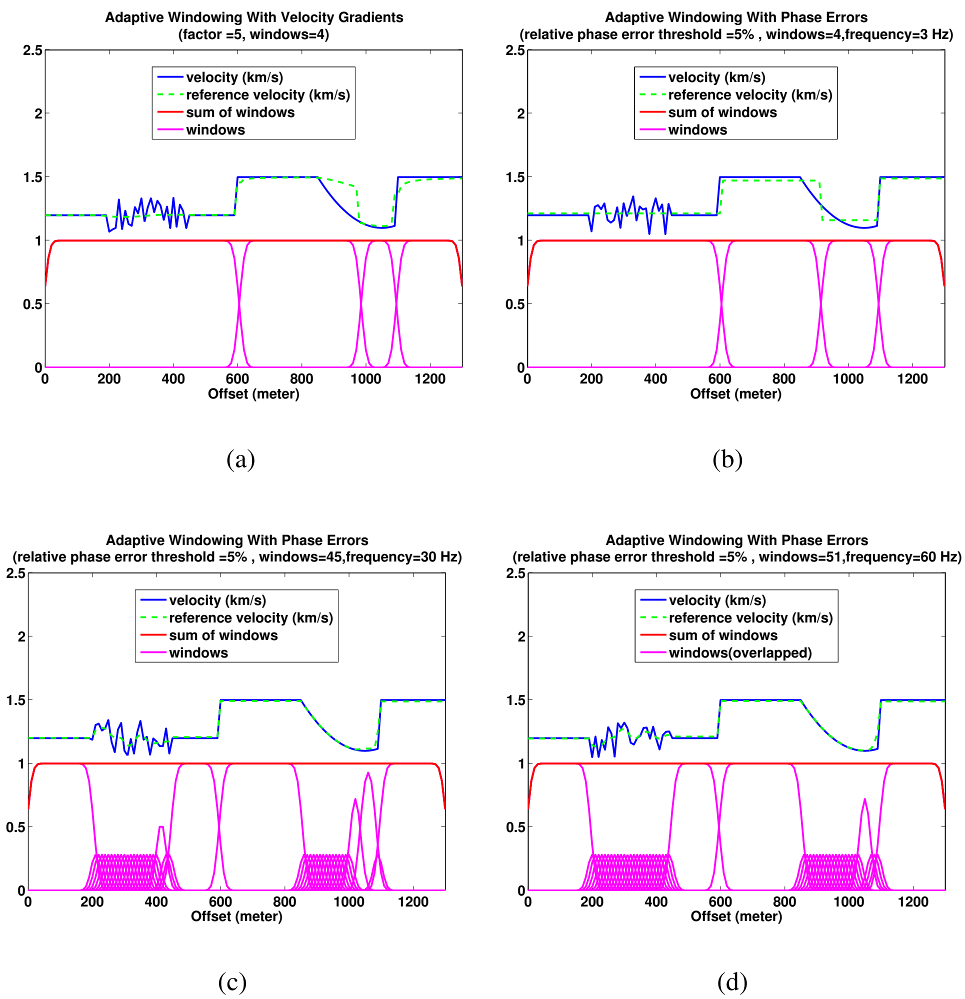

FIG. 3. Windowing with a Complicated Velocity Model

We use a velocity model recreated from Grossman et al. (2002), which includes lateral velocity structures of a few kinds. Such structures as homogeneous, random, step, steepingly varying velocity models are combined together. The results shown in Figure 3 are used to demonstrate the PEAW algorithm while changing the temporal frequency. A result using the VGAW algorithm is shown in Figure 3 (a) for comparison. Since the phase error of the Gabor wavefield extrapolator is related to the number of windows, we think the accuracy of the Gabor wavefield extrapolation can be directly related to the number of windows (without proof). As a result, we can compare the VGAW windowing (Figure 3 (a)) to the PEAW windowing (Figure 3 (b)) because they have almost the same set of windows, which means that they create Gabor extrapolators with comparable accuracies. Based on these, we conclude that with a factor of 5 used in the VGAW method, we can only get a comparable accuracy given by the PEAW with a relative phase error of 5% for wavefields with a frequency of about 3 Hz. If we increase the frequency, then we can not extrapolate wavefields with the same set of windows (for 3 Hz) at frequencies beyond 3 Hz and keep phase errors below 5%.

In Figure 3 (b), (c) and (d), we can see that the number of windows rises very quickly with the increment of the frequency, which indicates that more windows are needed in order to keep phase errors below the threshold 5% even the velocity profile remains unchanged.

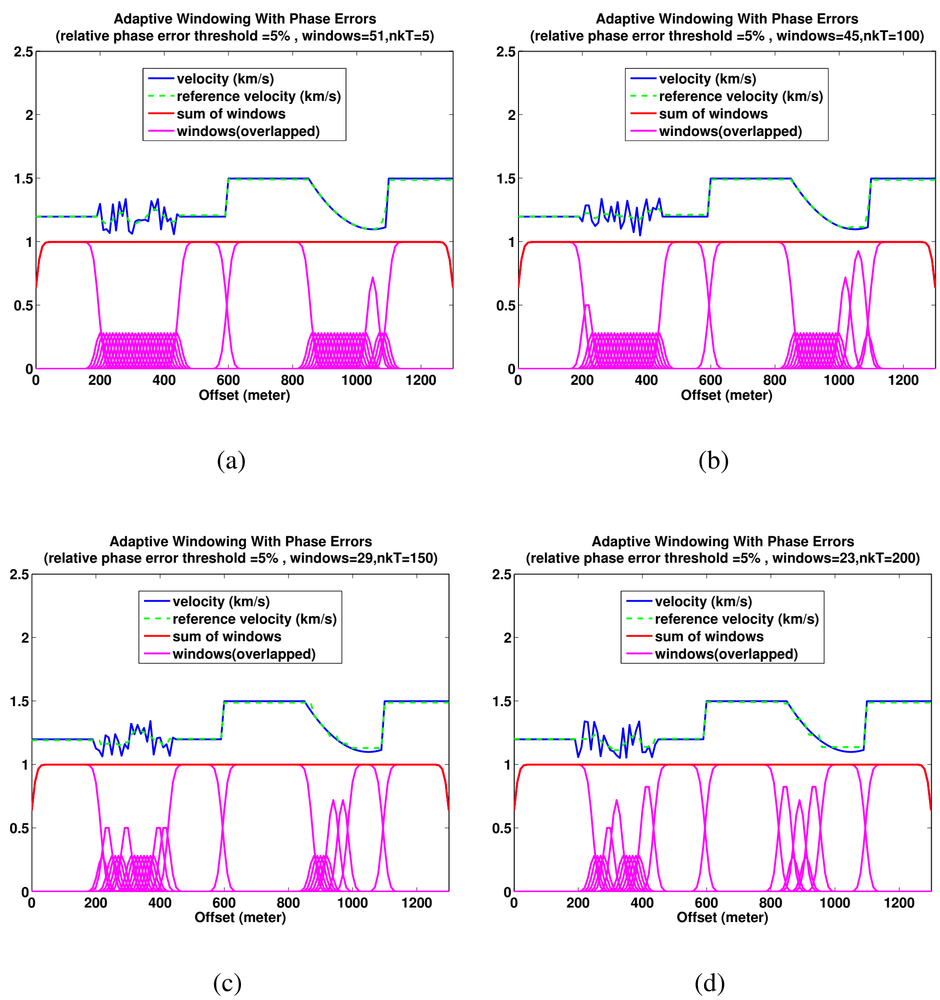

FIG. 4. Windowing with a Complicated Velocity Model

We have seen the windowing results varying with velocity models, showing how windows adapt to the lateral velocity structures. We have also observed that the temporal frequency has influence on the number of windows in the PEAW (see Figure 3). Figure 4 shows the windowing results using the PEAW when we employ different numbers of the transverse wavenumber k_T.

In Figure 4, we use the same the velocity model and the frequency (30 Hz). When we use more k_T's, the PEAW gives fewer windows to meet the phase error criterion (5% relative phase error). We explain this phenomenon in the way that wavefields propagating in various directions (corresponding to k_T) tend to average out phase errors. We can use many k_T's to make the redundency of windows as less as possible. In this way, efficient calculations in the Gabor extrapolation is expected if we use this property of the PEAW.

When we turn to the VGAW, we can only use the information on velocity variations. With the PEAW algorithm, the number of windows is controlled by three parameters, all of which are related to the physical properties of the wavefields and the Gabor extrapolator. In the VGAW, we can not relate the accuracy of wavefield extrapolations to the physical parameter such as the temporal (ω) and spatial frequencies (k_T). Therefore, we are not sure about phase errors to wavefields at various frequencies and propagating in directions. The VGAW may help us to get accurate phases for a certain wavefield but not all possible wavefields. The PEAW is one of the best windowing algorithms for the Gabor wavefield extrapolation.

CONCLUSIONS

The phase error adaptive windowing (PEAW) algorithm is a way better than the velocity gradient adaptive windowing (VGAW) method. The PEAW algorithm considers phase errors as criteria for window choosing, which gives a physically related windowing scheme and a possibility to get better and reliable imaging with the Gabor extrapolator.

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