Dispersion and apparent attenuation due to fine stratigraphy

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ABSTRACT

Since the seminal work of O'Doherty and Anstey, which describes a relationship between the spectra of the reflectivity of a one-dimensional random medium and of a transmitted pulse, a number of researchers have investigated the qualitative nature and the extent of the validity of this approximate relationship from different angles. A description of the design of an experiment to qualitatively illustrate the relationship is documented. The results of that experiment and some potential applications of the use of the effect then follow.

INTRODUCTION

O'Doherty and Anstey (1971) maintain that the power spectral density R of the reflection coefficients of a finely layered random medium is related to the magnitude spectrum of a transmitted spike through that medium in the following way:

$$T(\omega) \propto e^{-R\Delta}. \tag{1}$$

where Δ is the two-way travel time when the system is viewed according to Goupillaud layering (Goupillaud, 1961; Hsu and Burridge, 1991).

Original Derivation

The original authors explain the result by partitioning the amplitude of the one way transmitted response, that which a receiver at the base of the layers would receive, according to the delay l introduced into the response due to multiple reflections, and the number $2k$ of internal reflections. They insist that when $-m$ is the causal part of the autocorrelation of the reflectivity series, the contribution $s_k(l)$ of that part which has undergone $2k$ internal reflections and has been delayed by $k\Delta$ seconds satisfies the following relations:

$$\begin{aligned} s_1(l) &= m(l) \\ s_{k+1} &= \frac{1}{k} s_k * m. \end{aligned} \tag{2}$$

Note that s_0 is the direct arrival and that the crucial convolution above is discussed further in Appendix A. So the one way transmitted response is proportional to the expression

$$1 + s_1 + s_2 + \cdots + s_k + \cdots = 1 + s_1 + \frac{1}{2} s_1 * m + \frac{1}{3!} s_1 * m * m + \cdots$$

and that the magnitude spectrum of this response is proportional to e^M, where $M = \widehat{m}$. The magnitude spectrum of the two-way response is now proportional to

$$e^{2M} = e^{-R\Delta}.$$

Related work

The relationship between the O'Doherty-Anstey effect and apparent attenuation has been a topic of study (Richards and Menke, 1983; Schoenberger and Levin, 1978; Wennerberg and Frankel, 1989). Richards and Menke (1983), in particular, follows a numerical approach which is shared by Martínez, V. L., and Sacchi, M. D; Mateeva, A., Hart, D., and Mackay, S.; Schoenberger and Levin (1974, 1978).

Shapiro and Treitel (1997) treat the effect as the result of a first order approximation of exact fundamental polynomials developed by Robinson and Treitel (1977). * The idea of the formula being a truncated expansion is reiterated by Berlyand and Burridge (1995), who derive their results in great detail in the continuous and discrete cases, show a connection between the Ricatti equation and what they call the linear fractional recursion, and display bounds on the error of the approximation.

The fundamental polynomials of Robinson and Treitel (1977), which form the basis of a z-domain technique for computing multiples, are also the foundations of the perspective in which the amplitudes of multiples are viewed as an autoregressive time-series. This approach is taken by Kerner and Harris (1994); Shapiro and Hubral (1999); Walden and Hosken (1985); Hubral et al. (1980). Walden and Hosken (1985) go further by deriving an approximate relationship between the power spectrum of the logarithms of the acoustic impedance and the power spectrum of reflection coefficients.

More exotic approaches include mean field solutions of the stochastic wave equation versus the Born approximation (Banik et al., 1985), generalization of the Born approximation (Resnick et al., 1986), diffusion approximations in limiting solutions (Burridge and Chang, 1989; Burridge et al., 1988), invariant embedding with perturbation expansions and localization (Shapiro et al., 1996), Gabor wavelets (Morlet et al., 1982a,b), and radiative transfer Haney et al. (2005); van Wijk et al. (2003).

Other authors consider the distinction between regimes in which Backus averaging (Backus, 1962) and Eq. (1) are appropriate (Stovas and Arntsen, 2006) and angle dependency extensions to the typical normal incident case (Burridge and Chang, 1989; Shapiro et al., 1994; Kerner and Harris, 1994).

METHOD

To illustrate the presence of the effect described by Eq. (1), we decided to follow O'Doherty and Anstey (1971) and, in particular to reproduce figure 15 of O'Doherty and Anstey (1971) . This amounts to showing graphically that $\log T \propto -R$. To achieve this goal, a reflectivity series was chosen and a spike wavelet was applied to this series to produce a response with all multiples. The amplitude spectrum of the multiple train was then plotted as was the power spectral density of the reflectivity to effect a comparison as

*This latter paper is important for another reason. In it, the authors show that the recursive algorithm for carrying out the determination of the reflection coefficients is the same as the Levinson recursive method for the solution of normal equations.

in figure 15 of O'Doherty and Anstey (1971) .

The multiple generating code was `theo_simple`, which can be found in the CREWES toolbox. It was written by Margrave and was derived from an algorithm by Waters (1981).

RESULTS

Preliminary results did show broadening of the spike impulse, which is an indication of dispersion (FIG.2) Also, attenuation of higher frequencies is clear in FIG.4. However, as shown in FIG.7, where a positive correlation is seen, the effect was not demonstrated.

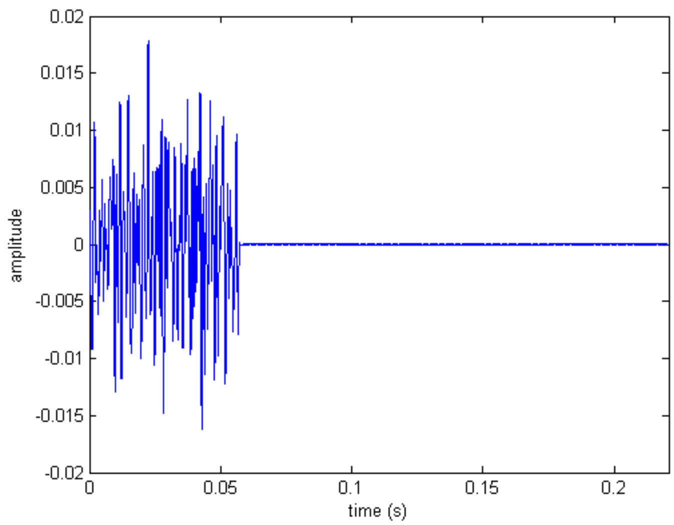

FIG. 1. A reflectivity series which has been padded with zeros to extend multiple train.

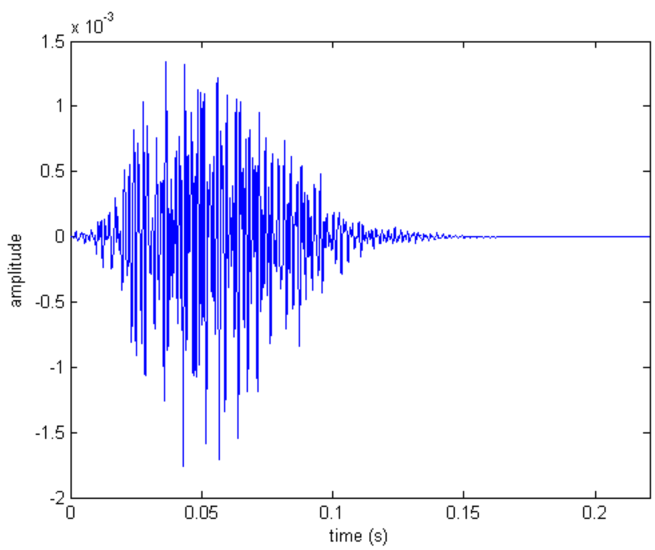

FIG. 2. The Green's function of the layers represented by FIG.1.

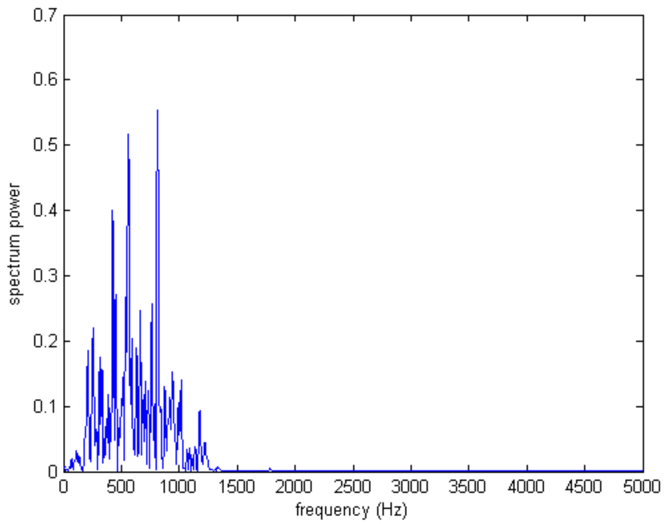

FIG. 3. The full power spectral density of the series in FIG.1.

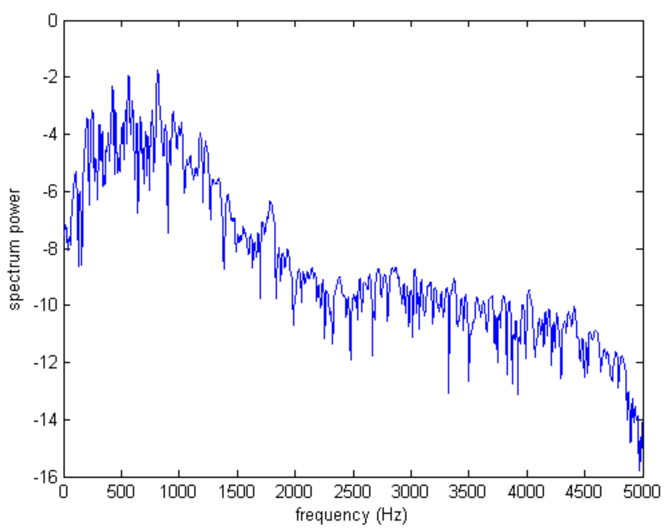

FIG. 4. The logarithm of the full amplitude spectrum of the multiple train in FIG.2.

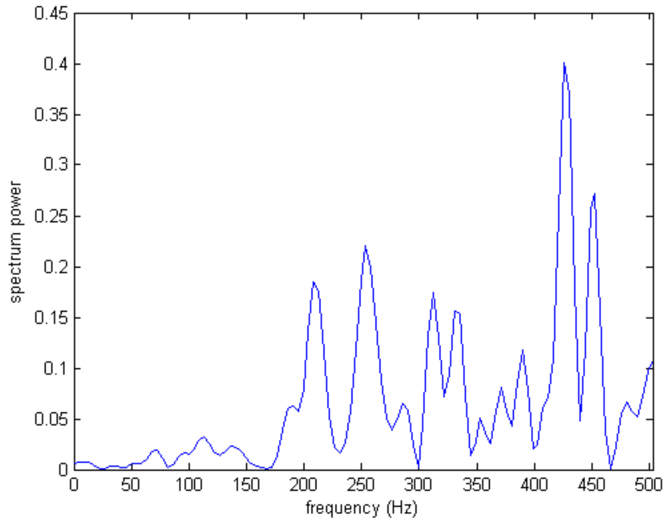

FIG. 5. The power spectral density of the series in FIG.1 restricted to below 500 Hz.

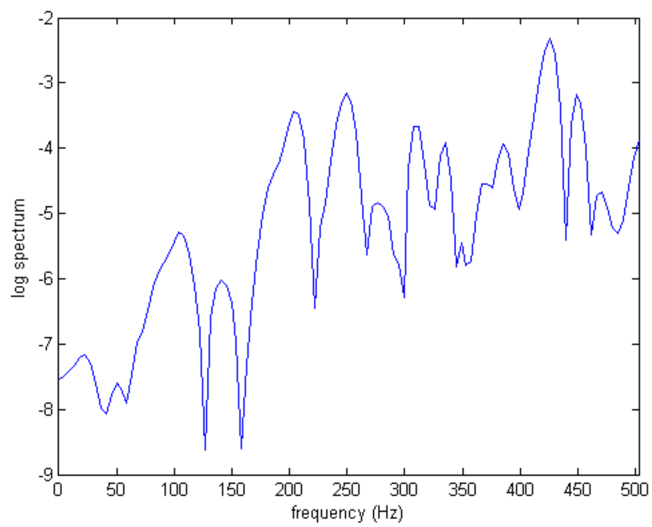

FIG. 6. The logarithm of the amplitude spectrum of the multiple train in FIG.2 restricted to below 500 Hz.

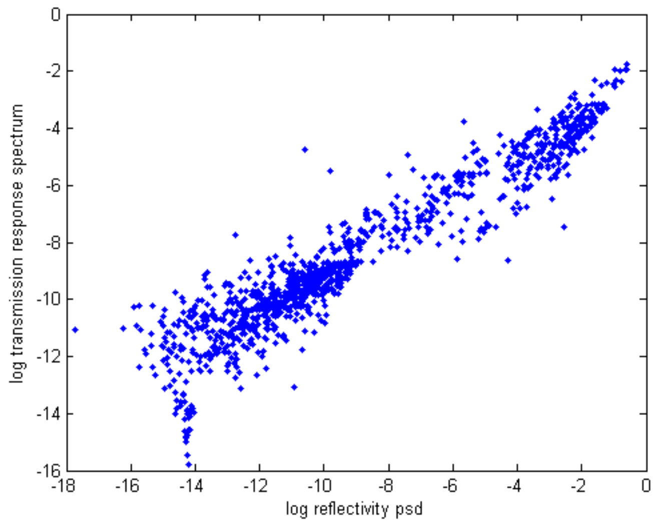

FIG. 7. A scatter plot of log(reflectivity psd) versus log(filter response spectrum) for frequencies below 500 Hz.

CONCLUSIONS

Of all the documents referred to as related work, only Serakiotou (1988, p. 389) and Resnick et al. (1986, p 361) produce a figure describing the effect with the clarity of figure 15 of O'Doherty and Anstey (1971) . This suggests that as a tool for applied work, the O'Doherty-Anstey relation is still evolving.

However, the appeal of a technique which would enable processors to see structure beyond wavelength limitations and to reconstruct reflectivities beneath VSP logs remains. Were we able to construct the power spectral density of the series from multiple transmission response data, the full reflectivity would be calculable as stratigraphic filters are minimum phase (Appendix B; Noted by O'Doherty and Anstey (1971)).

APPENDIX A

THE INTUITION BEHIND EQ. (2)

The essential part of the formula ignores the effect of transmission coefficients and focuses on reflections. This is one feature of the approximation, which may affect its accuracy.

As illustrated in FIG.A-1, the contribution to $s_k(l)$ due to k upward directed subpaths of length $l_1, l_2, \cdots, l_k$, where $l = \sum_{j=1}^{k} l_j$, is given by (A-1) below.

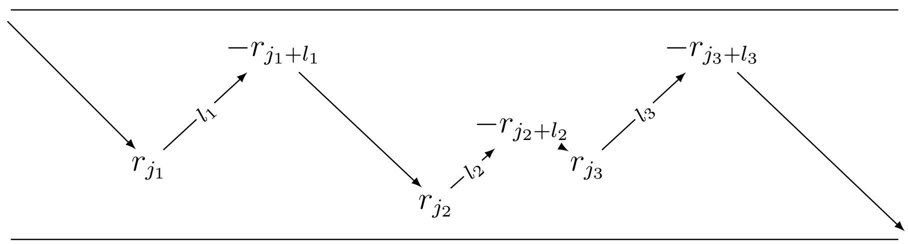

FIG. A-1. A raypath contributing to $s_k(l)$.

$$\underbrace{\left(-\sum_{j_1=1}^{N-l_1} r_{j_1} r_{j_1+l_1}\right)\left(-\sum_{j_2=1}^{\min(N-l_2, j_1+l_1-1)} r_{j_2} r_{j_2+l_2}\right)\left(-\sum_{j_3=1}^{\min(N-l_3, j_2+l_2-1)} r_{j_3} r_{j_3+l_3}\right)\cdots}_{k\text{-fold sum}} \tag{A-1}$$

The upper limits on the summations reflect the facts that rays are not transmitted across the surface, and that each upward subpath starts below a downward one. If we set all these upper limits to N, then the k-fold sum can be written as a k-fold product of terms like

$$-\sum_{j_i=1}^{N} r_{j_i} r_{j_i+l_i},$$

which is minus the autocorrelation of r at delay l_i. But the l_i's sum to l, so

$$s_k(l) = \frac{\overbrace{m * m * \cdots * m}^{k\text{-fold convolution}}}{k!},$$

where the $k!$ in the denominator captures the ordering imposed by the starting condition above.

APPENDIX B

THE FILTER IS MINIMUM PHASE

Since $-m$ is causal, the real and imaginary parts of its spectrum M form a Hilbert transform pair:

$$\Im M = H \Re M.$$

Thus the phase ϕ of $e^{-R\Delta}$ is

$$\phi = \Im(-R\Delta) = \Im(2M) = H(\Re(2M)) = H \log |e^{2M}|,$$

which means that $e^{-R\Delta}$ is minimum phase.

REFERENCES

Backus, G. E., 1962, Long-wave elastic anisotropy produced by horizontal layering: Jour. Geophys. Res, **67**, 4427–4440.

Banik, N. C., Lerche, I., and Shuey, R. T., 1985, Stratigraphic filtering part I: Derivation of the Odoherty-Anstey formula: Geophysics, **50**, No. 12, 2768–2774.

Berlyand, L., and Burridge, R., 1995, The accuracy of the O'Doherty-Anstey approximation for wave propagation in highly disordered stratified media: Wave Motion, **21**, 357–373.

Burridge, R., and Chang, H. W., 1989, Multimode, one-dimensional wave propagation in a highly discontinuous medium: Wave Motion, **11**, 231–249.

Burridge, R., Papanicolaou, G., and White, B. S., 1988, One-dimensional wave propagation in a highly discontinuous medium: Wave Motion, **10**, 19–44.

Goupillaud, P. L., 1961, An approach to inverse filtering of near-surface layer effects from seismic records: Geophysics, **26**, No. 06, 754–760.

Haney, M., Wijk, K. V., and Snieder, R., 2005, Radiative transfer in layered media and its connection to the O'Doherty-Anstey formula: Geophysics, **70**, No. 01, T1–T11.

Hsu, K., and Burridge, R., 1991, Effects of averaging and sampling on the statistics of reflection coefficients: Geophysics, **56**, No. 01, 50–58.

Hubral, P., Treitel, S., and Gutowski, P. R., 1980, A sum autoregressive formula for the reflection response: Geophysics, **45**, No. 11, 1697–1705, discussion in GEO-48-03-0400-0400; Reply in GEO-48-03-0401-0401; Discussion in GEO-50-08-1357-1357; Reply in GEO-50-08-1357-1357.

Kerner, C., and Harris, P. E., 1994, Scattering attenuation in sediments modeled by ArMa processes–validation of simple Q models: Geophysics, **59**, No. 12, 1813–1826.

Martínez, V. L., and Sacchi, M. D, 2003, Deconvolution in the presence of stratigraphic filtering, SBGf - Sociedade Brasileira de Geofísica, 8th International Congress of The Brazilian Geophysical Society.

Mateeva, A., Hart, D., and Mackay, S., 2003, Apparent attenuation from short-period multiples and instrinsic absorption in the seismic wavelet model, 65th Annual EAGE Conference.

Morlet, J., Arens, G., Fourgeau, E., and Giard, D., 1982a, Wave propagation and sampling theory - Part I: Complex signal and scattering in multilayered media: Geophysics, **47**, No. 02, 203–221, discussion in GEO-49-09-1562-1563; Reply in GEO-49-09-1564-1564.

Morlet, J., Arens, G., Fourgeau, E., and Giard, D., 1982b, Wave propagation and sampling theory - Part Ii: Sampling theory and complex waves: Geophysics, **47**, No. 02, 222–236.

O'Doherty, R. F., and Anstey, N. A., 1971, Reflections on amplitudes: Geophysical Prospecting, **19**, 430–458.

Resnick, J. R., Lerche, I., and Shuey, R. T., 1986, Reflection, transmission, and the generalized primary wave: Geophys. Jour. Roy. Astr. Soc, **87**, 349–377.

Richards, P. G., and Menke, W., 1983, The apparent attenuation of a scattering medium: Bull. Seis. Soc. Am, **73**, 1005–1021.

Robinson, E. A., and Treitel, S., 1977, The spectral function of a layered system and the determination of the waveforms at depth: Geophys. Prosp, **25**, 434–459.

Schoenberger, M., and Levin, F. K., 1974, Apparent attenuation due to intrabed multiples: Geophysics, **39**, No. 03, 278–291.

Schoenberger, M., and Levin, F. K., 1978, Apparent attenuation due to intrabed multiples Ii: Geophysics, **43**, No. 04, 730–737.

Serakiotou, N., 1988, Thin layer filtering and the O'Doherty-Anstey approximation, *in* SEP-57, Stanford Exploration Project: SEP-Report, 383–392.

Shapiro, S., and Hubral, P., 1999, Elastic waves in random media: Springer.

Shapiro, S. A., Hubral, P., and Ursin, B., 1996, Reflectivity/transmissivity for one-dimensional inhomogeneous random elastic media: dynamic-equivalent-medium approach: Geophys. Jour. Int, **126**, 184–196.

Shapiro, S. A., and Treitel, S., 1997, Multiple scattering of seismic waves in multilayered structures: Phys. Earth Planet. Int, **104**, 147–159.

Shapiro, S. A., Zien, H., and Hubral, P., 1994, Generalized O'Doherty-Anstey formula for waves in finely layered media: Geophysics, **59**, No. 11, 1750–1762.

Stovas, A., and Arntsen, B., 2006, Vertical propagation of low-frequency waves in finely layered media: Geophysics, **71**, T87–T94.

van Wijk, K., Haney, M., Snieder, R., and Scales, J. A., 2003, Radiative transfer in 1d, and the connection to the O'Doherty-Anstey formula: SEG Expanded Abstracts.

Walden, A. T., and Hosken, J. W. J., 1985, An investigation of the spectral properties of primary reflection coefficients: Geophys. Prosp, **33**, 400–435.

Waters, K. H., 1981, Reflection Seismology: John Wiley & Sons, New York.

Wennerberg, L., and Frankel, A., 1989, On the similarity of theories of anelastic and scattering attenuation: Bull. Seis. Soc. Am, **79**, 1287–1293.