Visualization of spherical tangency solutions for locating a source point from the clock time at four receiver locations

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ABSTRACT

Previous work (Bancroft and Du, 2006) derived the source point of an event from the first arrival times at four receiver locations. The 3D solution was based on the 2D problem in which the source was located from three receiver locations. That solution required the tangency of a source circle with three circles whose radii were proportional to the receiver clock-times. This 2D solution could be visualized, since the circles could be constructed and or drawn on a plane surface. However the 3D solution requires the tangency of a source sphere with four spheres whose radii are also proportional to the receiver clock-times. This paper demonstrates a method of visualizing the 3D solution.

This work has numerous applications that range over well fraccing, the monitoring of hazardous sites, sniper locating, global positioning, and Kirchhoff depth migrations which require the transfer of traveltimes, that are estimated along raypaths, to those on a grid.

INTRODUCTION

The 2D solution constructs three circles whose radii are proportional to the clock-times of the first arrival time, and whose centers are located at the receiver location. Two circles can be drawn tangent to the three receiver circles, one of which is located at the source point, and with a radius that is proportional to the clock-time of the event. The construction of the source circle was solved by Apollonius, a Greek mathematician, over two thousand years ago. This geometrical construction leads to simple algebraic solutions whose complexity is restricted to square-roots. An illustration of this 2D solution is shown in Figure 1. The algebraic solution uses the known velocity of the medium, the location of the three receivers, and the clock-times of the first arrival of the event.

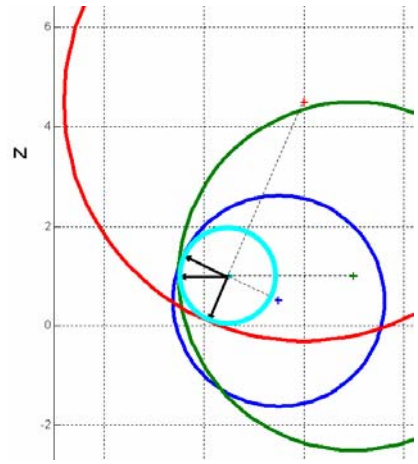

FIG. 1. The 2D solution illustrating the tangency the source circle (cyan) with the three receiver circles that are red, green, and blue.

The 3D solution was derived in a manner that follows the 2D solution and requires the traveltimes at four receiver locations. As in the 2D case, the solution requires simple algebra, with the square-root being the most complex operation. In this problem there are four unknowns, the three coordinates of the source location (x_0, y_0, z_0), and the clock-time t_0 of the source.

VISUALIZATION OF THE 3D SOLUTIONS

An illustration of the 3D solution begins by defining the source location and the clock-time of the event, along with the four receiver locations. The clock-times at the receiver locations were then computed using the know velocity of the medium. Then, only using the receiver locations and their clock-times of the event, the location of the source was computed. Two possible solutions are computed, and the one with the smallest radius is chosen as the solution.

Figure 2 shows a MATLAB display of four semi-transparent spheres that represent the clock-times of the four receivers. Figure 3 is similar to Figure 2, but now includes the source sphere that is tangent to the four receiver spheres. Verifying the tangency of the source sphere is difficult.

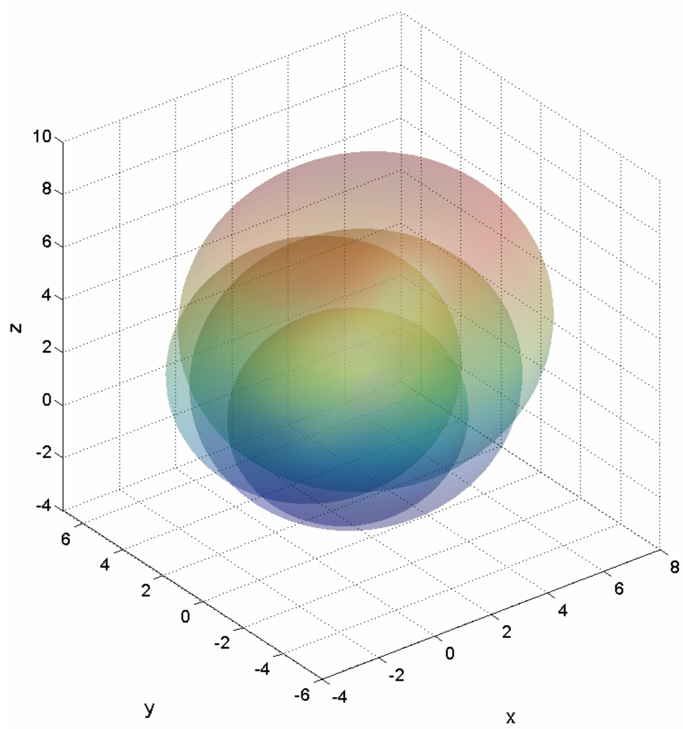

FIG. 2. A three dimensional plot which contains four semi-transparent spheres. Each sphere represents the clock-times of an event four receiver locations.

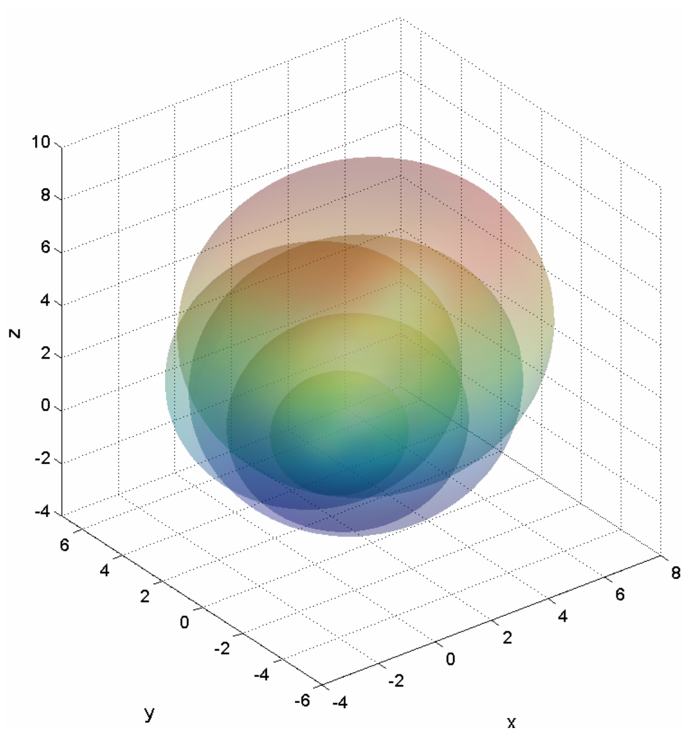

FIG. 3. This figure is similar to Figure 2, but now includes the source sphere. It is not possible from this plot to verify that the source sphere is tangent to the receiver spheres.

When viewing the actual MATLAB display, the 3D figure can be rotated to any viewing angle to verify that the smaller source circle (in this case), is tangent to the other spheres. As an aid in the verification, the source sphere is plotted separately with each receiver sphere in Figure 4. Two views are shown; the left is the original location, but in the right view, the image is 3D rotated to a position that illustrates the tangency. The numerical value of the source location is accurate to the machine round-off error.

COMMENTS

Clock times of the events can be very large. Consequently, the minimum clock-time of the four receivers can be subtracted from each receiver clock-time to improve the resolution of the computation and the construction.

CONCLUSIONS

MATLAB figures were created to display the tangency of a source sphere with four receiver spheres. The radius of each sphere is proportional to the source clock-time or the clock-time at each of the receivers.

ACKNOWLEDGEMENTS

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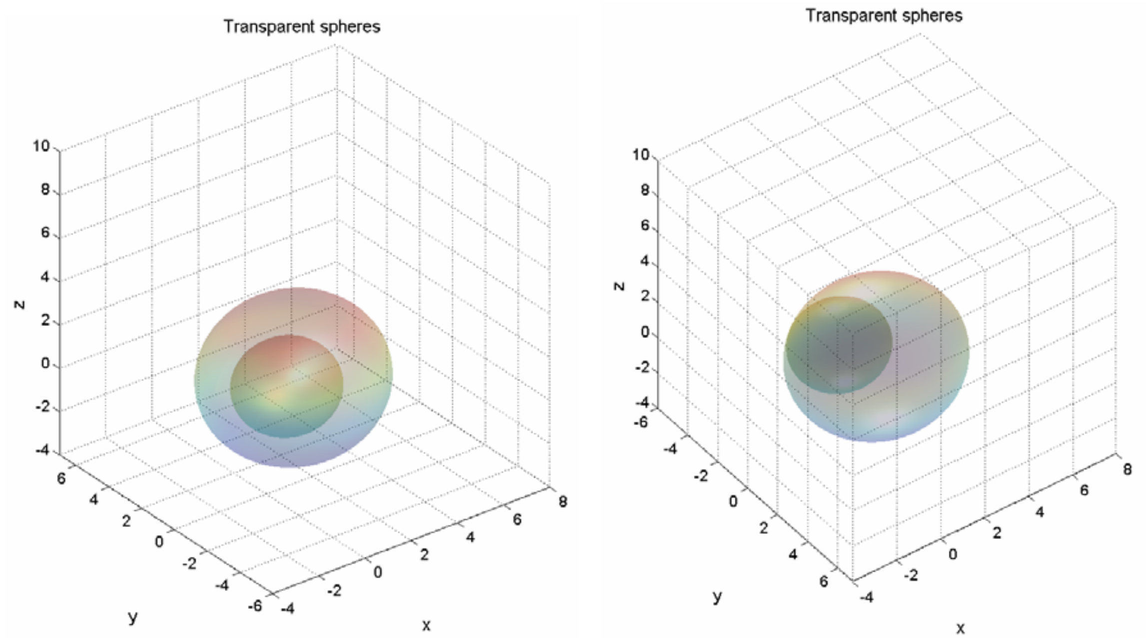
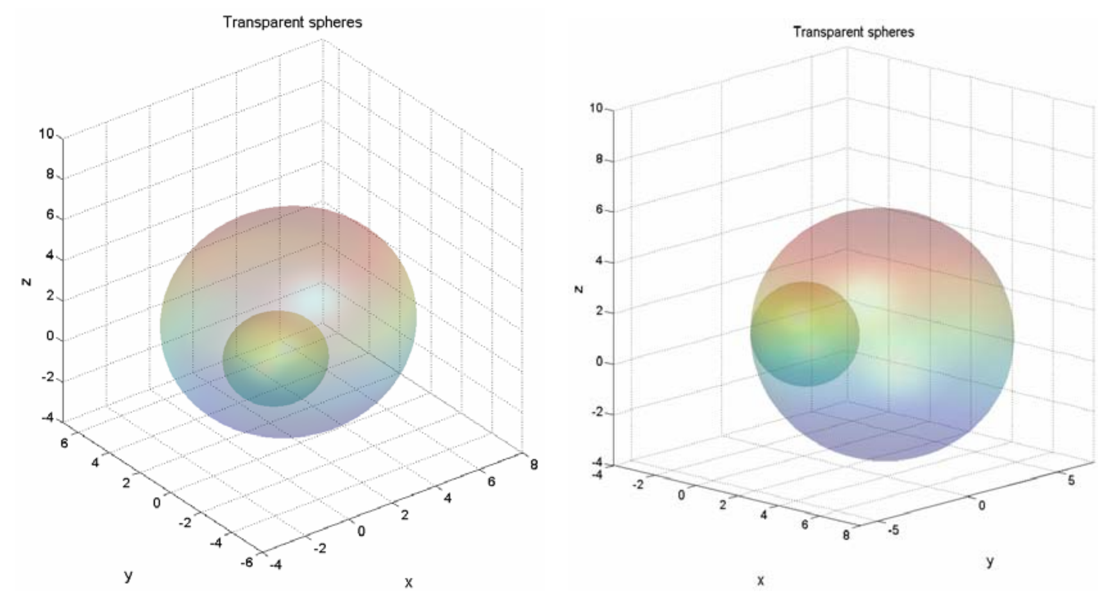
REFERENCES

Bancroft, J. C., and Du, x., 2006, Locating microseismic events and traveltime mapping using locally spherical wavefronts, CREWES Research Report, **18**.



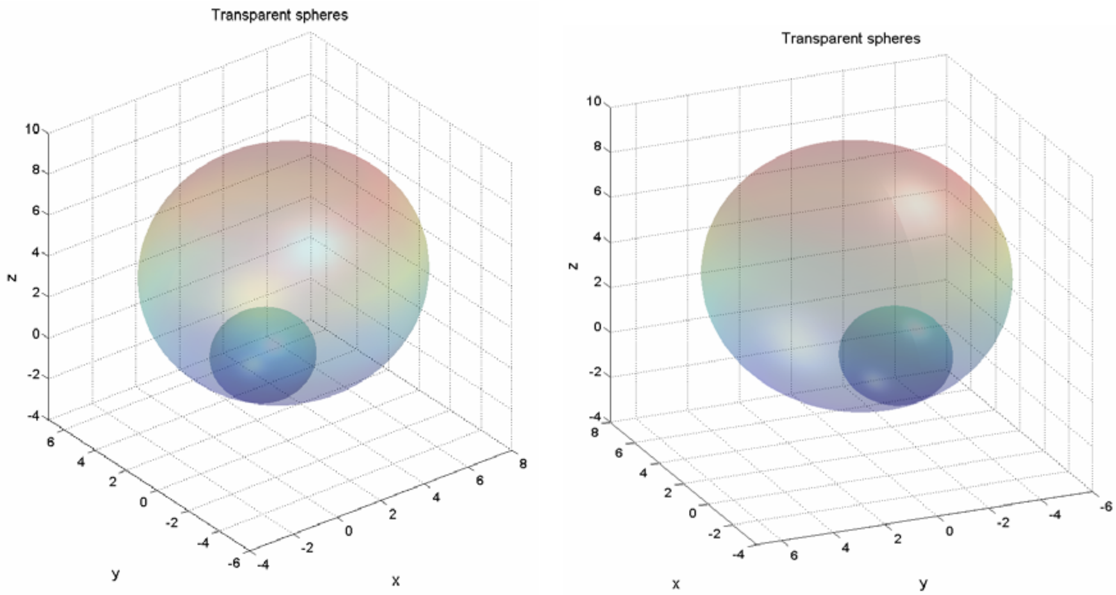
a)



b)

FIG. 4. Views of each receiver sphere with the source sphere (a – d). The left image is the original eye location, while the right image has been 3D rotated to view the area of tangency.

(Figure 4 continued on following page.)



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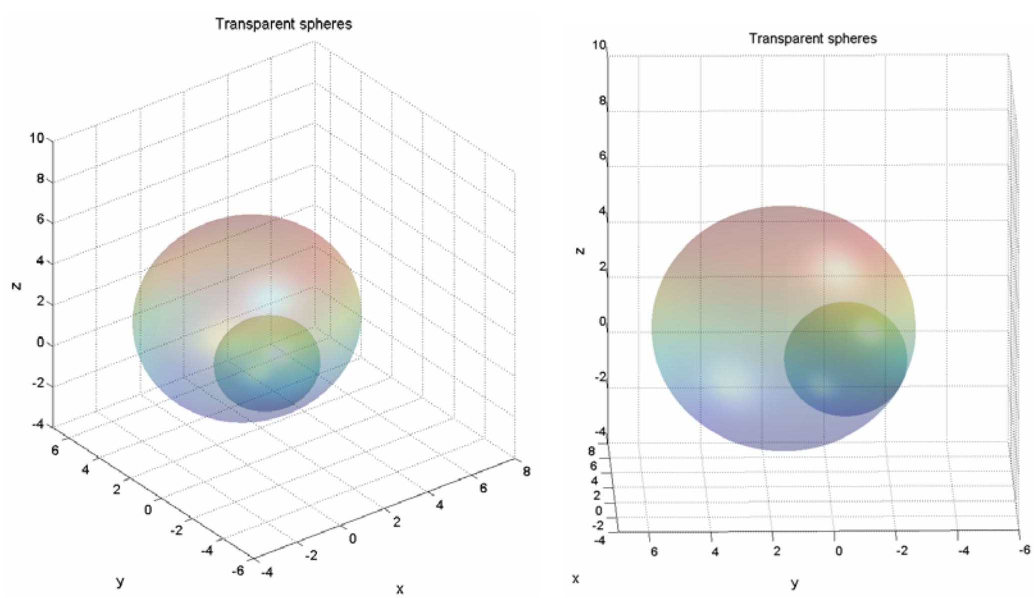

d)

Figure 4. Continued.