# Color correction for Gabor deconvolution

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## ABSTRACT

Reflectivity is usually assumed to be white for conventional deconvolution algorithms. However, the reflectivity series in practice usually demonstrates a characteristically colored spectrum, so deconvolution algorithms should be modified accordingly. This article presents a color correction method for Gabor deconvolution, and proposes different ways to conduct the color correction in case of incomplete well log information. Testing on synthetic data shows that the effectiveness of color correction is subject to the available frequency band and the completeness of well log information. In addition, the performance of proposed approaches to deal with an incomplete well log are evaluated using synthetic data.


## INTRODUCTION

The deconvolution of a seismic trace is derived from the corresponding convolutional model, which generally involves separating a seismic trace into two parts representing seismic wavelet and reflectivity respectively. Since both the seismic wavelet and reflectivity are unknown, some assumptions are necessary to develop the deconvolution algorithm. First, the seismic wavelet is assumed to be minimum-phase, which is theoretically expected from linearity and causality. Then, the reflectivity is supposed to be a random series, whose autocorrelation is a spike function at zero lag and whose power spectrum is constant over all frequencies. Such a reflectivity is the so-called white reflectivity. The final assumption is about the seismic trace itself, which is regarded as the convolution of reflectivity and embedded wavelet. Usually, the seismic trace is assumed to be stationary, which corresponds to a stationary convolutional model. Conventional Wiener spiking deconvolution is developed based on the stationary convolutional model. However, the seismic trace suffers attenuation during the propagation for various reasons such as Q attenuation and geometric spreading. This attenuation is frequency dependent and time-variant. So, the seismic trace is actually nonstationary. Margrave (1998) presented a nonstationary convolutional model, which takes the attenuation of seismic trace as a nonstationary filter applied to the stationary input. Based on this model, Margrave and Lamoureux (2002) proposed the Gabor deconvolution method, which honors the attenuation inherently and does not need additional gain correction.

Deconvolution algorithms usually assume that the reflectivity is white. In practice, the reflectivity is colored. Thus, the deconvolution algorithms should be modified accordingly to avoid producing distorted results. Montana and Margrave (2005) proposed a color correction method for Gabor deconvolution. This article presents a different color correction method using a smoothed Gabor spectrum of well-log reflectivity, and discusses the practical ways to conduct color correction in case of incomplete well log information.

The purpose of our work is to investigate a color correction method with potentially practical use for the Gabor deconvolution. This article is organized as follows: the first part introduces the Gabor deconvolution algorithm. The next section presents a color correction method and its practical implementation in case of incomplete well log information. Following the above theoretical material, numerical examples will be used to evaluate the influence of available frequency band for the color correction, and the performance of different practical color correction approaches. Finally, some basic conclusions are drawn from results of the examples.

## GABOR DECONVOLUTION

Gabor deconvolution is based on a nonstationary convolution model of the seismic trace. Margrave and Lamoureux (2002) presented a seismic trace model addressing the seismic wavelet and the nonstationary effect of constant-Q attenuation. The attenuated seismic trace is modeled as

$$\hat{s}(f) = \hat{w}(f) \int_{-\infty}^{\infty} \alpha_Q(\tau, f) r(\tau) e^{-i2\pi f\tau} d\tau, \tag{1}$$

where $\hat{s}(f)$ and $\hat{w}(f)$ are the Fourier spectra of the seismic trace $s(t)$ and seismic wavelet $w(t)$ respectively; $r(\tau)$ is the reflectivity, and $\alpha_Q(\tau, f)$ is the constant-Q transfer function given by

$$\alpha_Q(\tau, f) = e^{-\frac{i\pi f\tau}{Q} + iH\left(\frac{\pi f\tau}{Q}\right)}, \tag{2}$$

where $H$ denotes the Hilbert transform. Then, the Gabor transform of the attenuated seismic trace can be approximated by (Margrave and Lamoureux, 2002)

$$S_G(\tau, f) \approx \hat{w}(f) \alpha_Q(\tau, f) R_G(\tau, f)\ , \tag{3}$$

where $R_G(\tau, f)$ is the Gabor transform of reflectivity.

Based on equation (3), $|\hat{w}(f)||\alpha_Q(\tau, f)|$ can be estimated by smoothing $|S_G(\tau, f)|$ with an assumption that $|R_G(\tau, f)| \approx 1$.The simplest smoothing can be achieved by convolving $|S_G(\tau, f)|$ with a 2-D boxcar over $(\tau, f)$. Let $\overline{|S_G(\tau, f)|}$ be a proper smoothing of $|S_G(\tau, f)|$. With a minimum-phase assumption, the attenuated wavelet or propagating wavelet is estimated as

$$\hat{w}(f) \alpha_Q(\tau, f) \approx \overline{|S_G(\tau, f)|} e^{i\varphi(\tau, f)}\ , \tag{4}$$

where the phase $\varphi(\tau, f)$ is given by the Hilbert transform (over frequency),

$$\varphi(\tau, f) = H(ln\overline{|S_G(\tau, f)|})\ . \tag{5}$$

Therefore, an estimation of the reflectivity can be formulated in the Gabor spectral domain as

$$R_G(\tau, f)_{est} = S_G(\tau, f) \mathrm{D}(\tau, f), \tag{6}$$

where $\mathrm{D}(\tau, f)$ is the deconvolution operator formulated as

$$\mathrm{D}(\tau,f)=\frac{1}{\overline{|S_G(\tau,f)|}+\mu A_{max}}e^{-i\varphi(\tau,f)}, \tag{7}$$

in which $\mu$ is the stability factor, and $A_{max}$ is the maximum value of $\overline{|S_G(\tau,f)|}$.

## COLOR CORRECTION METHOD FOR GABOR DECONVOLUTION

If the assumption of white reflectivity is violated, i.e. if $|R_G(\tau,f)|$ deviates from unity significantly, then we should modify the above deconvolution algorithm, because it always gives a white reflectivity estimation with $|R_G(\tau,f)_{est}|\approx 1$, which may differ from the true nonwhite reflectivity apparently. If the regional well log information is available, we can conduct color correction to the Gabor deconvolution.

Suppose that $R_G^{'}(\tau,f)$ is the Gabor transform of the nonwhite reflectivity $r_c(t)$ calculated from a well log, and $\overline{|R_G^{'}(\tau,f)|}$ is the corresponding smoothed amplitude spectrum. The Gabor deconvolution operator with color correction can be formulated as

$$D_c(\tau,f)=\frac{\overline{|R_G'(\tau,f)|}}{\overline{|S_G(\tau,f)|}+\mu A_{max}}e^{i\varphi_c(\tau,f)}, \tag{8}$$

where $\mu$ is the stability factor, $A_{max}$ is the maximum value of $\overline{|S_G(\tau,f)|}$, and the phase $\varphi_c(\tau,f)$ is given by the Hilbert transform (over frequency),

$$\varphi_c(\tau,f)=H(\ln\ |\frac{\overline{|R_G'(\tau,f)|}}{\overline{|S_G(\tau,f)|}+\mu A_{max}}|). \tag{9}$$

The estimation of nonwhite reflectivity can be expressed in Gabor domain as

$$R'_G(\tau,f)_{est}=\frac{S_G(\tau,f)\overline{|R_G'(\tau,f)|}}{\overline{|S_G(\tau,f)|}+\mu A_{max}}e^{i\varphi(\tau,f)}. \tag{10}$$

While equation (10) uses well information to estimate the colored reflectivity, this information is only a smoothed Gabor amplitude spectrum. Neither detail nor phase information is needed. It is quite likely that this required well information is a very slowly changing function of position so that wells that are quite distant can be used.

The estimated result with the white reflectivity assumption can be viewed as a special case where $\overline{|R_G^{'}(\tau,f)|}$ is nearly constant. When the real $\overline{|R_G^{'}(\tau,f)|}$ has obvious amplitude fluctuations, a white estimation tends to enlarge some particular parts of reflectivity series, which correspond to the low amplitude areas of $\overline{|R_G^{'}(\tau,f)|}$. In addition, the effect of color correction depends on how much $\overline{|R_G^{'}(\tau,f)|}$ departs from unity or a constant and how reliable the used $\overline{|R_G^{'}(\tau,f)|}$ is, which is subject to the available frequency band and completeness of well log information.

The key point of the correction method is that how to obtain $\overline{|R_G^{'}(\tau,f)|}$. If sufficient well log information is available, $r_c(t)$ and $s(t)$ nearly have the same length in time, which can be denoted by a time interval $[0,t_{max}]$. $\overline{|R_G^{'}(\tau,f)|}$ can be directly obtained from the Gabor spectrum of $r_c(t)$. For this case, color correction can improve the reflectivity estimation most optimally.

However, in practice, the well log is usually incomplete and limited to some depth interval, which corresponds to only a part of the seismic trace. On this occasion, we need to use the limited well log to estimate a complete $\overline{\left|R_G^{'}(\tau,f)\right|}$, which should be of the same size with $S_G(\tau,f)$ in time-frequency domain as indicated by equation (9). There may be different ways to achieve this. One way assumes that the color feature of nonwhite reflectivity is temporally stationary, i.e. $\overline{\left|R_G^{'}(\tau,f)\right|}$ only changes with frequency $f$. Suppose that $\acute{r}_c(t)$ is the incomplete reflectivity series with a time interval $[t_1, t_2]$ ($0 < t_1 < t_2 < t_{max}$), and its' Fourier spectrum is $\acute{R}_c(f)$. Then, $\overline{\left|R_G^{'}(\tau,f)\right|}$ can be approximated by a smoothed version of $\acute{R}_c(f)$. There are various ways to do the smoothing. In this article, we use a polynomial approximation of $\acute{R}_c(f)$ as the smoothed result,

$$|\acute{R}_c(f)| \approx a_0 + a_1 f + a_2 f^2, \tag{11}$$

where $a_0$, $a_1$ and $a_2$ are constants determined using a least squares algorithm. So, $\overline{\left|R_G^{'}(\tau,f)\right|}$ can be modeled as

$$\overline{\left|R_G^{'}(\tau,f)\right|} = a_0 + a_1 f + a_2 f^2. \tag{12}$$

As an alternative, another way infers $\overline{\left|R_G^{'}(\tau,f)\right|}$ from the Gabor spectrum, $\acute{R}_G(\tau,f)$, of the incomplete reflectivity $\acute{r}_c(t)$, based on an assumption that the color feature of nonwhite reflectivity is smoothly time variant. First, $\acute{R}_G(\tau,f)$ is smoothed as following,

$$|\acute{R}_G(\tau,f)| \approx a_0'(\tau) + a_1'(\tau) f + a_2'(\tau) f^2,\ \tau \in [t_1, t_2], \tag{13}$$

where $a_i'(\tau)$, $i = 1, 2, 3$, is a coefficient curve and limited to the interval $[t_1, t_2]$ because of the incompleteness of $\acute{r}_c(t)$. Then, $\overline{\left|R_G^{'}(\tau,f)\right|}$ can be expressed as

$$\overline{|R_G'(\tau,f)|} = a_0(\tau) + a_1(\tau) f + a_2(\tau) f^2, \tau \in [0, t_{max}], \tag{14}$$

where $a_i(\tau)$, $i = 1, 2, 3$, can be modeled as

$$a_i(\tau) = \begin{cases} a_i'(t_1), & 0 \le \tau \le t_1 \\ a_i'(\tau), & t_1 < \tau < t_2 \\ a_i'(t_2), & t_2 \le \tau \le t_{max} \end{cases}. \tag{15}$$

When multiple well logs of one region are available, their Gabor spectra are smoothed using equation (13) first. Thus, we can get a set of coefficient curves. $\overline{|R_G'(\tau,f)|}$ is still modeled as equation (14), while $a_i(\tau)$ is calculated by combining all the coefficient curves of each well log through interpolation and extrapolation. Through this approach, we can approximate the true $\overline{|R_G'(\tau,f)|}$ very well if the well logs are well distributed in time and the color feature of nonwhite reflectivity is not drastically time-variant.

## EXAMPLES

A 0.85s long reflectivity series, calculated from a well log, was used to test the color correction method. Figure 1 shows the reflectivity series and its' amplitude spectrum. There is an obvious roll-off in the amplitude spectrum from 0 Hz to 100Hz, which indicates that the reflectivity is not white. The amplitude Gabor spectrum of the reflectivity series is shown in Figure 2. The low amplitude zone around 0.5s is apparent, which demonstrates that the color feature of the nonwhite reflectivity is time-variant.

According to equation (3), a synthetic attenuated seismic trace was created by applying a forward Q filter to the nonwhite reflectivity, and then convolving the result with a source wavelet. For the examples in this article, the Q value is 50, and the source wavelet is a minimum phase wavelet with a dominant frequency of 40Hz. Supposing that a complete well log is available and the effective frequency band for deconvolution is 10-150Hz, a testing on color correction method, directly conducted according to equation (10), is shown in Figure (3). We can see that conventional Gabor deconvolution, with white reflectivity assumption, gave an obviously enlarged estimation from 0.5s to 0.8s compared to the true reflectivity series, which corresponds to the low magnitude area of the Gabor spectrum shown in Figure 2. With color correction, the estimated result is very close to the true reflectivity. With sufficient well log information and wide frequency band for deconvolution, such a case is the most ideal one for color correction.

However, the available frequency band may be limited by the data quality of seismic traces in practice. Figure 4 and Figure 5 show the results of the Gabor deconvolution with a frequency band of 10-100Hz and 10-60Hz respectively. As demonstrated by Figure 3, 4, 5, the color correction method, even with complete well log information, gradually loses its advantage over conventional deconvolution method when the frequency band becomes narrower and narrower.

In addition, the well log is usually incomplete compared with the seismic trace. Figure 6 shows a truncated part of the nonwhite reflectivity shown in Figure 1, its amplitude spectrum and the polynomial approximation of amplitude spectrum. Assuming the color feature of nonwhite reflectivity is stable, color correction was conducted using equation (10) and (12). The result is shown in Figure 7. For this case, color correction improved the estimation by addressing the nonwhite Fourier spectrum shown in Figure 6, for example, the estimated reflectivity series is obviously more accurate around 0.19s and 0.36s, but it still gave an enlarged estimation around 0.5s because the time-variant color feature is not honored. Taking the time-variant color feature of nonwhite reflectivity into account, the Gabor spectrum can be smoothed using equation (13). For the complete nonwhite reflectivity series, the coefficient curves are shown in Figure 8, in which the smooth curves indicate the color feature of nonwhite reflectivity is slowly time-variant. For the incomplete reflectivity series shown in Figure 6, the coefficient curves can be created using equation (15). The color correction method was applied using equation (10) and (14). Figure 9 shows the deconvolved results. We can see that the reflectivity series around 0.5s is better estimated compared with the result in Figure 7, which results from the partially addressing of the time-variant color feature. When multiple incomplete well logs of one region are available, the coefficient curves can be obtained through interpolation and extrapolation. An example for this case is illustrated by Figure 10. The coefficient values for time interval 0.2s-0.4s and 0.6s-0.8s are obtained from two well logs using equation (13) respectively. Then, the coefficient values for other time intervals are obtained through interpolation and constant extrapolation. After modeling the Gabor spectrum of the nonwhite reflectivity, the color correction was applied according to equation (10), whose results are shown in Figure 11. The deconvolved trace with color correction matches the true reflectivity well because the time-variant feature of the nonwhite reflectivity is modeled with satisfied accuracy.

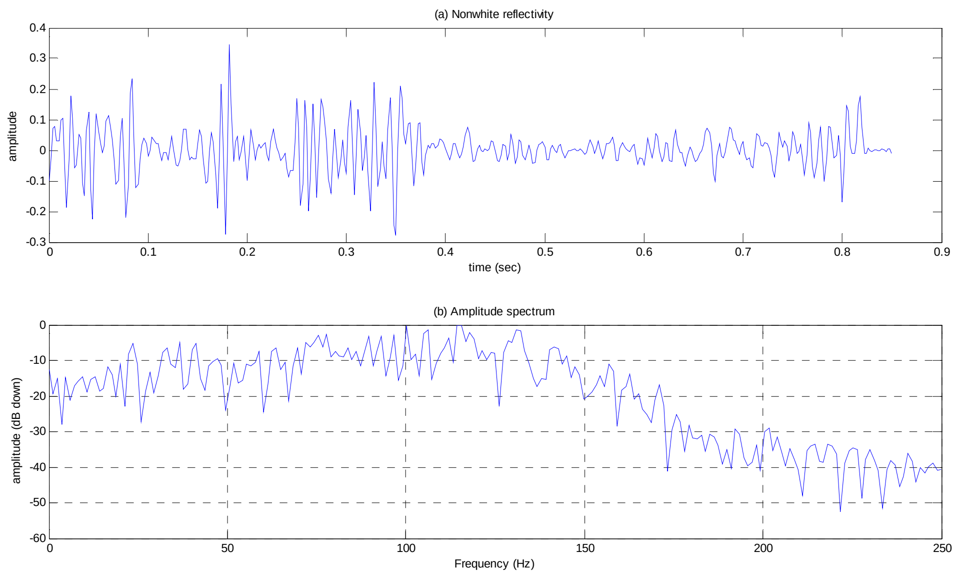


FIG. 1. (a) Nonwhite reflectivity calculated from a well log. (b) The amplitude Fourier spectrum of nonwhite reflectivity.

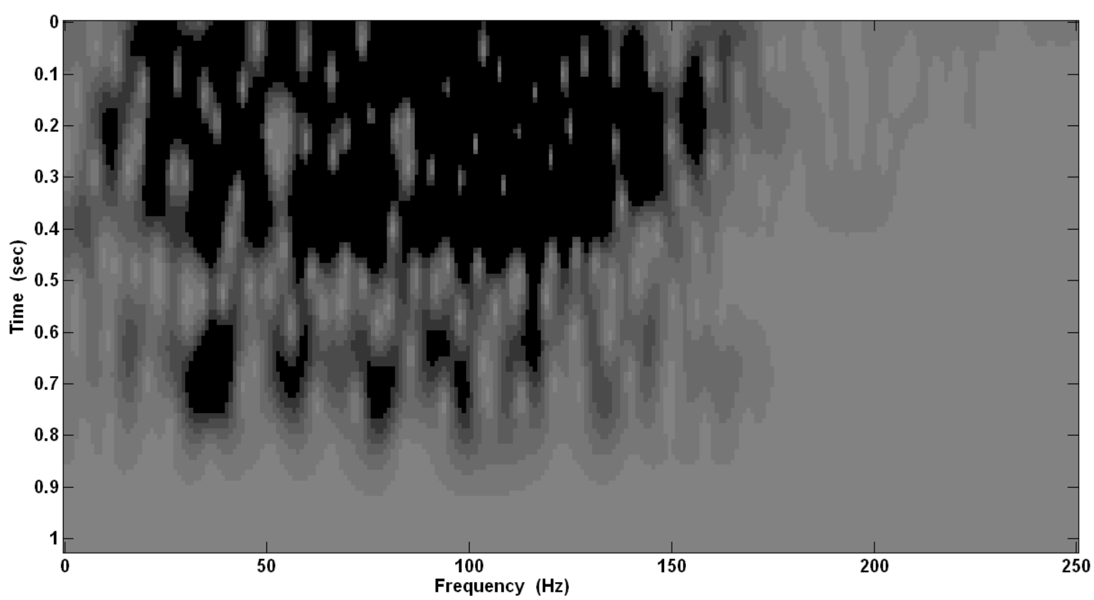


FIG. 2. Amplitude Gabor spectrum of the nonwhite reflectivity shown in Figure 1.

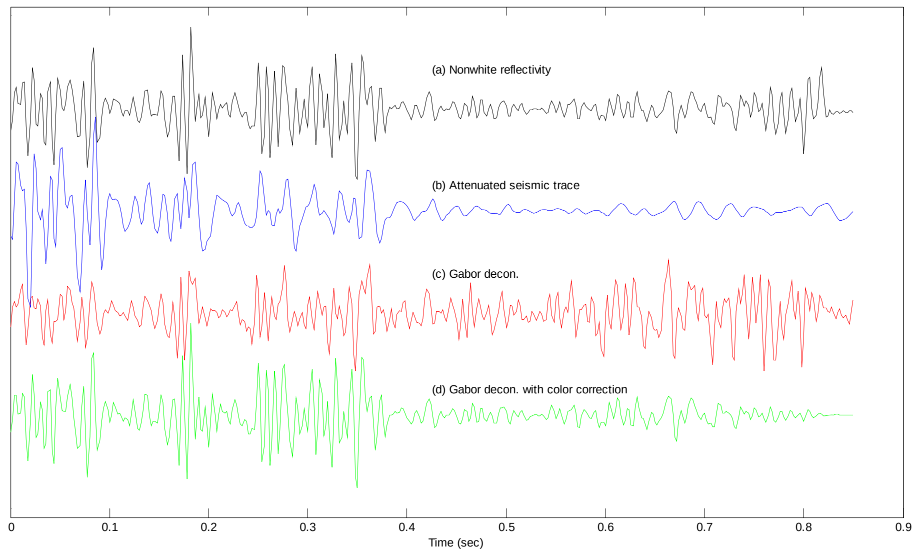


FIG. 3. Gabor deconvolution with a frequency band of 10-150 Hz. (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Gabor deconvolved trace without color correction. (d) Gabor deconvolved trace with color correction using a complete well log.

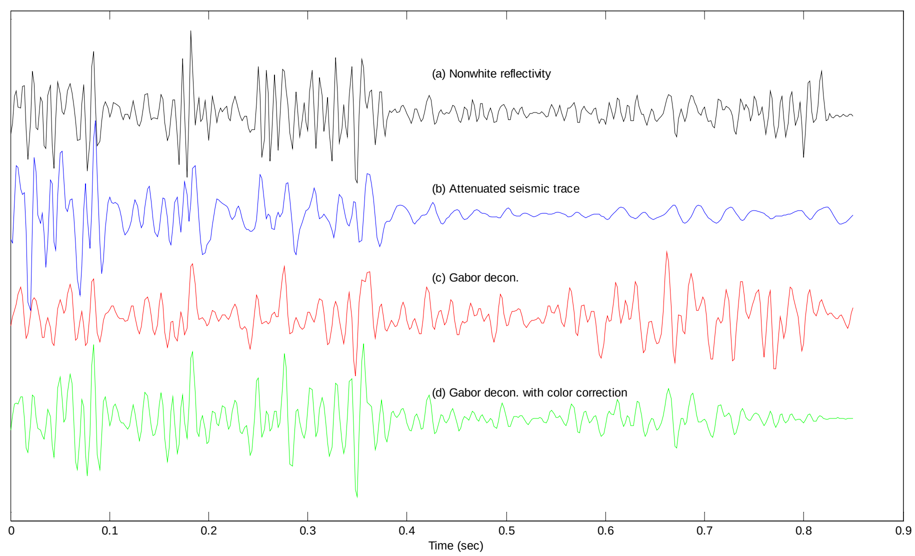


FIG. 4. Gabor deconvolution with a frequency band of 10-100Hz. (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Gabor deconvolved trace without color correction. (d) Gabor deconvolved trace with color correction using a complete well log.

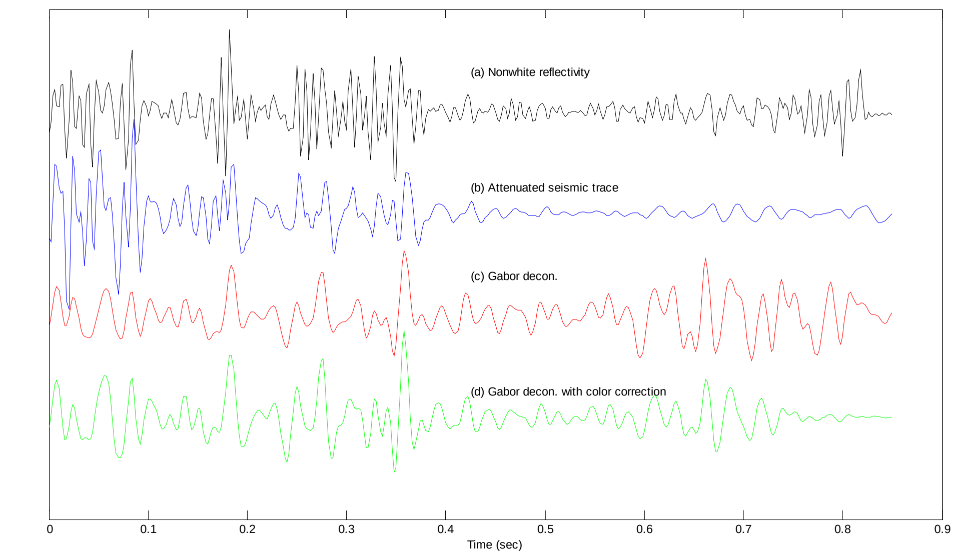


FIG. 5. Gabor deconvolution with a frequency band of 10-60Hz. (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Gabor deconvolved trace without color correction. (d) Gabor deconvolved trace with color correction using a complete well log.

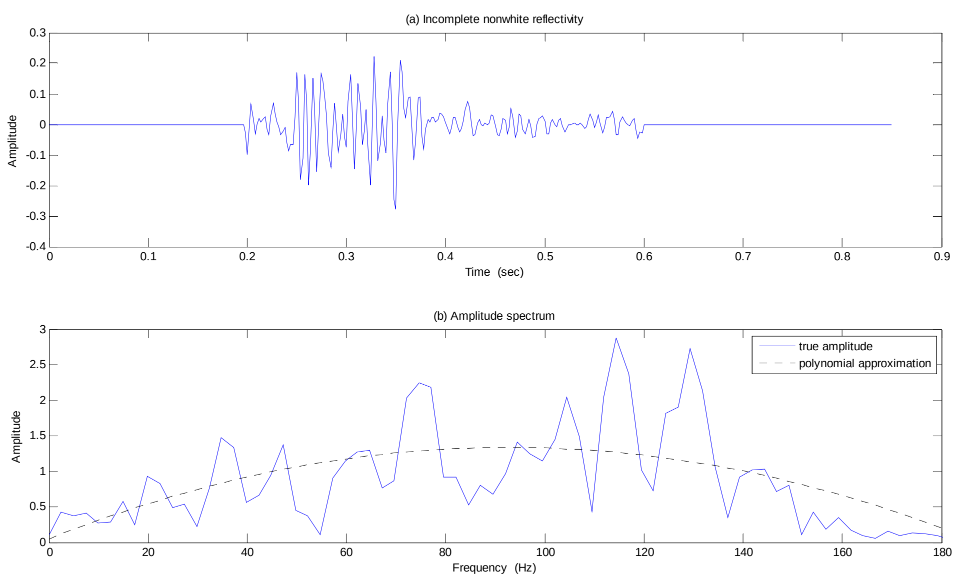


FIG. 6. (a) Incomplete nonwhite reflectivity: the 0.2s-0.6s part of the reflectivity series shown in Figure 1. (b) The amplitude spectrum of the incomplete reflectivity and its' polynomial approximation.

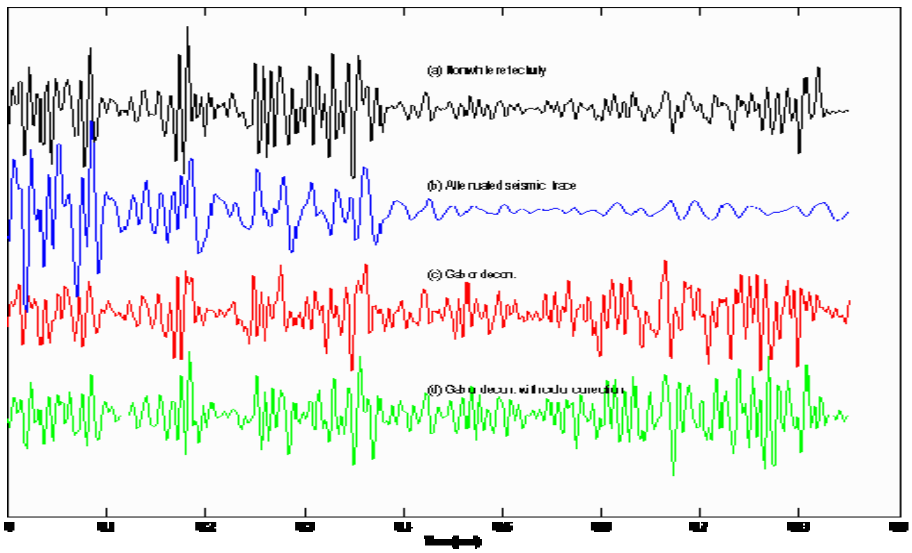

FIG. 7. Color correction using single incomplete well log (1). (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Gabor deconvolved trace without color correction. (d) Gabor deconvolved trace with color correction using equation (9) and (11).

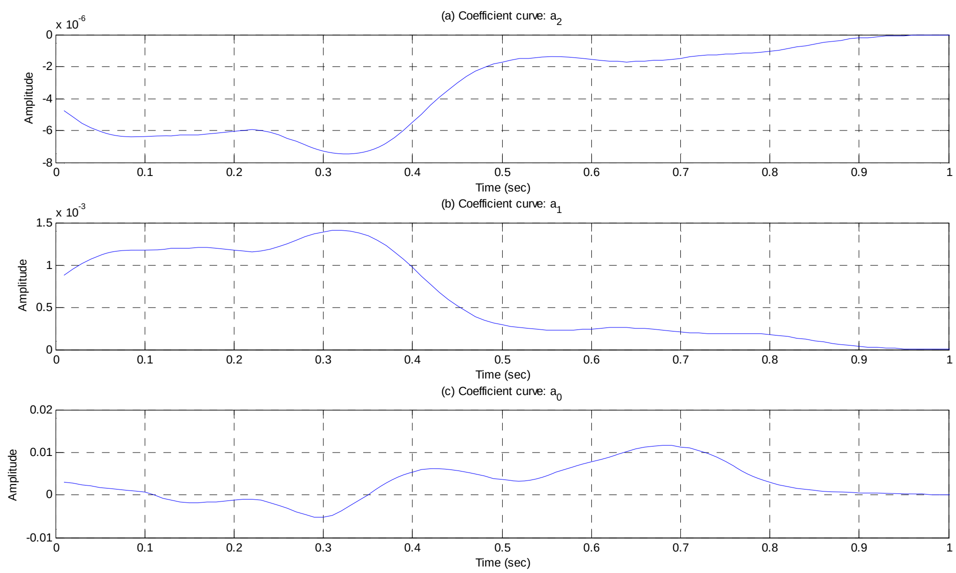


FIG. 8. Coefficient curves for polynomial approximation of Gabor spectrum of the complete nonwhite reflectivity. (a) Coefficient curve . (b) Coefficient curve . (c) Coefficient curve .

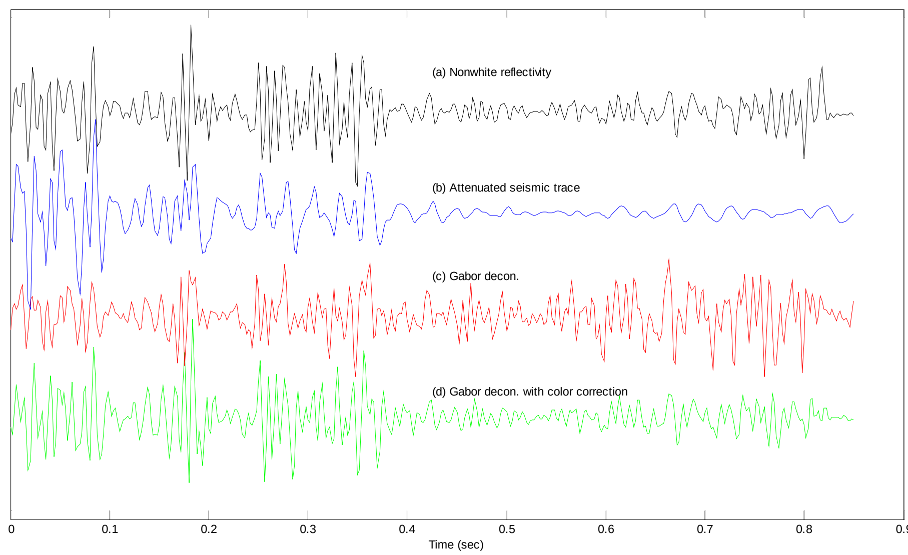


FIG. 9. Color correction using single incomplete well log (2). (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Gabor deconvolved trace without color correction. (d) Gabor deconvolved trace with color correction using equation (9) and (13).

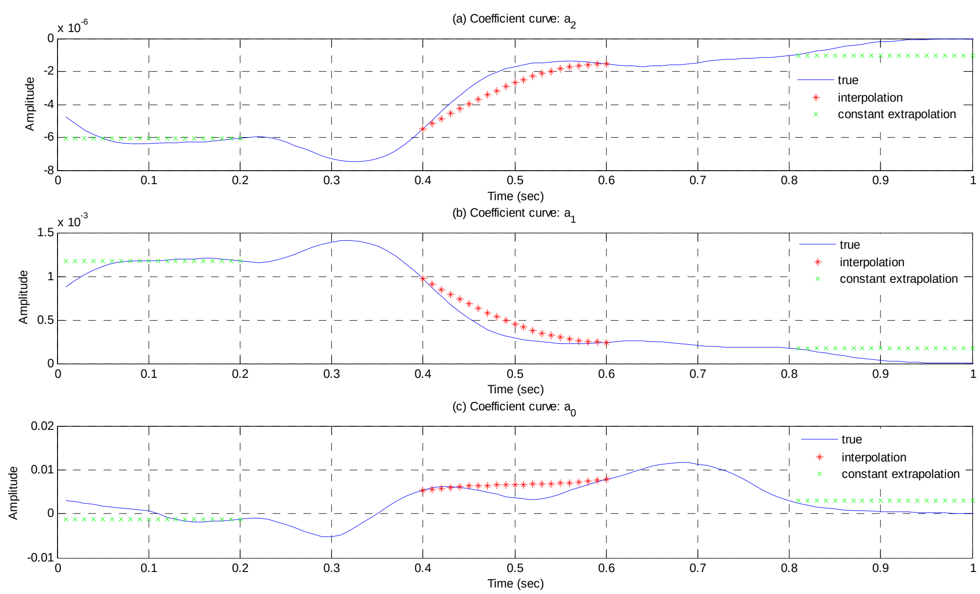


FIG. 10. Calculation of coefficient curves for polynomial approximation of the Gabor spectrum of the nonwhite reflectivity using two incomplete reflectivity series (One is from 0.2s to 0.4w, the other is from 0.6s to 0.8s) (a) Coefficient curve $a_2(\tau)$. (b) Coefficient curve $a_1(\tau)$. (c) Coefficient curve $a_0(\tau)$.

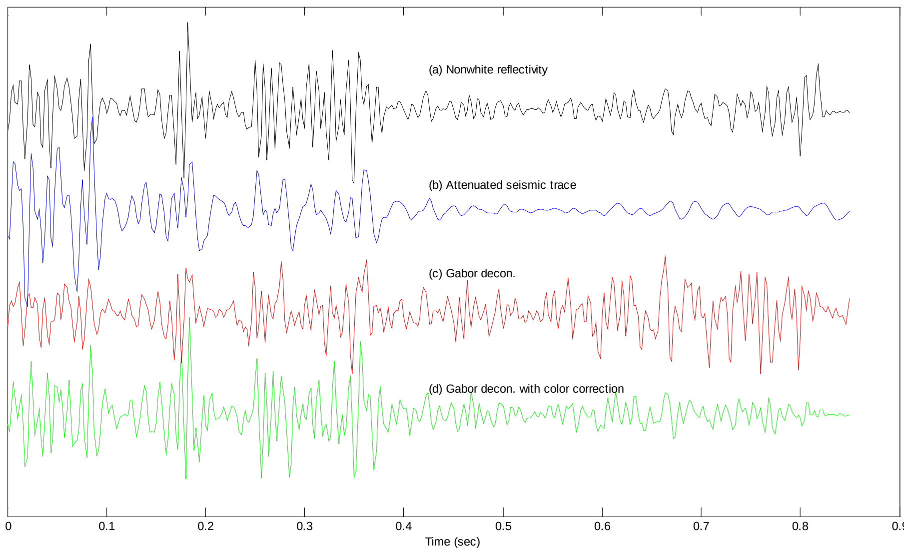


FIG. 11. Color correction using multiple well logs. (a) Nonwhite reflectivity. (b) Synthetic attenuated trace. (c) Gabor deconvolved trace without color correction. (d) Gabor deconvolved trace with color correction; The coefficient curves shown in Figure 10 are used to build the Gabor spectrum of nonwhite reflectivity according to equation (13).

## CONCLUSIONS

In practice, the reflectivity is usually nonwhite, and the color features can be time-variant. In presence of nonwhite reflectivity, conventional Gabor deconvolution gives a distorted estimation, which corresponds to the low amplitude area of the Gabor spectrum of nonwhite reflectivity. Color correction can significantly improve the reflectivity estimation for Gabor deconvolution, whose effect is subject to the available frequency band and completeness of well log information.

To address the incomplete well log information, different approaches are proposed to conduct the color correction, which are of practical use.Testing on synthetic data shows that all these approaches can improve the reflectivity estimation to some degree.

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