# Sommerfeld integral based spherical wave field computation applied to multi-interface VSP models for stratigraphic Q investigations

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## ABSTRACT

A method developed from the Ewing algorithm for *point sources in layered media* is employed to compute the complete spherical wave field for synthetic zero-offset VSPs. For the elastic case of six embedded Class 1 reservoir layers, the transmitted wave spectrum below the stack of layers shows a clear trend of amplitude decay with frequency which the *spectral ratio method* of Q-estimation interprets as a finite Q-factor; because of the purely elastic model this represents *stratigraphic attenuation*.

For a more realistic multi-layer situation Ross Lake well-logs are introduced. The depth dependent Q-factor estimated from a synthetic Ross Lake VSP modelled with a constant intrinsic Q of 100 resembles Q(z) estimated from actual Ross Lake VSP-data. This observation suggests a dominant role of *stratigraphic attenuation* in the Ross Lake area. When implementing a first-order estimation-error reduction by *spectral normalization*, model Q recovery is improved but still unsatisfactory.

## INTRODUCTION

The extension of Sommerfeld integral based spherical wave field computations beyond two layers is described in a previous report (Haase, 2008). The Ewing algorithm for *point sources in layered media* is utilized there to compute multi-layer AVO-responses. Spherical waves and multi-interface models describe the earth's response to seismic waves more accurately in complicated situations than plane waves and a single interface can. Multiples and reverberations, for example, are easily modelled in a stack of layers and with the Sommerfeld integral near-field, far-field and geometric spreading are introduced. It is also mentioned in a previous report that *stratigraphic attenuation* can be of the same order in magnitude as *intrinsic attenuation* (Haase and Stewart, 2007). In our quest for *intrinsic attenuation* we must isolate and remove the effects of *stratigraphic attenuation*. Haase and Stewart (2007) investigate Q-estimation errors caused by a simple density-step model. The multi-layer Ewing method, by contrast, allows the modelling of *stratigraphic attenuation* with velocity logs and density logs. The purpose of this study is to investigate Q-estimation errors in more realistic models and explore Q-estimation error reduction.

## STRATIGRAPHIC Q OF A STACK OF SIX LAYERS

The first step beyond single interface models is the introduction of a limited number of interleaved layers: Six Class 1 reservoir layers ($\alpha_{res}$ = 2933.33 m/s, $\beta_{res}$ = 1882.29 m/s, and $\rho_{res}$ = 2000 kg/m$^3$), increasing in thickness from 12.5 m to 75 m and separated by layers equal to the increasing reservoir thickness (but with overburden parameters $\alpha_o$ = 2000 m/s, $\beta_o$ = 879.88 m/s, and $\rho_o$ = 2400 kg/m$^3$), are placed between 100 m and 550 m depth. The transmitted wave arriving below this stack of reservoir layers is plotted in Figure 1. There is little change to the basic shape of the Ormsby wavelet as emitted by the source, but a ringing tail indicative of reverberations has developed. The amplitude spectrum computed from the trace shown in Figure 1 can be seen in Figure 2. An amplitude decay with frequency is clearly visible for this model. Q-estimation with a *spectral ratio method* interprets this frequency dependent amplitude decay as a finite Q-factor which is purely stratigraphic because intrinsic Q is not modelled in this case. Assuming the in-band spectrum of the Ormsby source wavelet arriving at the top of the shallowest reservoir layer is flat, the *spectral ratio method* estimates a Q-factor of 152 for the stack solely because of the frequency dependent amplitude loss across the six reservoir layers.

## STRATIGRAPHIC AND INTRINSIC Q OF A WELL-LOG MODEL

In a real earth model there are a multitude of layers well beyond the stack of six layers considered first in this report. For a more realistic model, two zero-offset VSPs are computed using velocity and density logs of the Ross Lake survey. Velocity and density values of these well-logs are displayed in Figure 3. Note the values filled in above and below the logs. The first synthetic VSP is computed without any intrinsic attenuation; any frequency dependent amplitude decrease/increase is caused by impedance changes representing *stratigraphic attenuation/amplification*. For the computation of the second VSP, *intrinsic attenuation* is represented by a constant Q-factor of 100; superimposed on the effects of *stratigraphic attenuation* are amplitude decays because of the additional *intrinsic attenuation*. Figure 4 shows maximum instantaneous amplitudes of the transmitted wave as a function of depth for the two VSPs. As expected, amplitudes are lower when *intrinsic attenuation* is added. There are several regions of relative amplitude increases in these curves. A time-domain Q-estimation algorithm such as the *analytical signal method* would return negative Q-values in these regions unless amplitude increases are smoothed away. The *stratigraphic imprint* on both maximum-instantaneous-amplitude curves is similar for this constant Q case; Q-estimation error reduction could be attempted by taking the ratio of the two curves (or by taking the difference in the logarithmic amplitude domain) which also compensates for geometrical spreading.

Figures 5 and 6 give amplitude spectra of the two synthetic zero-offset VSPs at selected depths. In both, Figure 5 and Figure 6, amplitudes decrease with depth and with frequency. This amplitude decay is more pronounced in Figure 6 because *intrinsic attenuation* is added to the model. Applying the *spectral ratio method* to the amplitude spectra shown in Figure 6 leads to the Q-factor plotted in Figure 7; the intrinsic model Q-factor curve is added for comparison purposes. There is no doubt that spectral averaging could improve this result somewhat, but a constant Q of 100 is not recovered. Figure 7 actually resembles Q(z) estimated from Ross Lake VSP-data (Haase and Stewart, 2005) which appears to confirm the role of *stratigraphic attenuation* in Ross Lake amplitude trends. Close inspection of Figures 5 and 6 reveals a similar *stratigraphic imprint* on amplitude spectra as is observed when comparing instantaneous amplitudes in Figure 4. This again suggests attempting a first-order Q-estimation error reduction by taking the ratio of same-depth amplitude spectra. The resulting *normalized* amplitude spectra can be seen in Figure 8. Geometrical spreading is removed and these *normalized* spectra appear somewhat smoothed when compared to their *un-normalized* equivalents in Figure 6. The *spectral ratio method* requires *log-spectral ratios* of magnitude spectra such as plotted in Figure 8. Three of these *log-spectral ratios* are displayed in Figure 9. They show clear linear trends and *least-square-error* straight line fitting can proceed. Q(z) obtained from a set of these straight lines is displayed in Figure 10 together with the curves of Figure 7 for comparison purposes. A first-order Q-estimation error reduction is far from ideal but at least a constant Q-trend is now discernable.

## CONCLUSIONS

The multi-layer Ewing method is employed to compute synthetic zero-offset VSPs from velocity logs and density logs with the Sommerfeld integral. For a stack of 6 Class 1 reservoir layers with increasing thicknesses it is found that amplitude spectra of the transmitted wave below the stack are decaying with frequency. The *spectral ratio method* estimates a Q-factor of 152 for this stack which is purely *stratigraphic attenuation* because no *intrinsic attenuation* is modelled in this case.

Ross Lake well-logs are introduced for a true multi-layer situation. The depth dependent Q-factor estimated from the synthetic VSP modelled with a constant intrinsic Q of 100 does not recover this constant model Q-factor. The quality factor as function of depth obtained above resembles Q(z) estimated from actual Ross Lake VSP-data suggesting a dominant role of *stratigraphic attenuation* here. When implementing a first-order estimation-error reduction by spectral normalization the constant Q-trend of the model is recovered, which represents an improvement; but these reduced errors are still unacceptable. An indirect Backus-Gilbert inversion method is expected to bring further improvement and this approach will be attempted next.

## ACKNOWLEDGEMENTS

Support from the CREWES Project at the University of Calgary and its industrial sponsorship is gratefully acknowledged.

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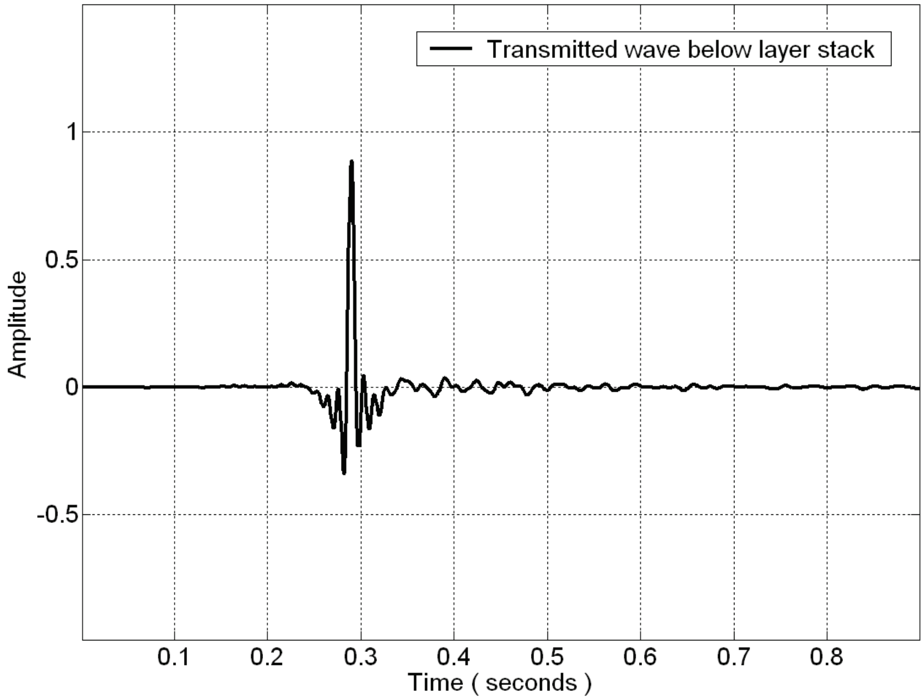

FIG. 1. Transmitted wave below a stack of 6 Class 1 reservoir layers.

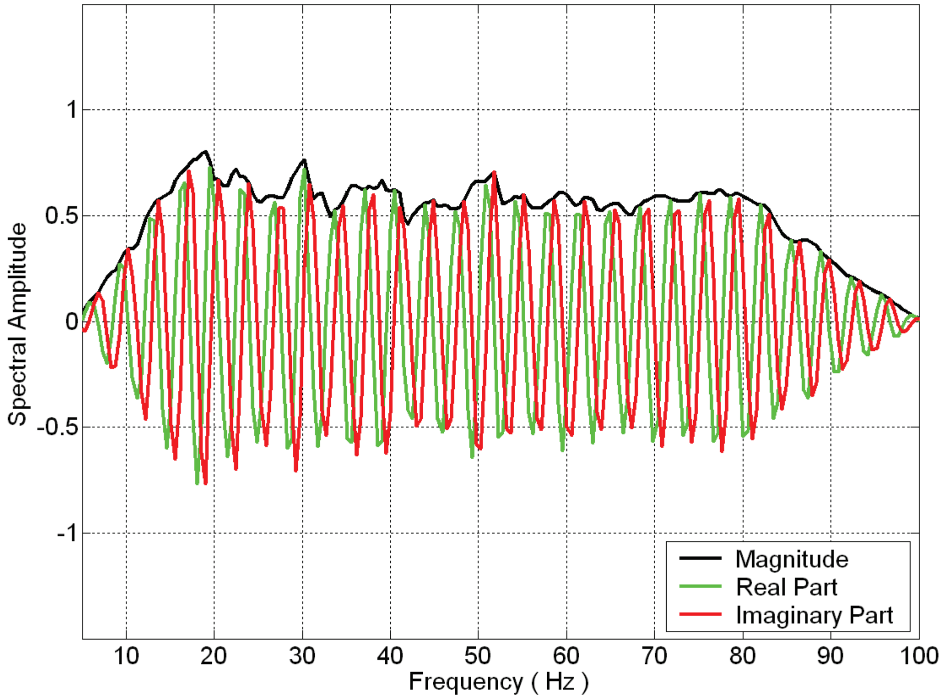

FIG. 2. Amplitude spectrum below a stack of 6 Class 1 reservoir layers.

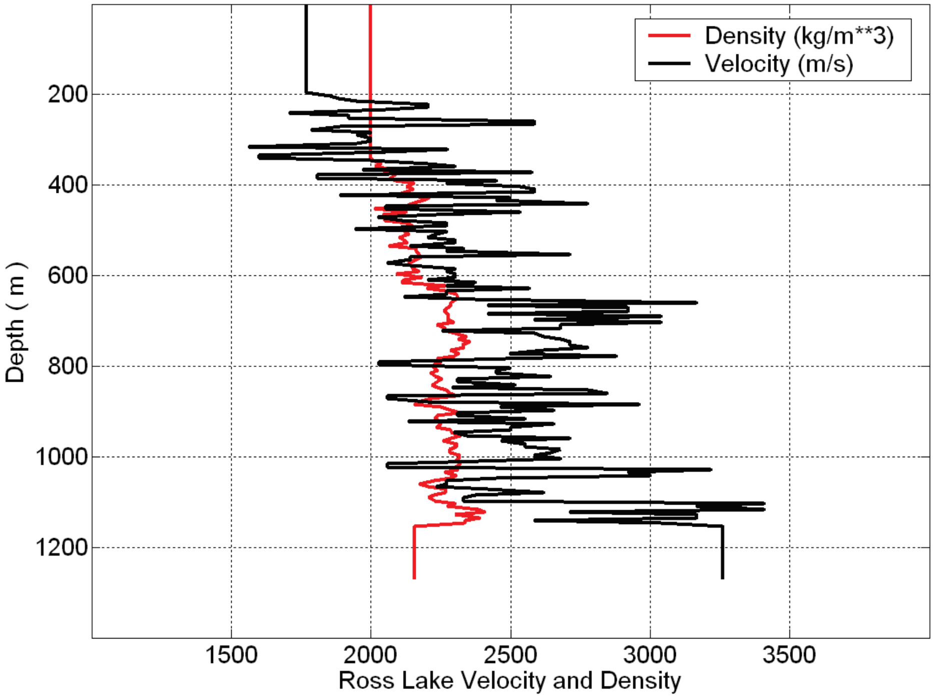

FIG. 3. Velocities and densities taken from Ross Lake well-log.

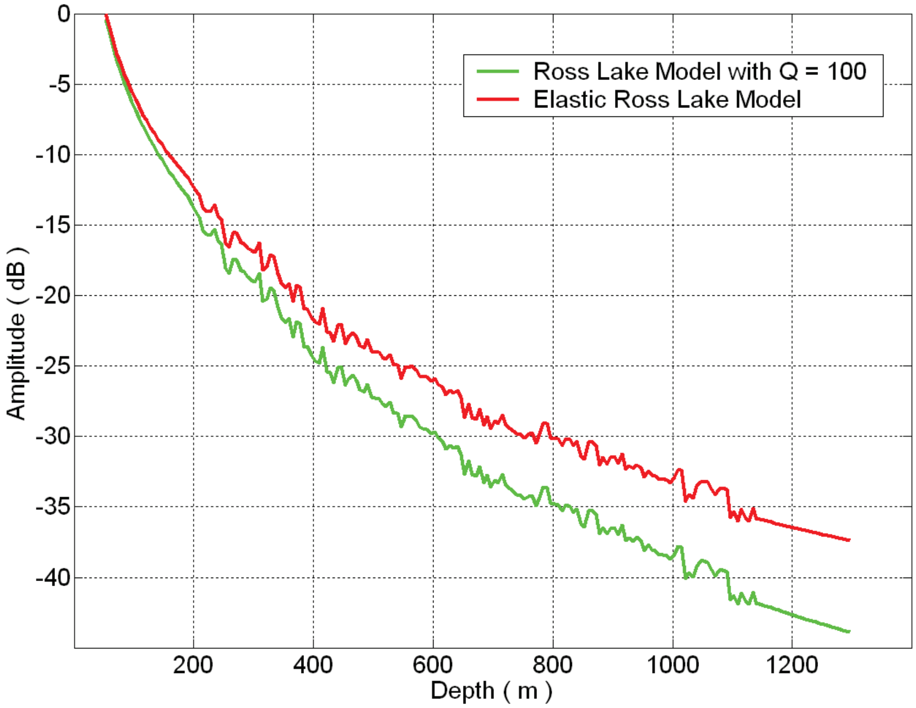

FIG. 4. Maximum instantaneous amplitudes as a function of depth.

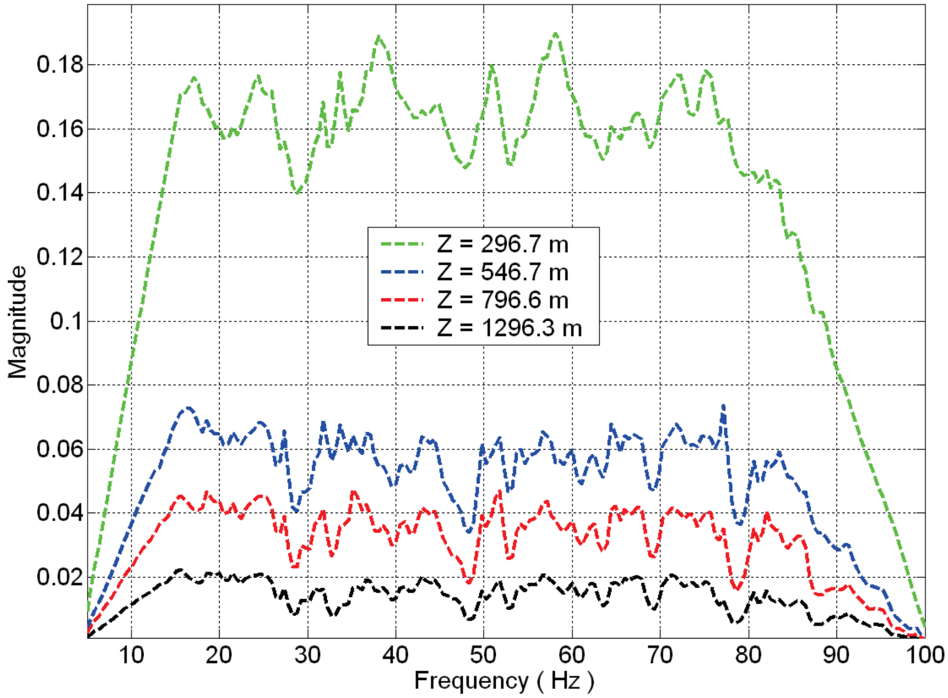

FIG. 5. Amplitude spectra for elastic synthetic VSP.

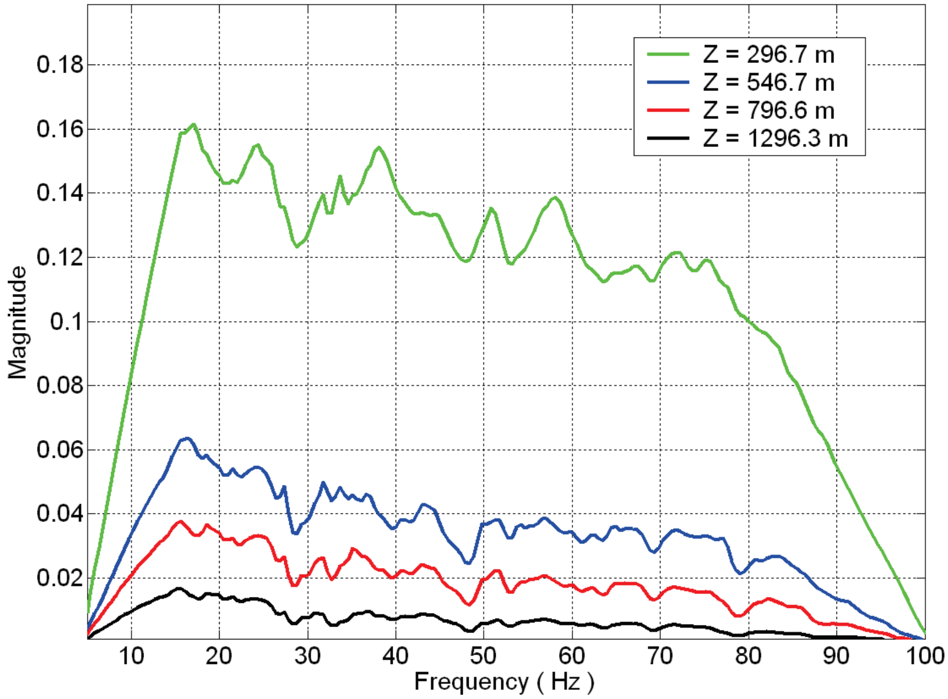

FIG. 6. Amplitude spectra for synthetic VSP modelled with Q = 100.

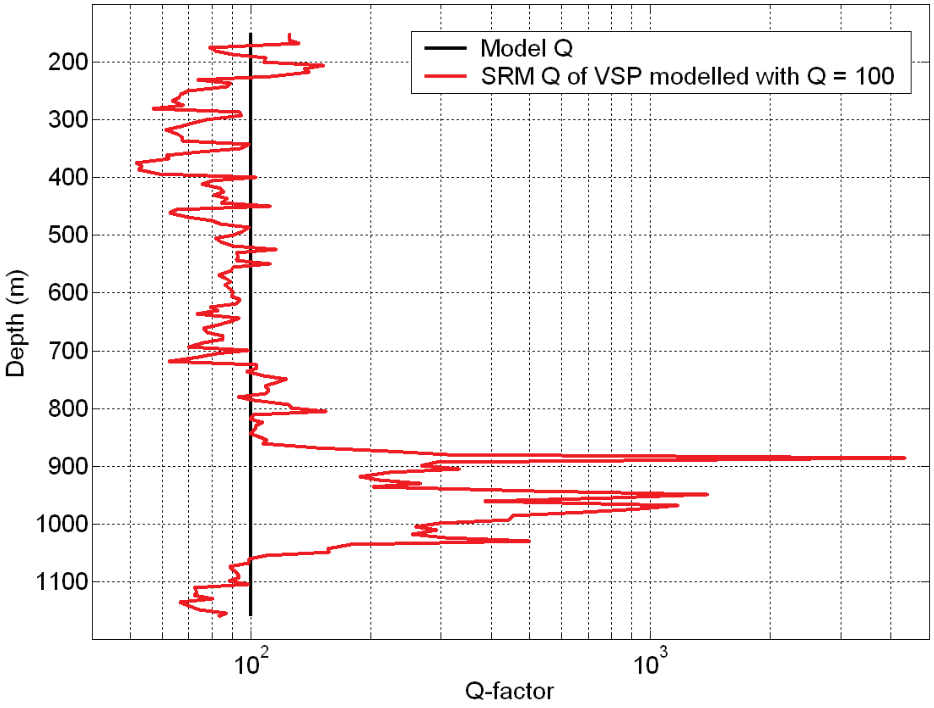

FIG. 7. Spectral ratio method applied to amplitude spectra of Figure 6 (note x-axis log-scale).

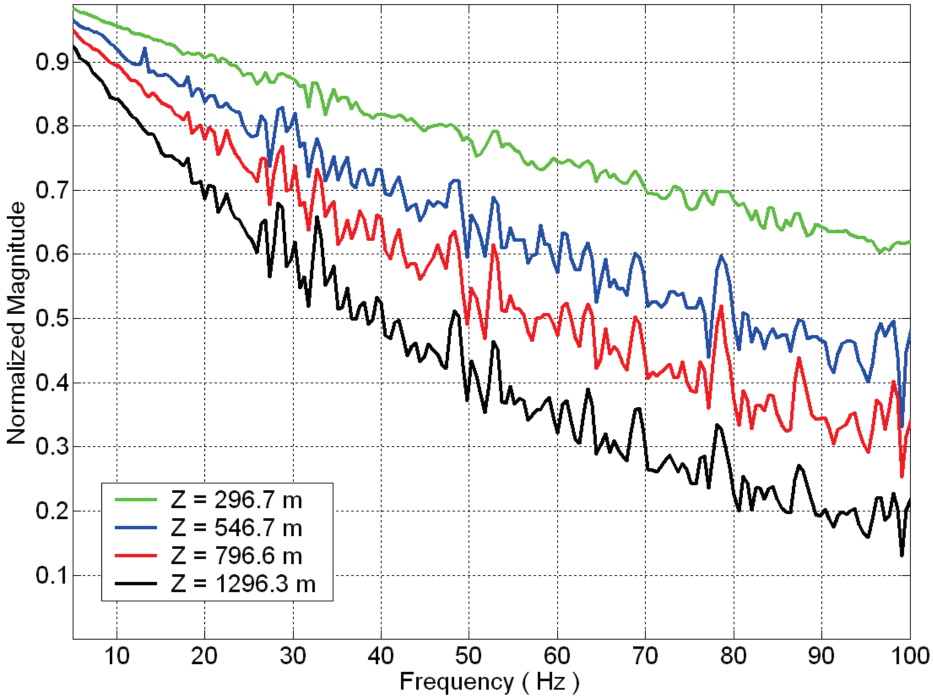

FIG. 8. Amplitude spectra of Figure 6 following normalization.

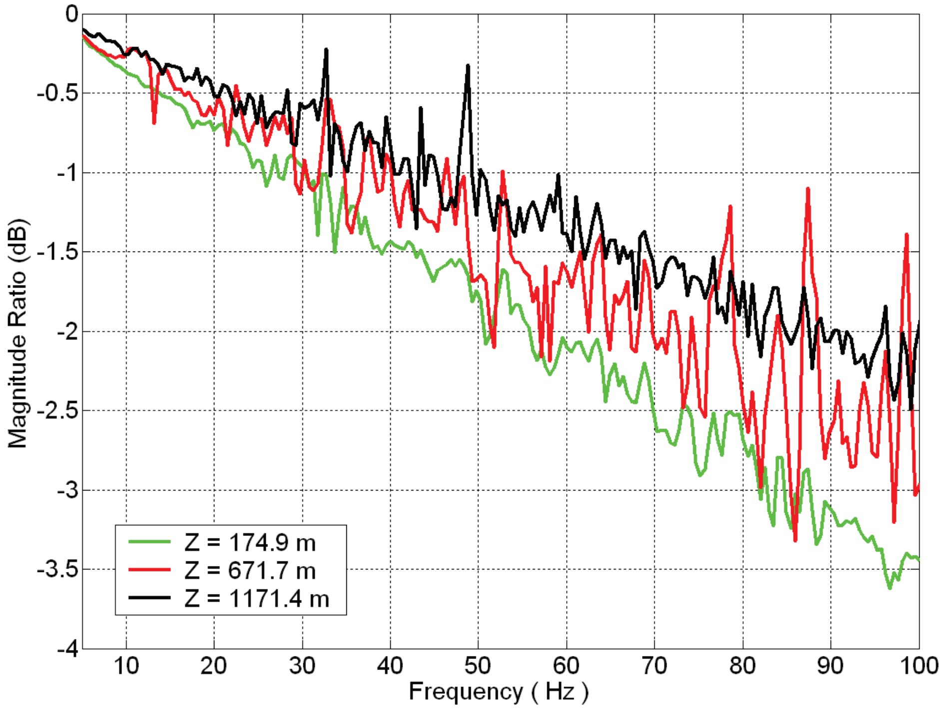

FIG. 9. Log spectral ratios of normalized amplitude spectra in Figure 8.

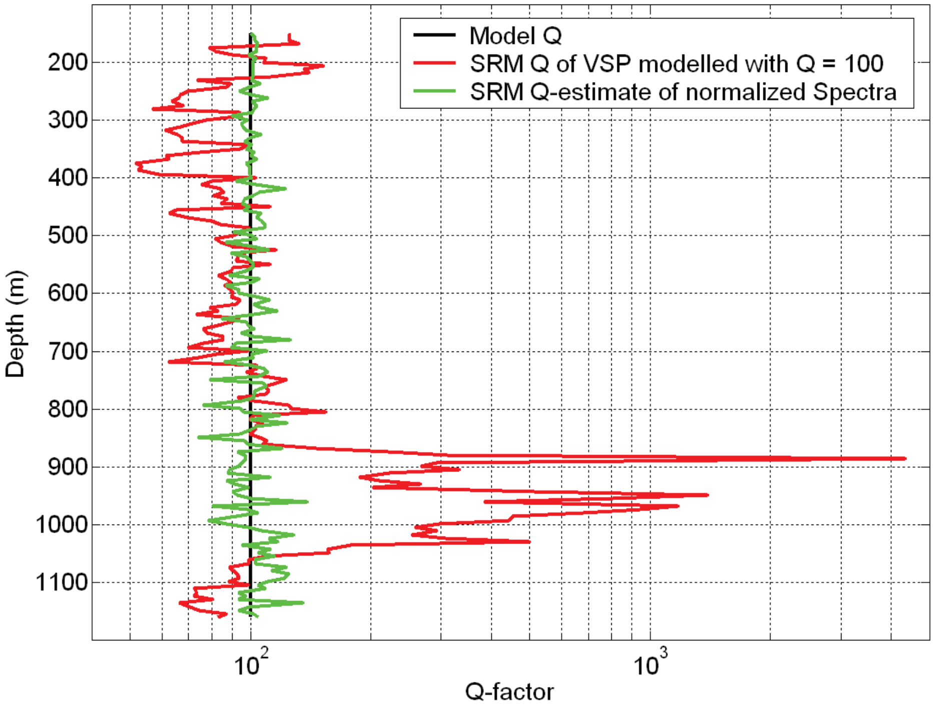

FIG. 10. Spectral ratio method applied to amplitude spectra of Figure 8 (note x-axis log-scale).