Enforcing minimum phase on nonstationary filters

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ABSTRACT

We discuss designing nonstationary filters that preserve the minimum phase property: minimum phase signals in, must produce minimum phase signals out. Such filters are important in deconvolution: by accurately representing the physics of wave propagation through the earth, one aims to build algorithms that give better seismic images. These filters model the observed physical phenomena that the minimum phase property of an impulsive source is preserved even as it passes through an attenuating, dispersive medium.

INTRODUCTION

When creating a mathematical simulation of physical phenomena, it is important to capture as much "physics" as possible in a reasonably simple, or computable, mathematical model. In modeling seismic phenomena, there are many physical laws that one should account for, including causality, conservation of energy, linearity, and others. In previous work in deconvolution (see Margrave and Lamoureux (2002), Margrave et al. (2003), Margrave et al. (2004), Montana and Margrave (2006b), Montana and Margrave (2006a)), we have used Gabor multipliers to model Q-attenuation effects, and an important consideration is creating models that respect the minimum phase characteristics of certain seismic signals.

Impulse sources such as dynamic blasts or weight-drop systems have the physical property that most of their energy is concentrated near the start of the signal. (They go BOOM.) This is called the minimum phase property, and it has been observed that such a signal maintains this property even as it travels through a complex, attenuating medium such as the earth. That is to say, even through the propagating wavelet from the source may get stretched, attenuated, and otherwise modified as it travels through the earth, it still maintains most of its energy at the front. (It does not suddenly go MOOB.)

A mathematical model of such propagation should at least respect this minimum phase property. We call such an operator a *minimum phase preserving operator*: it produces a min phase output for each min phase input. Equivalently, we call it a minimum phase preserving nonstationary filter.

We are interested in characterizing all possible minimum phase preserving nonstationary filters, and more specifically, we want to know how to build one to model seismic wave attenuation.

This paper shows what we know about such operators. To summarize what is shown here, a causal, minimum phase preserving nonstationary, linear operator is represented by a triangular matrix, each column of which forms a minimimum phase signal (possibly a dif-

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ferent signal for each column), and these column/signals have the spectral decay property – the Fourier amplitude spectrum of each successive column is decreasing.

We suspect, but are yet unable to prove, that an even stronger result is true: that a minimum phase preserving operator is characterized by exactly two minimum phase signals. The first signal generates a stationary filter, the other that generates a contraction of the unit disk in the complex plane, which is physically the decay operation. This second part results in a model of frequency-dependent Q-attenuation.

MINIMUM PHASE SIGNALS

Impulsive sources such as a dynamite blast have the physical property that most of their energy is concentrated near the start of the signal. Fixing the start time at $t = 0$, one can be more precise: such a signal is characterized by property that of all signals with a given amplitude spectrum, it is the unique one that maximizes its energy in any interval $[0, \epsilon]$ near the start time 0.

These signals are called "minimum phase" signals, as an extension of the concept of minimum phase delay filters which have a similar front-end loading property of the energy. For discrete time stationary filters, with a rational system response function, the minimum phase property is equivalent to the statement that the causal filter have no poles or zeros inside the unit disk on the complex plane. This is also equivalent to stating that the filter is causal and stable, with causal and stable inverse.

Signals, however, are more general than those functions that arise from rational system responses. The minimum phase property for a discrete causal signal $\mathbf{f} = (f_0, f_1, f_2, \ldots)$ is equivalent to the statement that the z-transform

$$F(z) = \sum_{0}^{\infty} f_n z^n \tag{1}$$

of the signal is an "outer function" in the Hardy space $H^2(\mathbb{D})$ of square integrable analytic functions on the unit disk. The precise definition of "outer" comes from complex analysis, and is discussed in detail in Lamoureux and Margrave (2007) as well as Helson (1995), Hoffman (1962). While these details are beyond what we need to discuss here, suffice to say that "outer" extends the notion that the z-transform has no zeros and poles in the interior of the unit disk, and also enforces no extreme singular behaviour on the boundary of the disk. We will use the fact that an outer function has a well-defined logarithm, which is analytic on the interior of the disk, and has no extreme singular behaviour on the boundary.

As a simple example highlighting the slight difference between min phase filters and min phase signals, note the causal signal

$$\mathbf{f} = (1, 1, 0, 0, 0, \ldots, 0, 0, \ldots) \tag{2}$$

has z-transform $F(z) = 1 + z$ which is outer, and hence the signal is minimum phase by our definition. Note however, it is not considered to represent a minimum phase filter, since its causal inverse

$$\mathbf{f}_{inv} = (1, -1, 1, -1, \ldots, 1, -1, \ldots) \tag{3}$$

is not a stable filter.

Every finite energy, causal signal has a minimum phase equivalent: that is, there is a minimum phase signal that has the same amplitude spectrum as the given signal. This minimum phase equivalent signal can be computed directly from the log amplitude spectrum, giving the z-transform as

$$F(z) = \exp\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{i\theta}+z}{e^{i\theta}-z}\log A(\theta)\,d\theta\right), \tag{4}$$

where the function $A(\theta)$ is the amplitude spectrum. This formula includes the convention that the first non-zero sample in the signal is positive. We observe that the amplitude spectrum cannot have too many zeros in it (for instance, it cannot be zero on an interval), otherwise the log singularities will cause the integral to diverge. Fortunately, every causal signal has this property of "few zeros in the spectrum."

STATIONARY FILTERS WITH MIN PHASE PROPERTY

It is a known how to characterize stationary filters that preserve minimum phase. We state the result here just for completeness. Stationary filters that respect minimum phase are as follows: these are exactly the linear operators obtained by convolution with a causal sequence

$$\mathbf{g} = (g_0, g_1, g_2, \ldots), \tag{5}$$

whose z-transform is a bounded outer function. Precisely, the function

$$G(z) = \sum_{0}^{\infty} g_n z^n \tag{6}$$

is an outer function in the Hardy space $H^\infty(\mathbb{D})$ of essentially bounded analytic functions.

This result follows from the fact that a stationary filter is obtained by convolution with some fixed sequence $\mathbf{g}$, and the fact that the filter applied to a delta function (which is a minimum phase signal) returns the sequence $\mathbf{g}$, which must also be minimum phase, hence its z-transform is outer. The fact that the z-transform must be bounded in essential sup norm, and hence lies in $H^\infty(\mathbb{D})$ follows from the observation that the operator norm of the filter equals the essential sup norm of the z-transform, which must be finite for a bounded filter.

Conversely, if $\mathbf{g}$ is such a sequence, and $\mathbf{f}$ is a minimum phase input signal, then the convolution $g * f$ has z-transform equal to the product $G(z)F(z)$. This product is outer, since the product of two outer functions is outer, hence output signal is minimum phase as well. Thus the filter preserve the minimum phase property.

The subtle difference between $H^2(\mathbb{D})$ and $H^\infty(\mathbb{D})$ is perhaps not too important in many applications, the really important result is that the z-transform of the sequence $\mathbf{g}$ is outer.

It is worth noting that the stationary filter given by the sequence

$$\mathbf{g} = (1, 1, 0, 0, \ldots, 0, 0, \ldots) \tag{7}$$

preserves the minimum phase property. However, in the standard systems theory it is not considered a "minimum phase filter" since its causal inverse is not stable.

SOME NONSTATIONARY FILTERS WITH MIN PHASE PROPERTY

The simplest nonstationary, minimum phase preserving filter is one with fixed decay. Fix a constant r with $0 < r < 1$ and define the decay operator by

$$D(f_0, f_1, f_2, \ldots) = (f_0, rf_1, r^2 f_2, \ldots, r^n f_n, \ldots). \tag{8}$$

It is easy to see that this linear operator preserves minimum phase.

For instance, suppose $\mathbf{f} = (f_0, f_1, f_2, \ldots)$ is a minimum phase signal corresponding to a rational function: hence the poles and roots of its z-tranform $F(z)$ lie outside the complex unit disk. The z-transform of the output signal $\mathbf{g} = (f_0, rf_1, r^2 f_2, \ldots, r^n f_n, \ldots)$ is given as

$$G(z) = \sum_0^\infty f_n r^n z^n = F(rz). \tag{9}$$

Now any pole or zero z_0 of $G(z)$ must correspond to a pole or zero $z_1 = rz_0$ of $F(z)$. Since z_1 has magnitude bigger than one, then z_0 also has magnitude bigger than $1/r > 1$, hence it is also outside the unit disk. So $G(z)$ is outer. And thus this linear operator preserves the minimum phase property.

More generally, suppose ϕ is an outer function, analytic on the closed unit disk, and strictly bounded by one. The generalized decay operator D can be defined on the z-transforms of signals as

$$(DF)(z) = F(z\phi(z)). \tag{10}$$

Which is to say, if $\mathbf{f} = (f_0, f_1, f_2, \ldots)$ is a input signal and $\mathbf{g} = (g_0, g_1, g_2, \ldots)$ is in output signal, $\mathbf{g} = D\mathbf{f}$, their z-transforms are related by

$$G(z) = F(z\phi(z)). \tag{11}$$

Such a linear operator is also minimum phase preserving.

To see this, again suppose $\mathbf{f} = (f_0, f_1, f_2, \ldots)$ is a minimum phase signal, hence its z-transform $F(z)$ is outer, and its logarithm $\log F(z)$ is analytic on the unit disk. The output has z-transform $G(z) = F(z\phi(z))$ with logarithm $\log G(z) = \log F(z\phi(z))$. This is a composition of analytic functions, hence itself is analytic on the whole unit disk, and so $G(z)$ is also outer. Thus the operator preserves minimum phase.

It is convenient to represent such an operator as an infinite matrix. We start by writing down the power series expansion for the function $\phi(z)$ and its powers, using coefficient

$a_{k,n}$ that are indexed in a special form:

$$\begin{aligned}
\phi^0(z) &= a_{0,0} + a_{1,0}z^1 + a_{2,0}z^2 + \ldots = 1 && (12)\\
\phi^1(z) &= a_{1,1} + a_{2,1}z^1 + a_{3,1}z^2 + \ldots && (13)\\
\phi^2(z) &= a_{2,2} + a_{3,2}z^1 + a_{4,2}z^2 + \ldots && (14)\\
\phi^3(z) &= a_{3,3} + a_{4,3}z^1 + a_{5,3}z^2 + \ldots && (15)\\
&\ldots && (16)\\
\phi^n(z) &= a_{n,n} + a_{n+1,n}z^1 + a_{n+2,n}z^2 + \ldots. && (17)
\end{aligned}$$

Now the z-transform $F(z) = \sum f_n z^n$ maps to

$$\begin{aligned}
G(z) &= \sum_n f_n (z\phi(z))^n && (18)\\
&= \sum_n f_n z^n \left[\phi^n(z)\right] && (19)\\
&= \sum_n f_n z^n \left[\sum_k a_{n+k,n} z^k\right] && (20)\\
&= \sum_{n,k} a_{n+k,n} f_n z^{n+k} \quad \text{change summation variables} && (21)\\
&= \sum_m \left[\sum_n a_{m,n} f_n\right] z^m && (22)\\
&= \sum_m \left[g_m\right] z^m, && (23)
\end{aligned}$$

and so we have the matrix form for the transform as

$$g_m = \sum_n a_{m,n} f_n. \tag{24}$$

That is, the coefficients for the transformation matrix $a_{m,n}$ just come from the power series expansion for $\phi(z)$ and its powers $\phi^2(z), \phi^3(z)$, etc. As a matrix, we write

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & a_{11} & 0 & 0 & 0 & \cdots \\ 0 & a_{21} & a_{22} & 0 & 0 & \cdots \\ 0 & a_{31} & a_{32} & a_{33} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots \end{bmatrix} \tag{25}$$

where the entries in the n-th column are the series coefficients for $\phi^n(z)$.

The reason $\phi(z)$ should be outer is that we want our minimum phase preserving filter to preserve not just min phase signals (that start at time $t = 0$) but also preserve time-shifted min phase signals (that start at time $t = n$). So, for instance, the signal

$$\mathbf{f} = (0, 1, 0, 0, \ldots) \tag{26}$$

is minimum phase, delayed by one time step. Using this as an input, the output signal is

$$(0, a_{11}, a_{21}, a_{31}, \ldots) \tag{27}$$

which has z-transform $G(z) = z\phi(z)$. This is the signal $\phi(z)$ shifted by one time step, which must also be minimum phase. So we want $\phi(z)$ to be an outer function.

THE GENERAL MINIMUM PHASE PRESERVING FILTER

We don't know what the general form is for a minimum phase preserving linear operator. However, we suspect these are just the stationary min phase filters composed with the general decay operators mentioned in the last section. Thus, we can fix one outer function $G(z)$ in $H^\infty(\mathbb{D})$, and an outer function $\phi(z)$ which is bounded by one, and analytic on the closed disk. The operator T is defined on the z-transforms as

$$(TF)(z) = G(z)F(z\phi(z)). \tag{28}$$

We don't have a proof of this fact. However, we have been unable to construct a single example that is of this form, despite a lot of work.

On the one hand, this seems a little discouraging – it is not a very rich family of operators. On the other hand, it does give an interesting mathematical justification for the physical observation that any signal, traveling through an attenuating medium, ends up looking more or less minimum phase. What happens is that the map $z \to z\phi(z)$ shrinks the domain of the unit disk, killing off zeros and poles that might exist near the edge of the disk. Iterating the maps kills more and more zeros and poles, until there are few, or none left. The result is a minimum phase output.

THE Q-ATTENUATION MODEL

The proposed general minimum phase preserving operator of the last section, given on the z-transform as

$$(TF)(z) = G(z)F(z\phi(z)) \tag{29}$$

can be considered a model for Q-attenuation. To see this, start with a delta pulse as input, with

$$\delta_n = (0, 0, 0, \ldots, 1, 0, \ldots), \quad \text{1 in the n-th place.} \tag{30}$$

Its z-transform is $F(z) = z^n$, and the z-transform of the output is a function

$$H(z) = G(z)(z\phi(z))^n. \tag{31}$$

The factor z^n is just a shift in time, so effectively the output signal $H_o(z) = G(z)(\phi(z))^n$. The Fourier transform of this output signal is obtained by evaluating the z-transform at $z = e^{i\theta}$, which gives

$$H_o(e^{i\theta}) = G(e^{i\theta})(\phi(e^{i\theta}))^n. \tag{32}$$

Since $\phi(z)$ is an outer function, we can write it as an exponential $\phi(e^{i\theta}) = e^{h(\theta)}$, so we have

$$\begin{aligned} H_o(e^{i\theta}) &= G(e^{i\theta})(e^{h(\theta)})^n & (33)\\ &= G(e^{i\theta})e^{nh(\theta)}. & (34) \end{aligned}$$

The factor $e^{nh(\theta)}$ represents the Q-attenuation, where the n represents decay in time, and the $h(\theta)$ represents decay in frequency. Note that the real part of h is negative, so this truly is a decay factor. (Real part of h is negative since $|\phi(e^{i\theta})| = e^{Re(h(\theta))} < 1$.)

If, as we suspect, Eqn (29) really is the general form of a minimum phase preserving linear operator, then this general form of Q-attenuation is actually a consequence of the requirement of min phase preservation.

THE COLUMN CONDITIONS: OUTER, AND SPECTRAL DECAY

Although we can't say exactly what all our minimum phase preserving filter are, we can state some restrictions on the possibilities.

First of all, we want a causal filter. This means the matrix of the operator should be lower triangular, in this form:

$$A = \begin{bmatrix} a_{00} & 0 & 0 & 0 & 0 & \cdots \\ a_{10} & a_{11} & 0 & 0 & 0 & \cdots \\ a_{20} & a_{21} & a_{22} & 0 & 0 & \cdots \\ a_{30} & a_{31} & a_{32} & a_{33} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{35}$$

Since we are working with real input signals, with real output signals, we can assume the coefficient a_{mn} are all real. We could normalize to assume the first arriving pulse (positive) produces a positive-going output signal, so the first entry in each column above is positive.

The next restriction is that each column comes from the series coefficients of an outer function. That is, the functions

$$\phi_n(z) = \sum_{k=0}^{\infty} a_{n+k,n} z^k \tag{36}$$

are outer. This restriction arises since the output of a shifted delta spike

$$\delta_n = (0, 0, 0, \ldots, 1, 0, \ldots), \quad \text{1 in the n-th place} \tag{37}$$

should be a shifted minimum phase signal; the n-th column of matrix A corresponds to this output.

The final restriction we can enforce is called the spectral decay condition: it requires that the amplitude spectrum of the signal in column $n + 1$ is smaller than the amplitude spectrum in column n. Mathematically, we require

$$|\phi_{n+1}(e^{i\theta})| \leq |\phi_n(e^{i\theta})|, \qquad \text{for all } \theta, n. \tag{38}$$

Physically, this says that the minimum phase preserving operators forces a spectral decay.

This statement requires a proof. The outline goes like this:

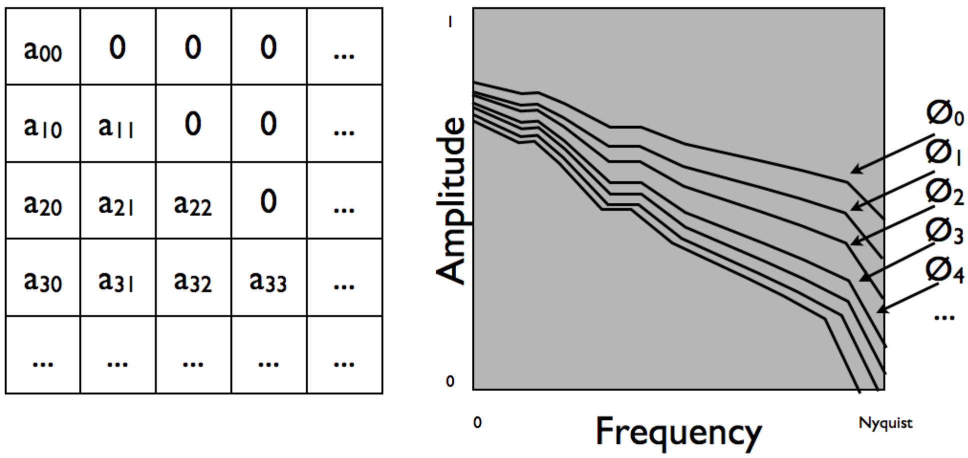

FIG. 1. Each column of the matrix represents a signal, with a Fourier amplitude spectrum. A plot of the spectra shows a decreasing sequence of functions.

For simplicity, let's take the first two adjacent columns, with outer functions $\phi_0(z), \phi_1(z)$ representing the coefficients of the columns of matrix A. If $|\phi_1(z)| > |\phi_0(z)|$ somewhere on the boundary of the disk, by continuity there is a nearby point z_0 in the interior of the disk with $|z_0\phi_1(z_0)| > |\phi_0(z_0)|$. Let $c_0 = -z_0\phi_1(z_0)/\phi_0(z_0)$, which is a complex number of modulus strictly bigger than one. The signal

$$(c_0, 1, 0, 0, 0, \ldots) \tag{39}$$

is minimum phase, and its output under the filter is a signal with z-transform

$$G(z) = c_0\phi_0(z) + z\phi_1(z). \tag{40}$$

But this function has a zero at z_0 since

$$G(z_0) = (-z_0\frac{\phi_1(z_0)}{\phi_0(z_0)})\phi_0(z_0) + z_0\phi_1(z_0) = 0, \tag{41}$$

which shows $G(z)$ is not outer. Hence the filter could not preserve minimum phase.

By this contraction, we conclude we must have the spectral decay condition on the columns.

It is easy to visualize the spectral decay condition, by plotting the amplitude spectrum of each column of the matrix A, as in Figure 1.

OPEN QUESTIONS ABOUT MIN PHASE OPERATORS

We can look at the simple case of a diagonal operator,

$$A = \begin{bmatrix} a_{00} & 0 & 0 & 0 & 0 & \cdots \\ 0 & a_{11} & 0 & 0 & 0 & \cdots \\ 0 & 0 & a_{22} & 0 & 0 & \cdots \\ 0 & 0 & 0 & a_{33} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{42}$$

If A preserves the minimum phase property, by the spectral decay property, we know the diagonal elements must be decreasing:

$$a_{00} \geq a_{11} \geq a_{22} \geq a_{33} \geq \ldots. \tag{43}$$

However, decreasing diagonal terms is not enough. For instance, the following matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1/2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1/2 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{44}$$

is not minimum phase preserving. For instance, the input signal $\mathbf{f} = (1, 2, 1, 0, 0, 0, \ldots)$ is minimum phase (its z-transform has a double root at $z = -1$) but its output

$$\mathbf{g} = A\mathbf{f} = (1, 2, .5, 0, 0, 0 \ldots) \tag{45}$$

is not minimum phase, since its z-transform has zeros at $-2 \pm \sqrt{2}$, one of which is inside the unit disk.

With a little work, an argument with three term signals shows the diagonal elements satisfy a convexity property: the map

$$n \mapsto a_{nn} \tag{46}$$

is a convex function. That is,

$$\frac{a_{nn} + a_{n+2,n+2}}{2} \geq a_{n+1,n+1} \qquad \text{for all indices } n. \tag{47}$$

An example of a diagonal map that satisfies the decreasing, convexity properties is the standard decay map, with

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & r & 0 & 0 & 0 & \cdots \\ 0 & 0 & r^2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & r^3 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{48}$$

However, we have been unable to find any other diagonal map (that satisfies both decreasing and convexity) which is actually minimum phase preserving.

For instance, consider the simple modification of the decay map:

$$A_\epsilon = \begin{bmatrix} 1+\epsilon & 0 & 0 & 0 & 0 & \cdots \\ 0 & r2 & 0 & 0 & 0 & \cdots \\ 0 & 0 & r^2 & 0 & 0 & \cdots \\ 0 & 0 & 0 & r^3 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{49}$$

For small values of $\epsilon \neq 0$, the diagonal map A_ϵ has decreasing, convex entries on the diagonal. However, we will show this map is not minimum phase preserving.

Take the signal $\mathbf{f} = (1, k/1!, k^2/2!, k^3/3!, \ldots)$, where for the moment the constant $k > 0$ is not specified. The z-transform of the signal is the exponential function $F(z) = \exp(kz)$. A simple calculation shows the map A_ϵ transforms signal $\mathbf{f}$ into signal $\mathbf{g} = (1+\epsilon, rk/1!, (rk)^2/2!, (rk)^3/3!, \ldots)$. The z-transform of the output is the function

$$G(z) = \exp(krz) + \epsilon. \tag{50}$$

This function has a zero at $z_0 = (\log(\epsilon) + \pi i)/kr$. By choosing k sufficiently large, this zero z_0 is in the interior of the unit disk. Thus the output signal is not minimum phase, and so the transform A_ϵ is not minimum phase preserving.

So, we don't know of any diagonal map, other than the decay map, that is minimum phase preserving.

Our conjecture is that the only diagonal maps which preserve minimum phase are in fact these decay maps, with decay coefficient r.

Similarly, we conjecture that the general minimum phase preserving operators are of the form

$$(TF)(z) = G(z)F(z\phi(z)), \tag{51}$$

as discussed in the last section.

SUMMARY

Minimum phase preserving operators can model an important physical property of seismic wave propagation, namely that an impulsive source such as a dynamite blast maintains a minimum phase characteristic as it travels through the earth. We have shown that when constructing such an operator, the columns in the matrix operators are coefficients of outer functions, and demonstrate spectral decay. We have conjectured a general form for such operators, which necessarily involved a general form of Q-attenuation. Further work will incorporate such operators to improve our deconvolution methods.

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