Error distribution when using first-break arrival times to locate microseismic events

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ABSTRACT

SVD method was used to estimate 3D hypocenter location and origin time of a microseismic event. Given variance of observed first-break arrival time, 3D error distribution of hypocenter location is calculated. It is shown that uncertainty in vertical direction is much bigger than that in horizontal directions.

INTRODUCTION

Microseismic monitoring has seen increasing application to hydraulic fracture monitoring and steam-assisted heavy oil production, it made possible to economically define treatment parameters and optimal well spacing.

First-break arrival times can be used to estimate location of a microseismic event and its origin time. Figure 1 shows the geometry of ray path from source to surface receivers, constant RMS velocity is assumed for simplification.

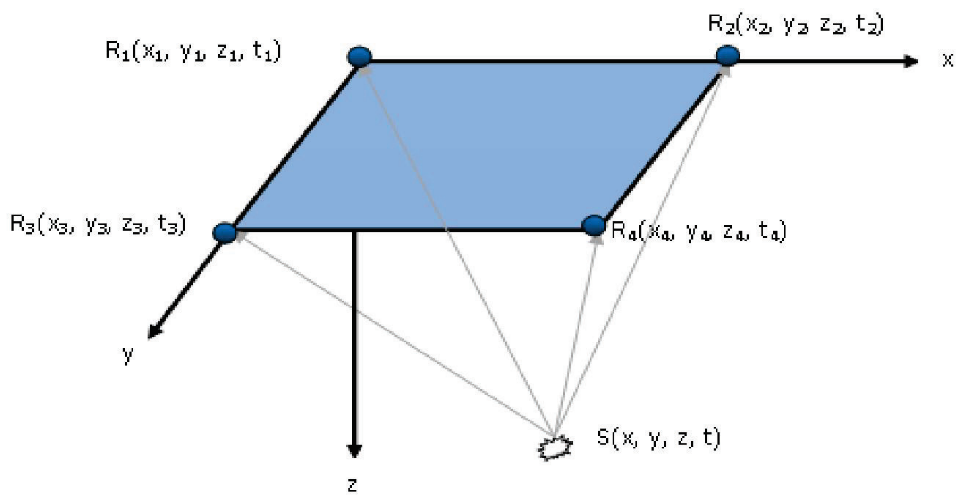

Figure 1. Geometry of ray path from source to receivers. $S(x, y, z, t)$ is a microseismic event with origin clock-time t and location (x, y, z). $R_i(x_i, y_i, z_i, t_i)$ is a surface receiver at (x_i, y_i, z_i) with observed first-break arrival time t_i. Constant RMS velocity is assumed.

Given first-break arrival times $t_1, t_2, t_3, t_4, \ldots, t_m$ at receivers $R_1, R_2, R_3, R_4, \ldots, R_m$, the location and origin time of a microseismic event can be estimated (Bancroft and Du, 2006). Suppose that all receivers start to record data at arbitrary clock-time t_0 and receiver R_i (x_i, y_i, z_i) record first-break arrival time at t_i, then the travel time from hypocenter to receivers satisfy

$$v^2(t_i - t_0)^2 = (x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2. \tag{1}$$

In order to change above quadratic equations into a linear parameter estimation problem, we apply above equation in receiver R_{i-1},

$$v^2(t_{i-1} - t_0)^2 = (x_{i-1} - x)^2 + (y_{i-1} - y)^2 + (z_{i-1} - z)^2. \tag{2}$$

Subtract (2) from (1) we will get:

$$\begin{aligned} &2(x_i - x_{i-1})x + 2(y_i - y_{i-1})x + 2(z_i - z_{i-1})z - 2v^2(t_i - t_{i-1})t_0 \\ &= v^2({t_i}^2 - {t_{i-1}}^2) + {x_i}^2 + {y_i}^2 + {z_i}^2 - ({x_{i-1}}^2 + {y_{i-1}}^2 + {z_{i-1}}^2). \end{aligned} \tag{3}$$

If we write it in matrix form, it will be

$$\begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) & 2(z_2 - z_1) & -2v^2(t_2 - t_1) \\ 2(x_3 - x_2) & 2(y_3 - y_2) & 2(z_3 - z_2) & -2v^2(t_3 - t_2) \\ 2(x_4 - x_3) & 2(y_4 - y_3) & 2(z_4 - z_3) & -2v^2(t_4 - t_3) \\ \vdots & \vdots & \vdots & \vdots \\ 2(x_m - x_{m-1}) & 2(y_m - y_{m-1}) & 2(z_m - z_{m-1}) & -2(t_m - t_{m-1}) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t_0 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{\mathrm{m}} \end{bmatrix}, \tag{4}$$

where

$$d_i = v^2({t_i}^2 - {t_{i-1}}^2) + {x_i}^2 + {y_i}^2 + {z_i}^2 - ({x_{i-1}}^2 + {y_{i-1}}^2 + {z_{i-1}}^2). \tag{5}$$

For convenience, we denote the coefficient matrix at the left-side of equation (4) as $\boldsymbol{G}$, the parameter vector to be estimated as $\boldsymbol{m}$ and known data at the right-side as $\boldsymbol{d}$.

$$\boldsymbol{Gm} = \boldsymbol{d}. \tag{6}$$

To solve this linear regression problem singular value decomposition (SVD) method (Aster et al., 2005) is used in this paper. First, $\boldsymbol{G}$ is factored into

$$\boldsymbol{G} = \boldsymbol{USV}^T, \tag{7}$$

where $\boldsymbol{U}$ is m by m orthogonal matrix with columns that are unit basis vectors, $\boldsymbol{S}$ is m by 4 diagonal matrix with nonnegative diagonal elements called singular values, $\boldsymbol{V}$ is 4 by 4 orthogonal matrix with columns that are basis vectors and T means transpose.

If only the first p singular values are nonzero, we can partition $\boldsymbol{S}$ as

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_p & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}. \tag{8}$$

Then the solution to $\boldsymbol{m}$ will be

$$\boldsymbol{m} = \boldsymbol{V}_p \boldsymbol{S}_p^{-1} \boldsymbol{U}_p^T \boldsymbol{d}, \tag{9}$$

where $\boldsymbol{V}_p$ and $\boldsymbol{U}_p$ mean the first p columns of $\boldsymbol{V}$ and $\boldsymbol{U}$, *-1* means inverse of a matrix.

Assume that measurement errors of t_i are independent and normally distributed, standard deviations σ_i can be incorporated into the solution by weighting $\boldsymbol{G}$ and $\boldsymbol{d}$,

$$\boldsymbol{G}_w = diag(\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \ldots, \frac{1}{\sigma_m})\boldsymbol{G}, \tag{10a}$$

$$\boldsymbol{d}_w = diag(\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \ldots, \frac{1}{\sigma_m})\boldsymbol{d}. \tag{10b}$$

If standard deviation for every t_i is identical, the covariance $\boldsymbol{C}$ of estimated parameters can be simply calculated by

$$\boldsymbol{C} = \sigma^2 \boldsymbol{V}_p \boldsymbol{S}_p^{-2} \boldsymbol{V}_p^T. \tag{11}$$

Covariance $\boldsymbol{C}$ can be used to estimate 95% confidence intervals for individual parameters which is given by

$$\boldsymbol{m} \pm 1.96\, diag(\boldsymbol{C})^{1/2}. \tag{12}$$

If we consider combinations of multi-parameter, the 95% confidence region is a 3D ellipsoid. This ellipsoid can be calculated by diagonalizing inverse of covariance, $\boldsymbol{C}^{-1}$,

$$\boldsymbol{C}^{-1} = \boldsymbol{P}^T \boldsymbol{\Lambda} \boldsymbol{P}, \tag{13}$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix of positive eigenvalues and the columns of $\boldsymbol{P}$ are orthonormal eigenvectors. The *i*th semimajor error ellipsoid axis direction is defined by $\boldsymbol{P}_{.,i}$, its length l is determined by

$$l = \Delta / \sqrt{\Lambda_{i,i}}, \tag{14}$$

where Δ is the 95$^{\text{th}}$ percentile of χ^2 distribution with 4 degrees of freedom.

EXAMPLE

A synthetic experiment was carried out to test the SVD method and estimate the error distribution of microseismic events. Suppose the source's position is $S(x=70,\ y=70,\ z=1000)$ and generate a microseismic event at $t=1.6s$.

Ten surface receivers were used to record first-break arrival times, they are located at $\boldsymbol{x}$= {10 20 30 40 50 60 80 90 100 110}, $\boldsymbol{y}$= {10 20 30 40 50 80 90 100 110 120}, $\boldsymbol{z}$= {0 2 1 4 3 3 5 7 3 2}.

The first-break arrival times are calculated by

$$\boldsymbol{t} = t + \frac{\sqrt{(\boldsymbol{x}-70)^2 + (\boldsymbol{y}-70)^2 + (\boldsymbol{z}-1000)^2}}{v}, \tag{15}$$

where constant RMS velocity v=3000m/s is used.

Arrival times ***t*** were perturbed with Gaussian distribution of zero mean and standard deviation of 10ms and 2ms respectively, the 10ms uncertainty is considered an upper bound when observing first-break arrival times (Eisner et al., 2009).

Figures 2 and 3 are 2D view of error distribution with 10ms and 2ms standard deviation respectively. It shows that uncertainty in vertical direction z is much bigger than that in horizontal directions x and y. For example, with deviation equals to 10ms, error in z direction is about 60 meters which is bigger than 8 meters in x direction.

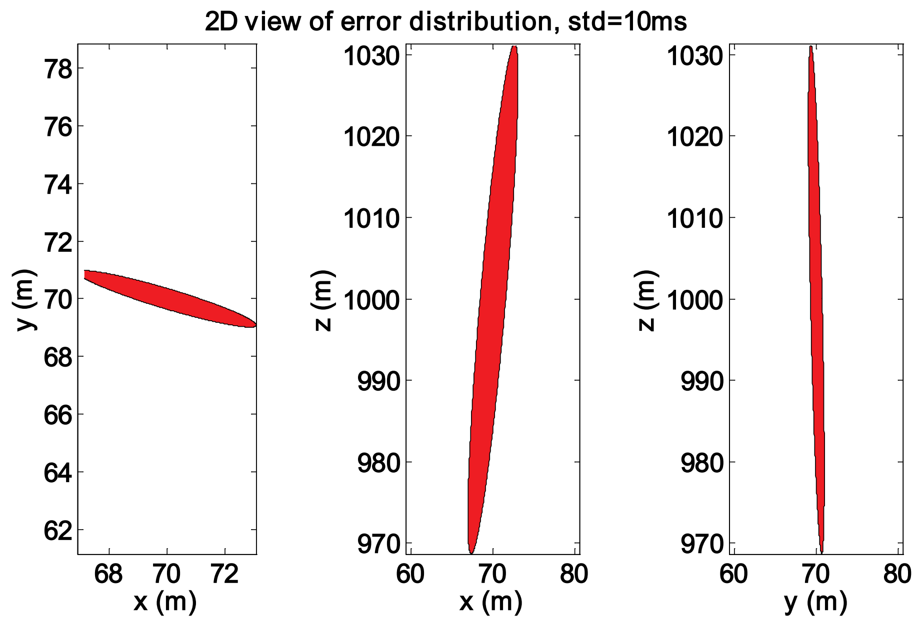

Figure 2. 2D view of error distribution of source with *std*=10ms.

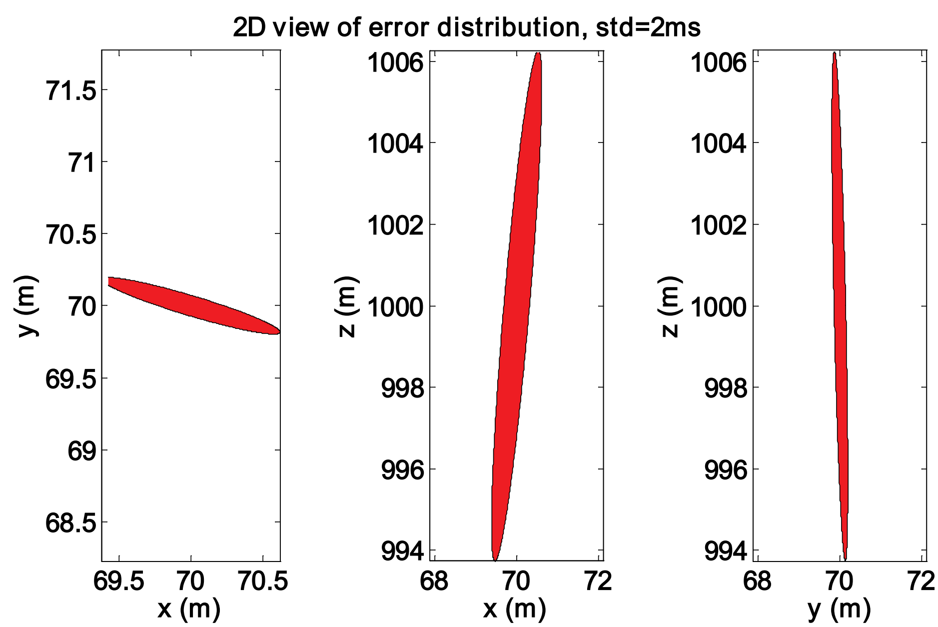

Figure 3. 2D view of error distribution of source with *std*=2ms.

Figure 4 is 3D view of error distribution of source with standared deviation equals to 10ms and 2ms respectively.

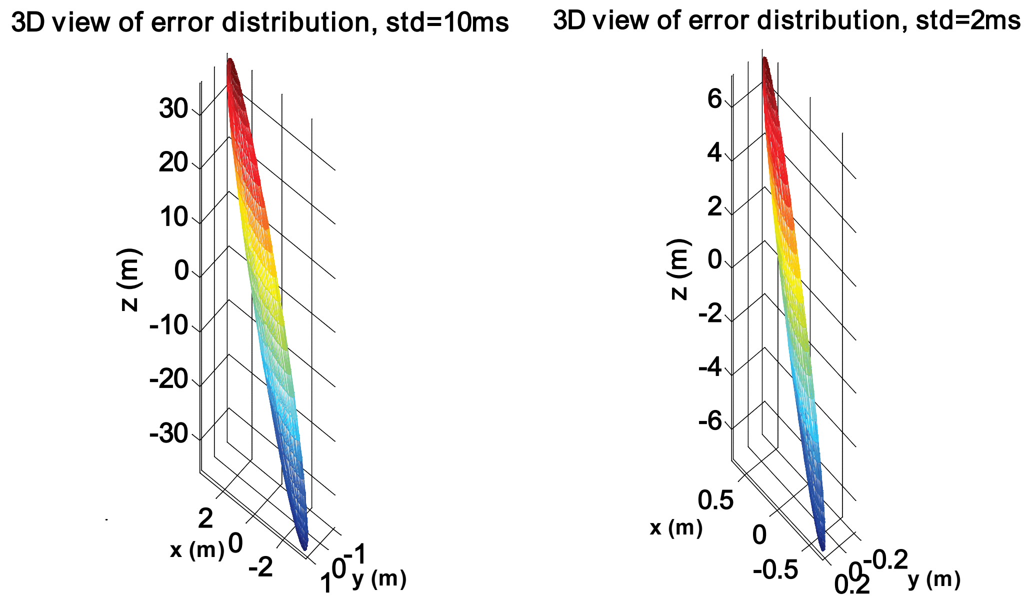

Figure 4. 3D view of error distribution of source with *std*=10ms (left) and *std*=2ms (right).

CONCLUSION

Given first-break arrival time at each receiver, SVD method can be used to estimate a microseismic event and its origin time. If standard deviation is known, which can be derived from observed visible arrival times, the error distribution of source can be estimated.

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