A POCS algorithm for spectral extrapolation

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ABSTRACT

Projection-onto-convex-sets or POCS algorithms are used to infill missing seismic data. Applications have generally been on multidimensional interpolation problems. We consider a different type of missing data: the low end of the frequency spectrum. We infill this spectral gap using a POCS algorithm, under the assumption that data events are lagged delta functions. A trace-by-trace implementation, tested on synthetics, confirms the applicability of the idea, and its resiliency to reduced data bandwidth, reasonable clustering-density of events, and uncorrelated noise. Testing on field data with well control is the next logical step. If successful, POCS spectral extrapolation could be a valuable preprocessing tool prior to seismic inversion.

INTRODUCTION

Projection onto convex sets, or POCS, algorithms, are simple and robust methods for completion of data sets. They have been widely used in seismic data processing for interpolation of missing traces (Abma et al., 2005; Abma and Kabir, 2006; Galloway and Sacchi, 2007), and their promise has led to attempts to extend their application to problems such as time-lapse data differencing (Naghizadeh and Innanen, 2011). POCS methods have also been used for phase reconstruction of seismic data (Ulrych et al., 2007).

Missing bandwidth in seismic data, particularly in the low end, is a critical obstacle for seismic inversion. Measurement of these low frequencies is of course ideal, and research towards providing—via sources and sensors in combination—the lowest possible spectral cutoff has been a large thrust of CREWES research this year (Margrave et al., 2011). Still, in the absence of measurement down to 0 Hz, spectral extrapolation methods (Oldenburg et al., 1983; Ulrych and Walker, 1984; Ulrych, 1989) may be extremely useful, if only to “finish the job” begun by an appropriate experiment. Extrapolation and well logs may provide a bridge for practical seismic inversion at CREWES (Lloyd and Margrave, 2011).

In this paper we examine the potential POCS-type algorithms have for extrapolation of low frequencies in seismic data. We will restrict our attention to synthetic data, but we will design these data sets to put some basic limits on how well the idea might work.

The assumptions of the method

The approach relies on a particular view of seismic signals as they appear in various transform spaces, in this case time and frequency. We will assume that a trace is sparse in the time domain, meaning that the signal, in its pure state, has a small number of large coefficients. Ideally, this would mean a trace that is predominantly zeros, but with a sequence of lagged spikes—something like a true reflectivity. When low frequencies are missing, the effect on the time domain is that a larger number of nonzero coefficients appear, in the form of sidelobes etc., but with amplitudes significantly smaller than the ones at and around the spike maxima.

Input to this algorithm resembles the input to other spectral extrapolation techniques. In particular, though bandlimited, the input should be source wavelet deconvolved; namely, within the band, the amplitude envelope should be maximally flat.

ALGORITHM AND SIMPLE SYNTHETIC EXAMPLES

With these conditions in place, the POCS algorithm can be carried out in a simple iterative fashion. We begin with a measured trace $x_0(t)$, which is deficient in frequencies below f_0. A threshold Υ_0 operator is formed, which generates $y_0(t) = \Upsilon_0 x_0(t)$, a trace which is equal to $x_0(t)$ for all values above the threshold, and zero everywhere else. This trace $y_0(t)$ is subject to a Fourier transform, and so is the original data trace, creating $X_0(f)$ and $Y_0(f)$ respectively.

A new spectrum is now generated, equal to $X_0(f)$ within the signal band, and equal to $Y_0(f)$ elsewhere:

$$X_1(f) = \Theta\ Y_0(f) + [1 - \Theta]X_0(f), \tag{1}$$

where $\Theta = H(f - f_0) - H(f + f_0)$ and H is the Heaviside or step function. This spectrum is inverse Fourier transformed to the time domain, forming $x_1(t)$. The process is then begun again, with a new threshold Υ_1 being chosen, and thus a $y_1(t)$ formed, etc.

The main input to the algorithm is the sequence of thresholds. If, for instance, two iterations are to be carried out, as an input a vector $\upsilon = [\upsilon_0, \upsilon_1]^T$ must be provided in order to construct the operators Υ_0 and Υ_1 etc.

In total then, using the symbol FT to denote the Fourier transform operator, the updated trace $x_{n+1}(t)$ is given in terms of $x_n(t)$ by

$$x_{n+1}(t) = \mathrm{FT}^{-1} \left\{ \Theta\ \mathrm{FT} \left[\Upsilon_n x_n(t) \right] + (1 - \Theta)\ \mathrm{FT} \left[x_n(t) \right] \right\}. \tag{2}$$

10-250Hz example

We begin with the simplest of our synthetic examples, to establish that a POCS type algorithm has the basic wherewithal to fill in the missing low frequencies of a spiky reflectivity. In Figure 1a–b we illustrate a three event reflectivity series and its spectrum respectively. We then set the lowest 10 Hz of the spectrum to zero, and inverse Fourier transform back to time, in Figure 1c–d respectively. Figure 1d constitutes our input synthetic.

We next iterate the procedure discussed in the previous section. In Figure 2 each row represents an iteration, with the top row being the input. On the top left is the input spectrum (black) overlain with the correct spectrum (dashed). In the top middle the input trace (red) is overlain on the idealized trace (black).

Most importantly, on the right panel is the integral of the bandlimited trace (red) overlain on the exact integral (black). The trace integral is a fundamental ingredient of seismic inversion, and it makes visible issues of bandlimitation much more vividly than does the trace itself. The difference between red and black in this top right panel is about the clearest illustration of the need for low frequencies one could arrange for.

In the middle row of Figure 2 is POCS iteration 1. On the left, on top of the input and exact spectra is now overlain the infilled spectrum in red. The updated trace is plotted in the middle, as is its integral on the right. On the bottom is the second iteration. By this time a very satisfactory result is being obtained. Visually, the input trace (top middle in red) and the output trace (bottom middle in red) do not seem very different, but, in comparing their respective integrals (top right vs. bottom right), the possible influence of spectral extrapolation on inversion is clear.

Very broadly this example makes the POCS spectral extrapolation seem possible. The answer is not perfect after two iterations, but it is close. Several additional iterations were attempted with little further improvement.

Thresholds used were $\upsilon = [0.1, 0.05]^T$.

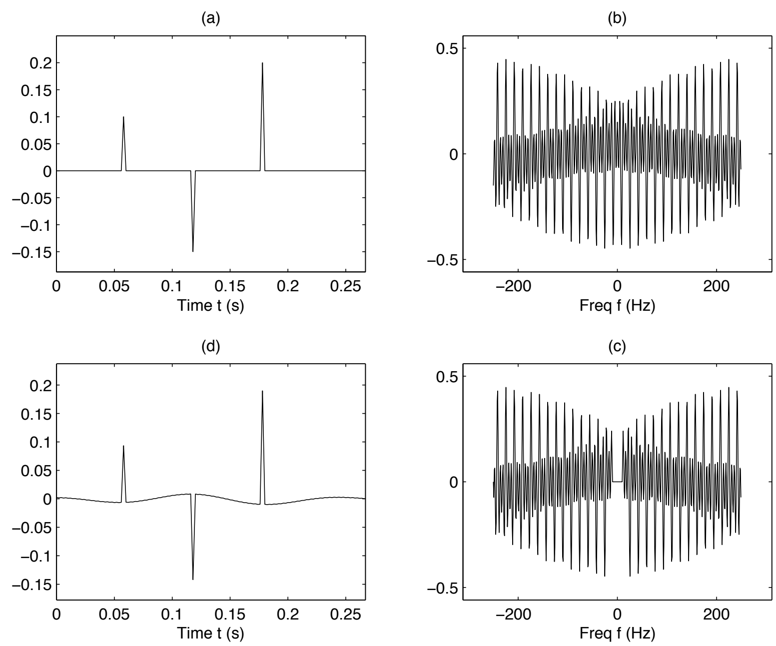

FIG. 1. Input data for synthetic test of POCS spectral extrapolation. (a) Three event reflectivity at full bandwidth. (b) Spectrum of reflectivity. (c) Spectrum with lowest 10 Hz removed. (d) Bandlimited trace: input to the POCS algorithm.

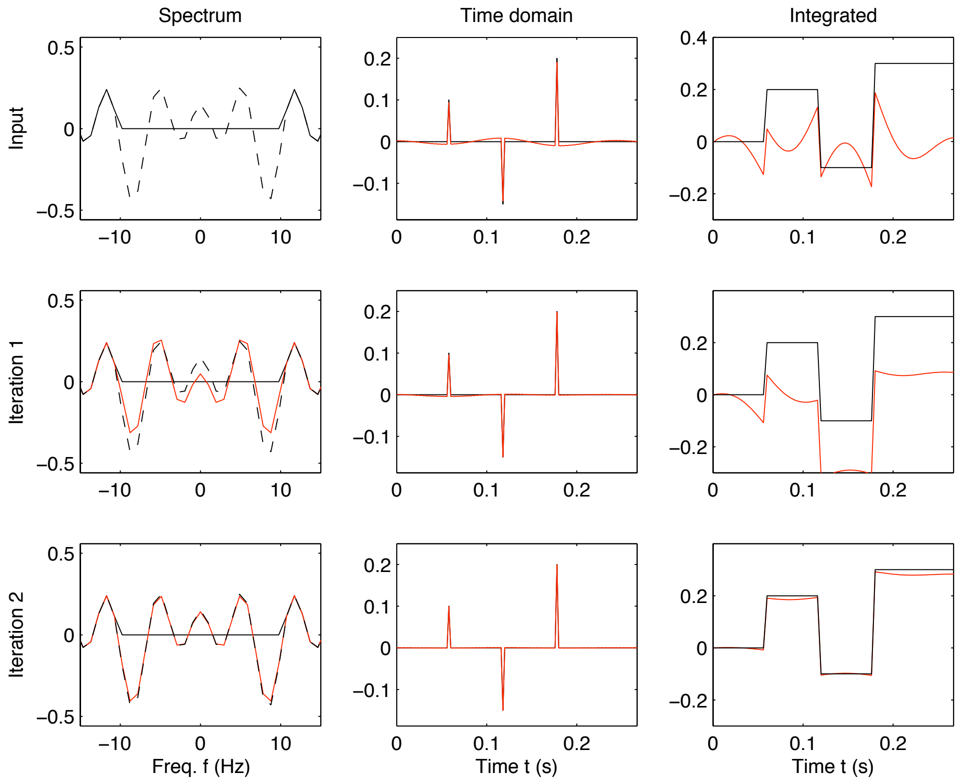

FIG. 2. Iterations of POCS. Top row: left; input trace (black) vs. idealized (dashed); middle; input trace (red) vs. idealized trace (black); right; integrated traces, input (red) vs. idealized (black). Middle row, first iteration of POCS; bottom row, second iteration of POCS.

25-250Hz example

We next aggravate the problem by removing a greater interval of low frequencies. In Figure 3 we repeat the construction of input data seen in Figure 1, but this time we zero out 0-25 Hz. Repeating the POCS iterations we again recover reasonably good results. Having chosen a sampling interval of 0.002s, however, we point out that our synthetic bandwidth extends to 250 Hz, which is significantly greater than that normally encountered in seismic data. This "helps" POCS, by providing lots of signal within which patterns can establish themselves.

Nevertheless the algorithm shows itself at least in principle to be capable of extrapolating several tens of Hz of low frequency.

Thresholds used were again $\upsilon = [0.1, 0.05]^T$.

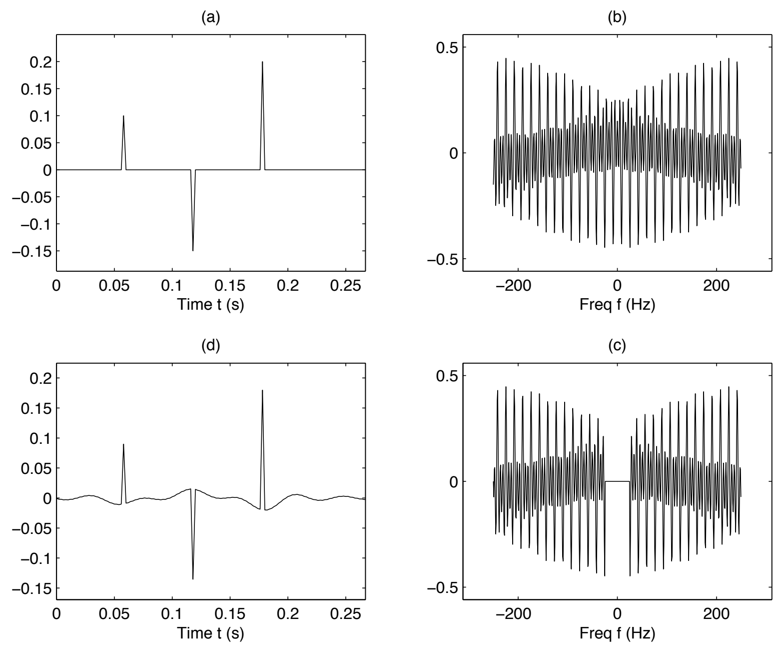

FIG. 3. Input data for synthetic test of POCS spectral extrapolation. (a) Three event reflectivity at full bandwidth. (b) Spectrum of reflectivity. (c) Spectrum with lowest 25 Hz removed. (d) Bandlimited trace: input to the POCS algorithm.

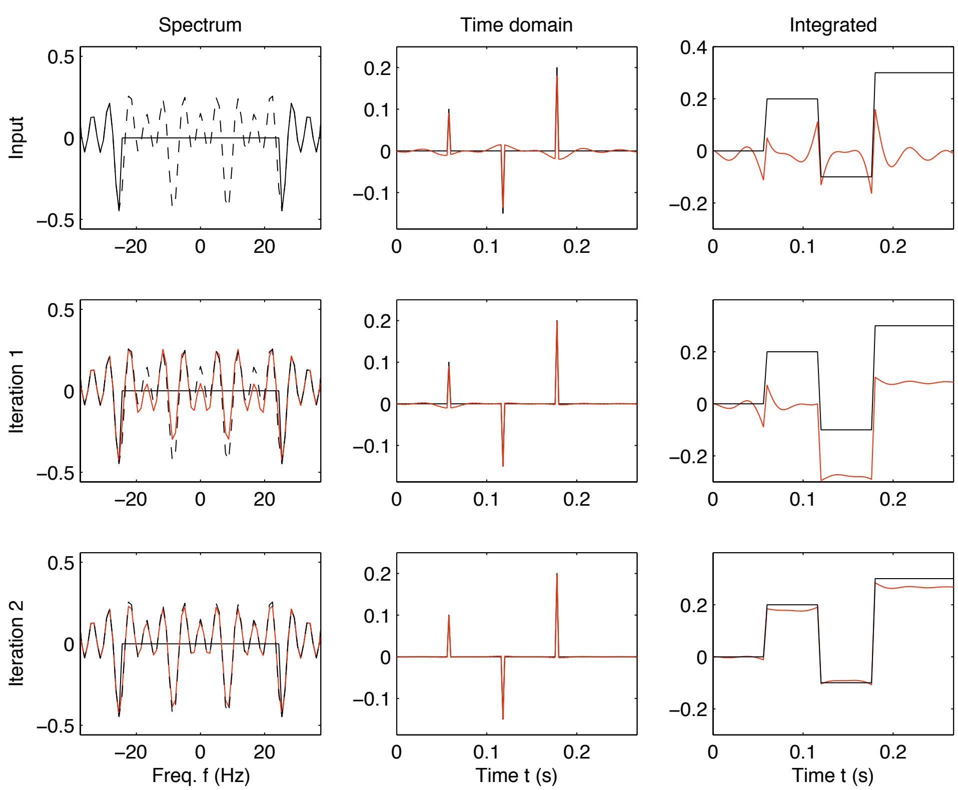

FIG. 4. Iterations of POCS. Top row: left; input trace (black) vs. idealized (dashed); middle; input trace (red) vs. idealized trace (black); right; integrated traces, input (red) vs. idealized (black). Middle row, first iteration of POCS; bottom row, second iteration of POCS.

More events and less bandwidth

In this and the next section we will expose the algorithm to some mild stresses. Here we will do so by increasing the number of events and reducing the bandwidth of the data. In Figure 5a–d we again display the input data, now with a larger number of events, and with a high cut of 120 Hz. The POCS extrapolation is illustrated in Figure 6 in the same format as before. The results exhibit some increased amplitude sensitivity, but capture the essential behaviour of the desired full bandwidth traces. This is especially visible in the integrated traces (right column).

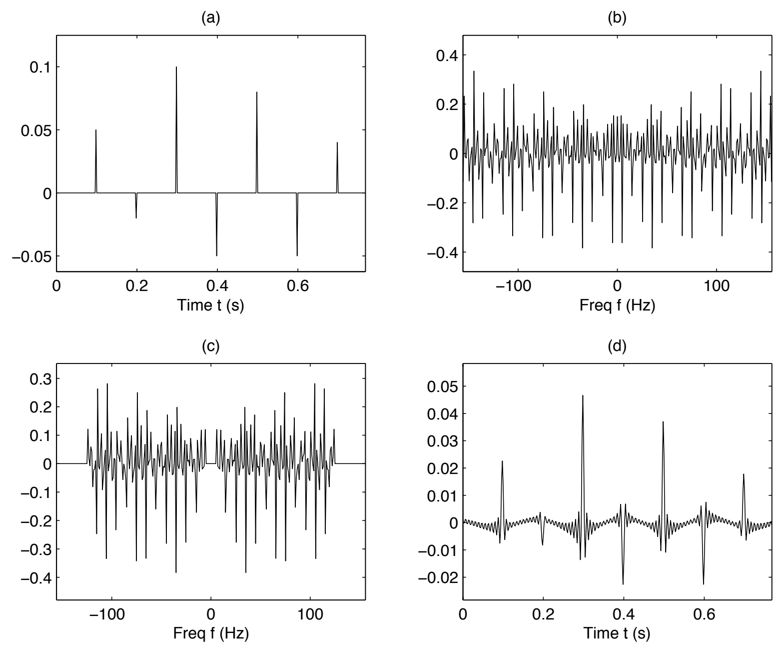

FIG. 5. Input data for synthetic test of POCS spectral extrapolation. (a) Three event reflectivity at full bandwidth. (b) Spectrum of reflectivity. (c) Spectrum with lowest 5 Hz removed. (d) Bandlimited trace: input to the POCS algorithm.

Uncorrelated noise I: %1 noise

Here we add %1 uncorrelated noise, drawn from a Gaussian distribution, primarily to demonstrate that there is no extreme sensitivity to imperfect data to be concerned about. Figures 7-8 repeat the above synthetic data/POCS spectral extrapolation exercise, showing basic insensitivity to the low level noise.

Uncorrelated noise II: %5 noise

Finally in Figures 9-10 we repeat the same exercise with %5 noise drawn from a Gaussian distribution. A slightly greater deviation of the reconstruction from the idealized integrated trace (i.e., red vs. black lines in the bottom right of Figure 10) is in evidence; nevertheless the differential benefit in comparison to straight integration of the bandlimited trace (i.e., red vs. black lines in the bottom right of Figure 10) is clear.

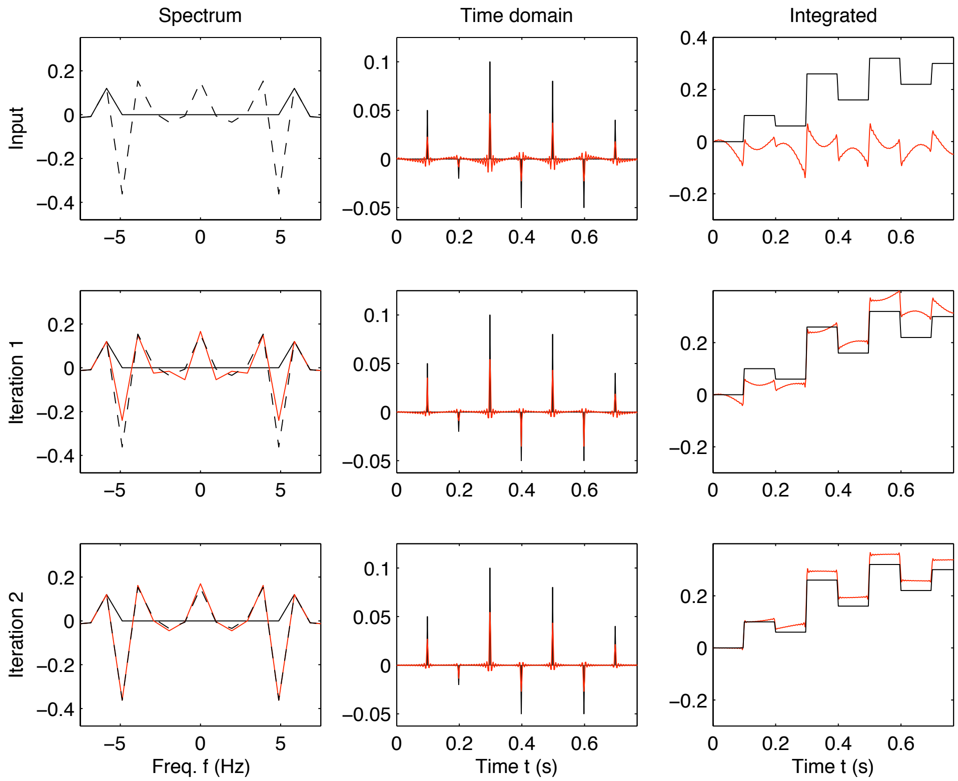

FIG. 6. Iterations of POCS. Top row: left; input trace (black) vs. idealized (dashed); middle; input trace (red) vs. idealized trace (black); right; integrated traces, input (red) vs. idealized (black). Middle row, first iteration of POCS; bottom row, second iteration of POCS.

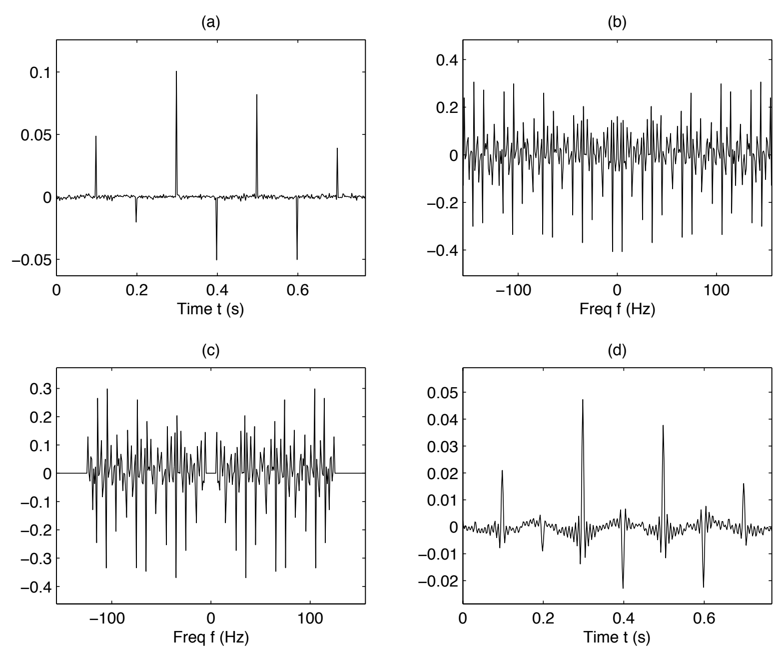

FIG. 7. Input data for synthetic test of POCS spectral extrapolation with %1 noise drawn from a Gaussian distribution. (a) Three event reflectivity at full bandwidth. (b) Spectrum of reflectivity. (c) Spectrum with lowest 5 Hz removed. (d) Bandlimited noisy trace: input to the POCS algorithm.

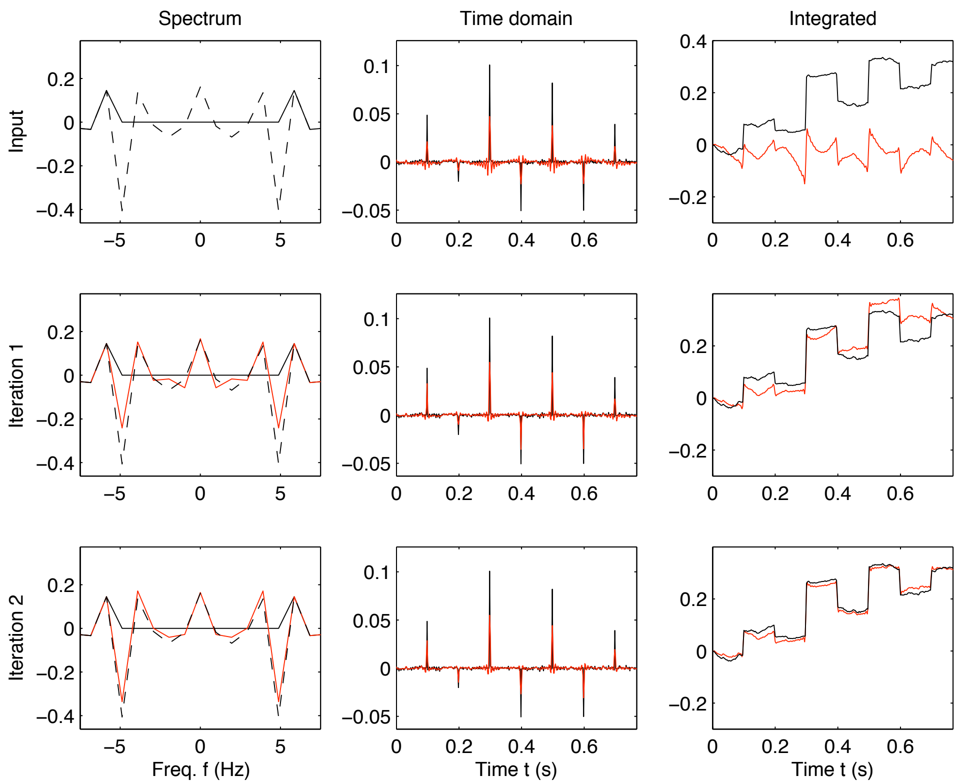

FIG. 8. Iterations of POCS with %1 noise drawn from a Gaussian distribution. Top row: left; input trace (black) vs. idealized (dashed); middle; input trace (red) vs. idealized trace (black); right; integrated traces, input (red) vs. idealized (black). Middle row, first iteration of POCS; bottom row, second iteration of POCS.

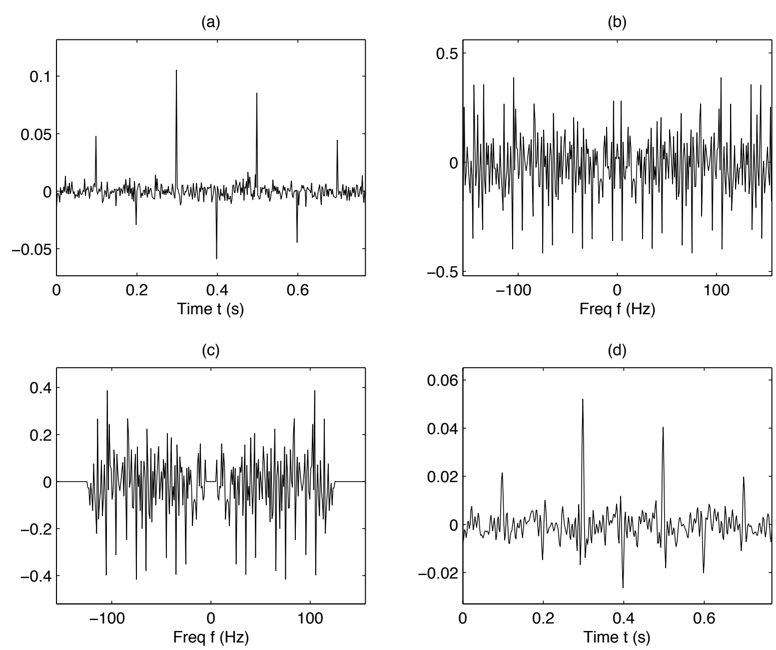

FIG. 9. Input data for synthetic test of POCS spectral extrapolation with %5 noise drawn from a Gaussian distribution. (a) Three event reflectivity at full bandwidth. (b) Spectrum of reflectivity. (c) Spectrum with lowest 5 Hz removed. (d) Bandlimited noisy trace: input to the POCS algorithm.

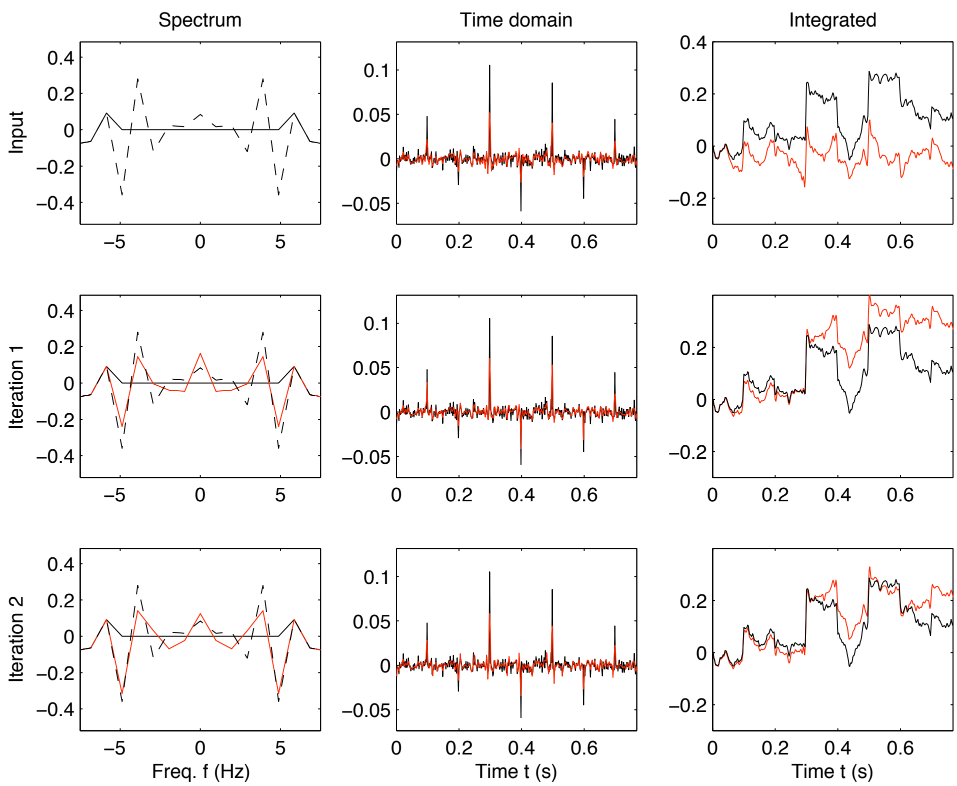

FIG. 10. Iterations of POCS with %5 noise drawn from a Gaussian distribution. Top row: left; input trace (black) vs. idealized (dashed); middle; input trace (red) vs. idealized trace (black); right; integrated traces, input (red) vs. idealized (black). Middle row, first iteration of POCS; bottom row, second iteration of POCS.

CONCLUSIONS

Projection-onto-convex-sets or POCS based algorithms have a demonstrated record of completing seismic data in a cheap, robust and effective manner. Most efforts have been in the direction of multidimensional interpolation, i.e., the problem of supplying missing traces using nearby ones. It is natural to ask whether such an algorithm might complete a different type of missing data, the low end of the frequency spectrum, under the relatively simple and plausible assumption that the data events are intrinsically "spike like".

A trace by trace implementation, tested on synthetics, appears to confirm the basic applicability of the idea. Indeed, mild stressing of the problem by (1) limiting the number of data points which provide the pattern for extrapolation, (2) increasing the complexity of these patterns with a larger number of events, and (3) adding uncorrelated noise with amplitudes of up to %5 of the signal maxima, does not appear to obstruct its use.

Clearly, systematic testing on field data with comparison to well control is the next step. If successful, POCS spectral extrapolation could be seen as a useful preprocessing step prior to various types of seismic inversion.

ACKNOWLEDGMENTS

The sponsors of CREWES and NSERC are gratefully acknowledged for their support. Mostafa Naghizadeh and Tad Ulrych are thanked for teaching me about POCS.

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